Materials Today: Proceedings xxx (xxxx) xxx

Contents lists available at ScienceDirect

Materials Today: Proceedings

journal homepage: www.elsevier.com/locate/matpr

Temperature stresses in Functionally graded (FGM) material plates using deformation theory – Analytical approach

D.M. Sangeetha [a,*], D.T. Naveenkumar [b], V. Vınaykumar [c], KE. Prakash [d]

[a] Department of Civil Engineering, A. J. Institute of Engineering & Technology, Mangalore, India
[b] Department of Civil Engineering, SJB Institute of Technology, Bangalore, India
[c] Department of Anatomy, A. J. Institute of Medical Sciences, Mangalore, India
[d] Department of Civil Engineering, ShreeDevi. Institute of Technology, Mangalore, India

ARTICLE INFO

Article history:
Received 29 May 2021
Received in revised form 12 August 2021
Accepted 16 August 2021
Available online xxxx

Keywords:
Functionally graded plates (FGM)
Analytical formulation
Power-law function
Thermal stresses
First Order Shear Deformation Theory (FSDT)

ABSTRACT

The advancement in the field of composites lead to the development of a new material called Functionally graded materials (FGMs). The concept of grading two distinct materials is introduced in FGMs to withstand high temperature variations across thin section. In this study analytical formulations and theoretical solutions were developed for the FGM plates subjected to thermal loads using refined computational model based on First Order Shear Deformation Theory (FSDT). Material properties are based on Power-law function. The effects of thermal stresses are studied for constant, linear and nonlinear variation of temperatures across the thickness of the plate, whereas in-plane is assumed to be sinusoidal. The accuracy of solutions are established with the literature. Parametric studies are performed and numerical results are presented for the simply supported FGM plates. Comparative studies are performed between various temperature profiles and bench mark results are presented for displacements, in-plane and transverse stresses.

1. Introduction

Functionally graded materials (FGMs) are the advanced materials in the field of composites, which are extensively used in aerospace, nuclear, automotive fields etc., involving high temperature fluctuations. In this material, the concept of layers in composite laminated plates is replaced by adopting continuous gradation of materials. Due to this property of FGM, the effect of debonding or delamination failures in laminates are suppressed. This idea of gradually changing the properties of the materials was first proposed by Shen and Bever [1] for composites and polymeric materials in 1972. Most of these materials were used as coating materials to resist thermal stresses. The first application was carried out at National Aerospace Laboratories of Japan in 1984, to create fuselage and nosecone parts of a space vehicle which can withstand a temperature gradient of 1600 K across a 10 mm thick section [2]. These FGMs inherit the physical and chemical properties of both the materials and found to exhibit high bond strength with excellent insulation properties, thermal resistivity, high strength and stiffness.

Most of the theories that are proposed for the analytical evaluation of FGM plates are extended from laminate plate theories. The approximations in non-symmetric plates and its effects on bending-stretching coupling terms are discussed by Mian and Spencer [3]. Exact solutions are the most accurate methods for predicting the plate responses, but due to mathematical complexities involved, it has to be reduced to two dimensional forms. Reddy and Cheng [4] and Wang et al. [5] predicted thermo-elastic responses in FGM plates using asymptotic methods. Yihunie M B et al. [6] discussed the effect of volume fraction index on thermal properties like conductivity and expansion coefficient in radial directions in a thick walled FGM cylinders. It was observed that the temperature distribution is not homogeneous. Anthony Xavier et al., [7] manufactured a synthesized FGM by varying its densities along width using additive manufacturing technology. Chung and Chang [8] investigated the elastic behaviour of rectangular FGM plates subjected to constant temperature throughout the plate, using Power law (P-FGM), Sigmoidal (S-FGM) and exponential (E-FGM) varia-

* Corresponding author.
E-mail addresses: dr7795787676@gmail.com, sangeetha@ajiet.edu.in (D.M. Sangeetha).

https://doi.org/10.1016/j.matpr.2021.08.130
2214-7853/

tions. Nosier and Fallah [9] used First Order Shear Deformation Theory (FSDT) to investigated the responses in circular FGM plates subjected to mechanical and thermal loads using power law variation of material properties. Temperature is assumed to vary according to 1-D steady state heat equation. It was observed that the material properties and temperature profiles had a significant effect on the stress. In the present study, analytical evaluation of thermal stresses in a simply supported rectangular FGM plate subjected to constant, linear and nonlinear variation of thermal loads is carried out. Comparative studies were performed on various material properties, temperature profiles and plate parameters.

2. Theoretical Formulations:

The First Order Shear Deformation Theory (FSDT) is based on classical plate theory and was first proposed by Whitney and Pagano [10] in 1970, the displacement model at any point in an FGM plate is expressed as given in Eq. (1),

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y), \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y), \\ w(x,y,z) &= w_0(x,y). \end{aligned} \tag{1}$$

The terms u, v and w are the displacements of a general point (x, y, z) in ×, y and z directions respectively. The parameters u_0, and v_0 are the in-plane displacements and w_0 is the transverse displacement of a point in the middle plane. The functions θ_x and θ_y are rotations of the normal to the middle plane about y and × axes respectively.

In-plane/transverse thermal stresses (σ, τ) and strains (ε, γ) in an FGM plate are related using generalized Hooke's law, and is given by Eq. (2),

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_z\Delta T \\ \varepsilon_y - \alpha_z\Delta T \\ \varepsilon_z - \alpha_z\Delta T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{2}$$

Where,

$$Q_{11} = Q_{22} = Q_{33} = \tfrac{E_z}{(1-\nu^2)};$$
$$Q_{12} = Q_{21} = Q_{13} = Q_{31} = Q_{23} = Q_{32} = \tfrac{\nu E_z}{(1-\nu^2)};$$
$$Q_{44} = Q_{55} = Q_{66} = \tfrac{E_z}{2(1+\nu)};$$

In the above equation, material properties are defined according to Power-Law function as given in Eq.(3).

$$E_z = E_m + (E_c - E_m)V_f^p$$

$$\alpha_z = \alpha_m + (\alpha_c - \alpha_m)V_f^p$$

$$k_z = k_m + (k_c - k_m)V_f^p$$

$$V_f = \left(\frac{z}{h} + \frac{1}{2}\right) \tag{3}$$

The volume fraction V_f and power-law parameter p defines the gradation of material from metal at bottom surface to the ceramic at the top surface of the plate. Through the thickness variation of physical properties like young's modulus of elasticity E_z, thermal coefficient of expansion α_z, thermal conductivity k_z and volume fraction V_f are evaluated using the relation given in Eq. (3). The change in temperature from stress free state ΔT as given in Eq. (4), is assumed to vary sinusoidally across the plane of the plane T(x,y) and nonlinearly across the thickness direction T_z and is given by

$$\Delta T = T(x,y,z) = T_z \times T(x,y) \tag{4}$$

The nonlinear variation of temperature across the thickness direction T_z, is obtained by solving one dimensional steady state heat conduction equation and is given by Eq. (5),

$$T_z = T_{NL} = T_0 + (T_1 - T_0)\frac{\sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{(k_c-k_m)}{k_m}\right)^n}{(np+1)}V_f^{(np+1)}}{\sum_{n=0}^{\infty}\frac{(-1)\left(\frac{(k_c-k_m)}{k_m}\right)^n}{(np+1)}} \tag{5}$$

Constant and linear variations of temperature across the plate thickness are given by Eq. (6) and Eq. (7) respectively.

$$T_z = T_C = T_0 = T_1 \tag{6}$$

$$T_z = T_L = T_0 + (T_1 - T_0)V_f \tag{7}$$

Where;

p = Power law parameter

V_f = Volume fraction.

E_m, E_c = Young's modulus of elasticity of metal and ceramic respectively.

k_m, k_c = Thermal conductivity of metal and ceramic respectively.

α_m, α_c = Thermal coefficient of expansion of metal and ceramic respectively.

T_0, T_1 = Temperature at bottom and top surface of the plate respectively.

T_{NL}, T_C, T_L = Nonlinear, constant and linear variation of temperatures across the thickness direction.

3. Equilibrium equations

The governing equations of equilibrium are obtained using Principle of Minimum Potential Energy (PMPE). The corresponding equations related to first order computational model are,

$$(1)\delta u_0 : \left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y}\right) = 0$$

$$(2)\delta v_0 : \left(\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x}\right) = 0$$

$$(3)\delta w_0 : \left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p_z^+\right) = 0$$

$$(4)\delta\theta_x : \left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x\right) = 0$$

$$(5)\delta\theta_y : \left(\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y\right) = 0 \tag{8}$$

Here (N_x, N_y, N_{xy}), (M_x, M_y, M_{xy}) and (Q_x, Q_y) respectively denotes in-plane, bending and shear stress resultants due to thermal loads, which are expressed in Eq. (9)

$$\begin{Bmatrix} Q_x \\ Q_x^* \end{Bmatrix} = [D]\begin{Bmatrix} \theta_x \\ \frac{\partial w_0}{\partial x} \end{Bmatrix} + [D']\begin{Bmatrix} \theta_y \\ \frac{\partial w_0}{\partial y} \end{Bmatrix}, \begin{Bmatrix} Q_y \\ Q_y^* \end{Bmatrix} = [E]\begin{Bmatrix} \theta_y \\ \frac{\partial w_0}{\partial y} \end{Bmatrix} + [E']\begin{Bmatrix} \theta_x \\ \frac{\partial w_0}{\partial x} \end{Bmatrix},$$

$$\begin{Bmatrix} N_x \\ N_y \\ M_x \\ M_y \end{Bmatrix} = [A]\begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \end{Bmatrix} + [A']\begin{Bmatrix} \frac{\partial u_0}{\partial y} \\ \frac{\partial v_0}{\partial x} \\ \frac{\partial \theta_x}{\partial y} \\ \frac{\partial \theta_y}{\partial x} \end{Bmatrix} - \alpha_z \Delta T \quad [C_T]$$

$$\begin{Bmatrix} N_{xy} \\ M_{xy} \end{Bmatrix} = [B']\begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \end{Bmatrix} + [B]\begin{Bmatrix} \frac{\partial u_0}{\partial y} \\ \frac{\partial v_0}{\partial x} \\ \frac{\partial \theta_x}{\partial y} \\ \frac{\partial \theta_y}{\partial x} \end{Bmatrix} \tag{9}$$

$[A]$, $[A']$, $[C_T]$, $[B]$, $[B']$, $[D]$, $[D']$, $[E]$, $[E']$ are the matrices of plate stiffness whose elements are defined in Eq. (10),

$$[A] = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{11}z & Q_{12}z \\ Q_{12} & Q_{22} & Q_{12}z & Q_{22}z \\ Q_{11}z & Q_{12}z & Q_{11}z^2 & Q_{12}z^2 \\ Q_{12}z & Q_{22}z & Q_{12}z^2 & Q_{22}z^2 \end{bmatrix} dz$$

$$[C_T] = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} + Q_{12} \\ Q_{12} + Q_{22} \\ Q_{11}z + Q_{12}z \\ Q_{12}z + Q_{22}z \end{bmatrix} \mathrm{dz}$$

$$[B] = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{44} & Q_{44} & Q_{44}z & Q_{44}z \\ Q_{44}z & Q_{44}z & Q_{44}z^2 & Q_{44}z^2 \end{bmatrix} dz$$

$$[D] = \int_{-h/2}^{h/2} [Q_{66} \quad Q_{66}] dz$$

$$[E] = \int_{-h/2}^{h/2} [Q_{55} \quad Q_{55}] dz$$

$$[A'] = [B'] = [D'] = [E'] = 0 \tag{10}$$

4. Analytical solutions

In order to solve the boundary value problem for thermal stresses in a rectangular simply supported FGM plate, Navier's solution technique using the double Fourier series is adopted. The solution for the Fourier amplitudes is obtained using Eq. (11),

$$[X]_{5\times5}\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_x \\ \theta_y \end{Bmatrix}_{5\times1} = \begin{Bmatrix} 0 \\ 0 \\ P_z^+ \\ 0 \\ 0 \end{Bmatrix}_{5\times1} + \{F_T\}_{5X1} \tag{11}$$

For any fixed values of m and n. The elements of the coefficient matrix [X] is given by Eq. (12),

$$X_{1,1} = A_{1,1}\alpha^2 + B_{1,1}\beta^2$$

$$X_{1,2} = A_{1,2}\alpha\beta + B_{1,2}\alpha\beta$$

$$X_{1,3} = 0$$

$$X_{1,4} = A_{1,3}\alpha^2 + B_{1,3}\beta^2$$

$$X_{1,5} = A_{1,4}\alpha\beta + B_{1,4}\alpha\beta$$

$$X_{2,2} = A_{2,2}\beta^2 + B_{1,2}\alpha^2$$

$$X_{2,3} = 0$$

$$X_{2,4} = A_{2,3}\alpha\beta + B_{1,3}\alpha\beta$$

$$X_{2,5} = A_{2,4}\beta^2 + B_{1,4}\alpha^2$$

$$X_{3,3} = D_{1,2}\alpha^2 + E_{1,2}\beta^2$$

$$X_{3,4} = D_{1,1}\alpha$$

$$X_{3,5} = E_{1,1}\beta$$

$$X_{4,4} = A_{3,3}\alpha^2 + B_{2,3}\beta^2 + D_{1,1}$$

$$X_{4,5} = A_{3,4}\alpha\beta + B_{2,4}\alpha\beta$$

$$X_{5,5} = A_{4,4}\beta^2 + B_{2,4}\alpha^2 + E_{1,1} \tag{12}$$

where $\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$, m,n = 1,3,5,7 odd.

The thermal force matrix $\{F_T\}$ is given by Eq. (13),

Table 1
Properties of FGMs.

Material Set	Properties	Metal	Ceramic
Monel-Zirconia (M1), [11]	E, (GPa)	227.24	125.83
	α, (/K)	15×10^{-6}	10×10^{-6}
	k, (W/mK)	25	2.09
	ν	0.3	0.3
Aluminium-Alumina (M2), [12]	E, (GPa)	70	380
	α, (/K)	23×10^{-6}	7.4×10^{-6}
	k, (W/mK)	204	10.4
	ν	0.3	0.3
Titanium Alloy – Zirconia (M3), [13]	E, (GPa)	66.2	117
	α, (/K)	10.3×10^{-6}	7.11×10^{-6}
	k, (W/mK)	18.1	2.036
	ν	0.322	0.322

Table 2
In-plane/transverse displacements in a square FGM (M1) plate.

a/h	Model	$\bar{u}_{@\ z=h/2}$	$\bar{w}_{@\ z=h/2}$
4	3D-Exact[$]	0.004021	−0.0135
	FSDT	0.004228 [5.15][*]	−0.0142 [5.83]
10	3D-Exact[$]	0.02617	−0.1689
	FSDT	0.026425 [0.97]	−0.1719 [1.79]
50	3D-Exact[$]	0.6603	−20.3200
	FSDT	0.6606 [0.05]	−20.3334 [0.07]

[*] Parentheses values are percentage differences to exact solutions.
[$] Reddy, J. N. and Cheng, Z. Q.[4].

Table 3
In-plane/transverse Stresses in a square FGM (M1) plate.

a/h	Model	$\bar{\sigma}_{x@\ z=h/2}$	$\bar{\tau}_{xz@\ z=0}$
4	3D-Exact[$]	−3.1540	−0.9500
	FSDT	−2.8652 [-9.16][*]	−0.9597 [1.03]
10	3D-Exact[$]	−18.1700	−2.3960
	FSDT	−17.9078 [-1.44]	−2.3994 [0.14]
50	3D-Exact[$]	−447.9000	−12.0000
	FSDT	−447.6940 [-0.05]	−11.9967 [-0.03]

[*] Parentheses values are percentage differences to exact solutions.
[$] Reddy, J. N. and Cheng, Z. Q.[4].

Table 4
Non-dimensionalised In-plane/Transverse displacements and stresses for various temperature profiles in an FGM plates. (M2, a/h = 4 and p = 2).

a/b		z	T_{NL}	T_L	T_C
1.5	$\bar{u}$	h/2	−1.0382	−1.3779	−1.1706
	$\bar{w}$	h/2	1.1133	0.9784	−2.8299
	$\bar{\sigma}_x$	0	−583.84	−1164.88	−2078.37
	$\bar{\sigma}_y$	0	−316.04	−722.19	−1061.66
	$\bar{\tau}_{xy}$	h/2	−1430.12	−1897.85	−1612.40
2.5	$\bar{u}$	h/2	−0.4654	−0.6176	−0.5247
	$\bar{w}$	h/2	0.4990	0.4386	−1.2686
	$\bar{\sigma}_x$	0	−702.04	−1360.28	−2527.13
	$\bar{\sigma}_y$	0	−197.83	−526.80	−612.91
	$\bar{\tau}_{xy}$	h/2	−1068.48	−1417.93	−1204.67
3.5	$\bar{u}$	h/2	−0.2547	−0.3380	−0.2871
	$\bar{w}$	h/2	0.2731	0.2400	−0.6941
	$\bar{\sigma}_x$	0	−745.53	−1432.17	−2692.23
	$\bar{\sigma}_y$	0	−154.34	−454.91	−447.80
	$\bar{\tau}_{xy}$	h/2	−818.50	−1086.19	−922.82

$$\{F_T\}^t = \{\ -\alpha N_{x_T} \quad -\beta N_{y_T} \quad 0 \quad -\alpha M_{x_T} \quad -\beta M_{y_T}\ \} \tag{13}$$

5. Numerical results and discussion

In this section, the numerical examples solved are described and discussed for establishing the accuracy of the solutions obtained using FSDT model. A shear correction factor of 5/6 is used in obtaining the results. For all problems described and discussed below, a simply supported rectangular FGM plate with SS-1 boundary conditions is considered. Closed form solutions obtained using *Navier's* solution technique for the above geometry and loading are presented. The material sets used for the study are listed in Table 1 and are used in obtaining numerical results,

All the reported results in tables and figures are non-dimensionalized using the relations given in Eq. (14)

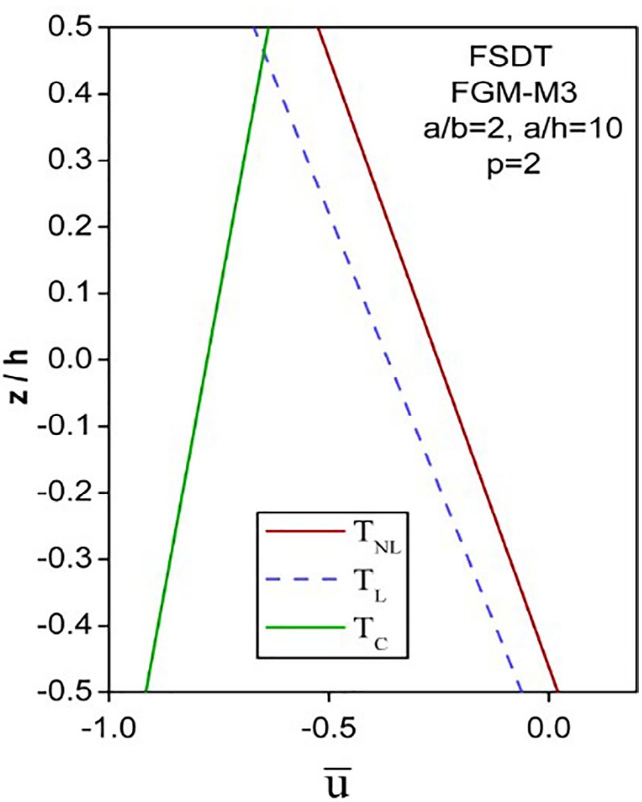

Fig. 1. Non-dimensionalised in-plane displacement ($\bar{u}$) in a rectangular Titanium Alloy- Zirconia (M3) FGM plate.

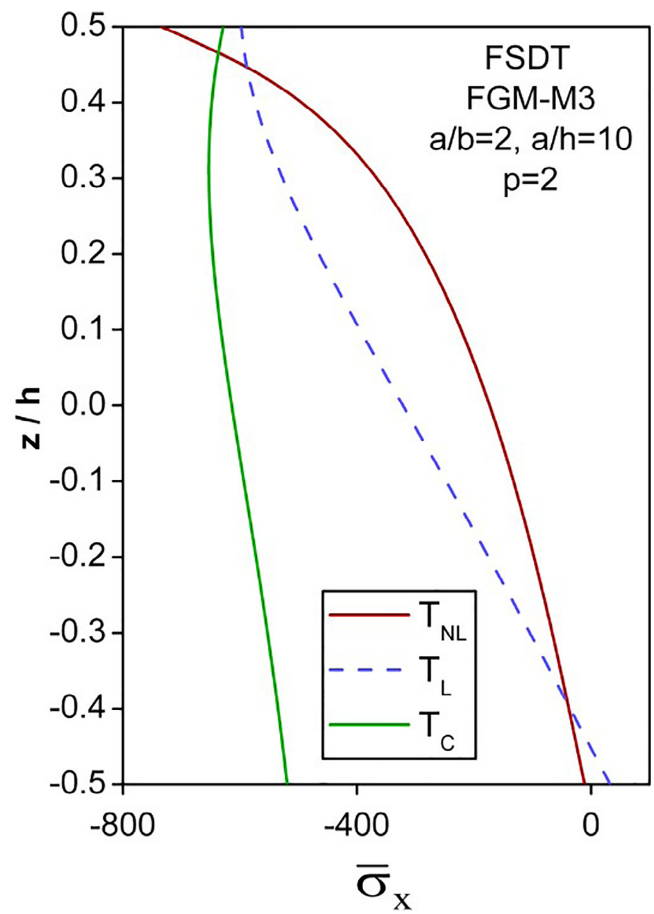

Fig. 2. Non-dimensionalised in-plane normal stress ($\bar{\sigma}_x$) in a rectangular Titanium Alloy- Zirconia (M3) FGM plate.

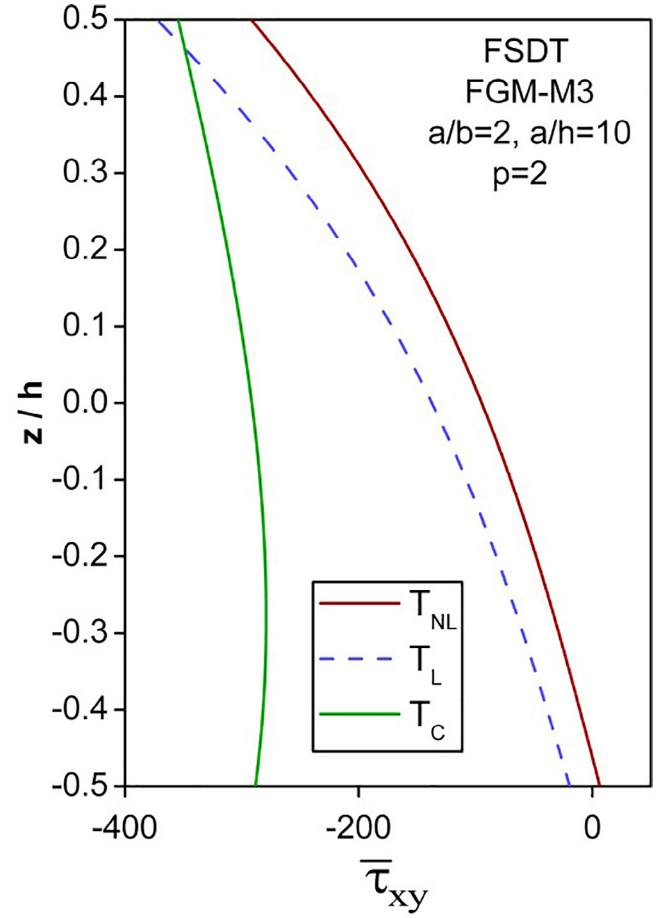

Fig. 3. Non-dimensionalised transverse normal stress ($\bar{\tau}_{xy}$) in a rectangular Titanium Alloy- Zirconia (M3) FGM plate.

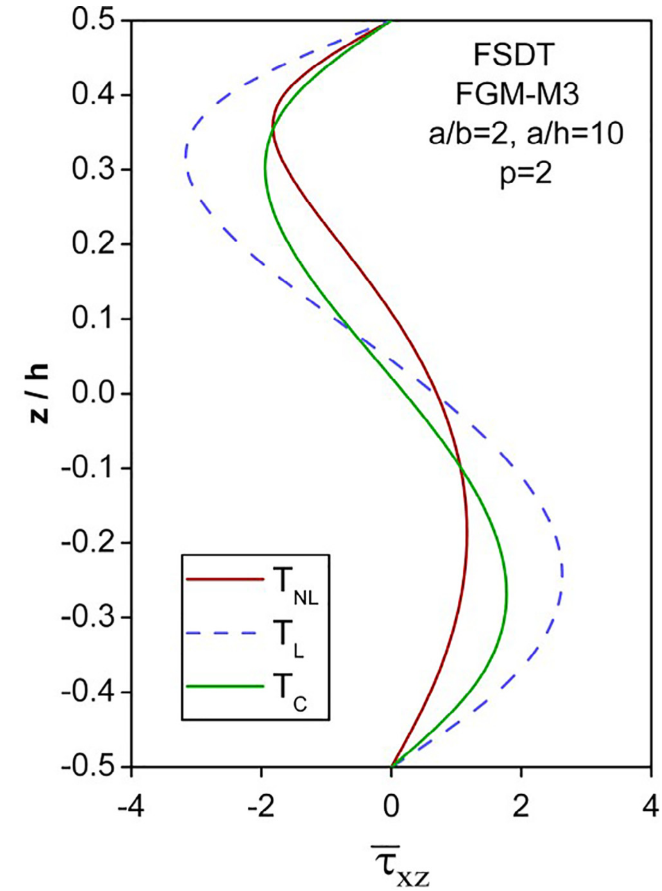

Fig. 4. Non-dimensionalised transverse shear stress ($\bar{\tau}_{xz}$) in a rectangular Titanium Alloy- Zirconia (M3) FGM plate.

$$(\bar{u}, \bar{v}, \bar{w}) = \frac{(u, v, w)E^*}{P \; a}$$
$$(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\sigma}_z, \bar{\tau}_{xy}, \bar{\tau}_{xz}, \bar{\tau}_{yz},) = \frac{(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz})}{P \; E^*} \quad (14)$$
$$\bar{T} = \frac{\alpha^* \; T_z}{P}$$

Where, $P = p_0/E^*$for applied transverse mechanical load p_0 and $P = \alpha^*/T_1$ for applied temperature T_1 at the top surface of the FGM plate. The scale factors corresponding to coefficient of thermal expansion and young's modulus of elasticity are $\alpha^* = 10^{-6}/K$ and E^* = 1GPa respectively.

Example 1.. *A simply supported square Monel-Zirconia* ($M1$) *FGM plate is considered. The plate is subjected to transverse sinusoidal mechanical load of intensity* p_0 *at the top surface of the plate. The accuracy of the solutions obtained is established by comparing the results with the* 3-*D exact solutions reported by Reddy and Cheng* [4] *for in-plane/transverse displacements and stresses in* Table 2 *and* Table 3 *respectively.*Table 4

It is observed that the percentage difference decreases with increase in a/h ratio and the maximum difference observed is found to be 9% for in-plane stress with a/h ratio 10. The results predicted by FSDT model are in good agreement with the 3-D elasticity, for the parameters considered.

Example 2:. *A simply supported rectangular FGM plate subjected to constant, linear and nonlinear variation of thermal loads across the plate thickness is considered. Material set* 2 *and Material set* 3 *is used. The non-dimensionalized in-plane/transverse displacement and stresses for varying a/b ratios are given in* Table 2.

It is observed that the assumed constant variation of temperature predicts higher values of displacements and stresses as compared to linear and nonlinear variations of temperatures. Though the linear temperature profile predicts closer results to that of nonlinear variation, accurate evaluation is possible only by adopting nonlinear temperature profile. This can be clearly seen in Figs. 1-4, where the displacements and stresses are plotted across the plate thickness for a Titanium Alloy- Zirconia (M3) FGM plate. Generally in an FGM plate the top is a ceramic surface and is exposed to temperatures, whereas bottom is a solid metal which cannot withstand temperature fluctuations. In constant temperature condition, both the surfaces will be exposed to same temperatures, which is not an appropriate approximation for analysis of FGM plates. Therefore choosing for an accurate variation of temperature profile plays a vital role during analytical evaluation of FGM plates subjected to thermal loads.

6. Conclusion

Analytical formulations and theoretical solutions are presented for thermal displacements and stresses in a simply supported FGM plate subjected to constant, linear and nonlinear variation of thermal loads using refined computational model based on First Order Shear Deformation Theory (FSDT). From the present investigation it is observed that the effect of assumed temperature profile plays a crucial role in predicting the thermal plate responses. The temperature field based on steady state heat conduction equation predicts accurate plate responses as compared to constant and linear variation. For all the parameters considered in the study, nonlinear displacements and stresses are found deviate more than that of linear and constant variations. Maximum percentage difference is observed in rectangular plate and the least in square plates. In FGM thin plates with a/h ratio 50 and above, effect of nonlinearity is found to be the least. The least differences with the temperature field is noticed in Titanium Alloy-Zirconia FGM plates. The results presented here will serve as benchmark solutions for researchers working in this area.

CRediT authorship contribution statement

D.M. Sangeetha: Conceptualization, Data curation, Investigation, Methodology, Writing - original draft. **D.T. Naveenkumar:** Formal analysis, Methodology, Software. **V. Vınaykumar:** Writing - review & editing. **KE. Prakash:** Formal analysis, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] M. Shen, M.B. Bever, Gradients in polymeric materials, J Mater Sci. 7 (1972) 741–746.

[2] M. Koizumi, FGM activities in Japan, Compos Part B Eng 28 (1-2) (1997) 1–4.

[3] M.A. Mian, A.J.M. Spencer, Exact solutions for functionally graded and laminated elastic materials, J Mech Phys Solids 46 (1998) 2283–2295.

[4] J. Reddy, Z.Q. Cheng, Three-dimensional thermomechanical deformations of functionally graded rectangular plates, Eur J Mech A/Solids 20 (2001) 841–855.

[5] Y.Z. Wang, D. Liu, Q. Wang, J.Z. Zhou, Asymptotic analysis of thermoelastic response in functionally graded thin plate subjected to a transient thermal shock, Compos Struct 139 (2016) 233–242.

[6] M.B. Yihunie, E.W. Dereje, T.C. Ewnetu, A.A. Solomon, K.S. Senthil, P. Velmurugan. Effect of volumetric fraction index on temperature distribution in thick-walled functionally graded material made cylinder. Mater. Today Proc. 2021. https://doi.org/10.1016/j.matpr.2021.01.042.

[7] M.A. Xavior, D. Nishanth, N. Navin Kumar, P. Jeyapandiarajan, Synthesis and Testing of FGM made of ABS Plastic Material, Mater. Today Proc. 22–4 (2020) 1838–1844.

[8] Y.L. Chung, H.X. Chang, Mechanical behavior of rectangular plates with functionally graded coefficient of thermal expansion subjected to thermal loading, J Therm Stress 31 (2008) 368–388.

[9] A. Nosier, F. Fallah, Reformulation of Mindlin-Reissner governing equations of functionally graded circular plates, Acta Mech 198 (2008) 209–233.

[10] J.M. Whitney, N.J. Pagano, Shear deformation in heterogeneous anisotropic plates, ASME J Appl Mech 37–4 (1970) 1031–1036.

[11] A. Alibeigloo, Exact solution for thermo-elastic response of functionally graded rectangular plates, Compos Struct 92 (1) (2010) 113–121.

[12] G.N. Praveen, J.N. Reddy, Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates, Int J Solids Struct 35 (1998) 4457–4479.

[13] M. Jabbari, E. Shahryari, H. Haghighat, M.R. Eslami, An analytical solution for steady state three dimensional thermoelasticity of functionally graded circular plates due to axisymetric loads, Eur J Mech A/Solids 47 (2014) 124–142.