SYLLABUS

Oscillations and Waves

Module -I: Oscillations and Shock waves: Oscillations: Simple Harmonic motion (SHM), differential equation for SHM (No derivation), Sprigs: Stiffness Factor and its Physical Significance, series and parallel combination of springs (Derivation), Types of spring and their applications. Theory of damped oscillations (Qualitative), Types of damping (Graphical Approach). Engineering applications of damped oscillations, Theory of forced oscillations (Qualitative), resonance, and sharpness of resonance. Shock waves: Mach number and Mach Angle, Mach Regimes, definition and characteristics of Shock waves, Construction and working of Reddy shock tube, Applications of Shock Waves, Numerical problems.

Pre requisites: *Basics of Oscilations*

Self-learning*: Simple Harmonic motion, differential equation for SHM*

Oscillations and vibrations play a more significant role in our lives than we realize. When you strike a bell, the metal vibrates, creating a sound wave. All musical instruments are based on some method to force the air around the instrument to oscillate. Oscillations from the swing of a pendulum in a clock to the vibrations of a quartz crystal are used as timing devices. When you heat a substance, some of the energy you supply goes into oscillations of the atoms. Most forms of wave motion involve the oscillatory motion of the substance through which the wave is moving. Despite the enormous variety of systems that oscillate, they have many features in common with the simple system of a mass on a spring. The harmonic oscillators have close analogy in many other fields; mechanical example a weight on a spring, oscillations of charge flowing back and forth in an electrical circuit, vibrations of a tuning fork, vibrations of electrons in an atom generating light waves, oscillation of electrons in an antenna etc.,

Periodic Motion: Periodic motion is any motion that repeats itself in equal intervals of time.
Examples:

- a swing in motion
- a vibrating tuning fork
- the Earth in its orbit around the Sun
- simple pendulum

A special type of periodic motion is **simple harmonic motion**

Simple Harmonic Motion: SHM is the type of periodic motion in which the net restoring force F acting on the body is proportional to the displacement x from the equilibrium position and is

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directed opposite to the displacement, i.e., towards the equilibrium point. The body performing SHM is known as a simple harmonic oscillator (SHO).

There are two types of SHM

1. Linear SHM

Eg: 1. The vertical oscillations of a loaded spring suspended from a rigid support.

2. Motion of needle of sewing machine

2. Angular SHM

Eg : 1. Motion of pendulum

2. Vibration of tuning fork

Characteristics of SHM:

1. Motion must be periodic.
2. The acceleration developed in the motion due to the restoring force is directly proportional to its displacement from the equilibrium position.
3. The Force or acceleration is always directed opposite to the displacement i.e., towards the mean position (F α -x or F = – kx). The displacement can be represented by a sine or cosine function such as
   $x = a\ sin\omega t$, where, a is the amplitude and ω is the angular frequency.
4. The velocity of the body is maximum at the centre and minimum at extreme position.

Concepts of Simple Harmonic Motion (S.H.M):

Displacement (x): The distance covered by the body in SHM from its mean position.

Amplitude (a): The maximum displacement of the body from its equilibrium position or mean position is its amplitude.

Period (T): The time taken by the body to complete one oscillation is its period.

$$T = \frac{2\pi}{\omega}$$

Frequency (ν): Frequency of S.H.M. is the number of oscillations that a oscillating body performs per unit time.

$$\boldsymbol{\nu} = \frac{1}{T}$$

$$\boldsymbol{\nu} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi\boldsymbol{\nu}$$

Angular frequency (ω): It is the angular displacement per unit time.

$$\omega = \frac{angle\ covered}{time\ taken} = \frac{2\pi}{T} = 2\pi\nu$$

Derivation of differential equation of SHM starting from Hooke's Law:

Consider a body of mass 'm' executing simple harmonic motion. Let the restoring force be F and the displacement of the body from its equilibrium position be *'x'*.

Then, for an oscillating body,the restoring force is directly proportional to the displacement from the mean position.

i.e., F = – kx --------(1)

which is commonly known as Hooke's Law where 'k' is the spring constant or force constant, [i.e. the restoring force/unit displacement]. The negative sign shows that restoring force acts in the direction opposite to the displacement 'x'.

The system must also obey Newton's second law of motion which states that the force is equal to mass 'm' times acceleration 'a', i.e. F = ma.

F = ma

$$F = m\frac{dv}{dt}$$

$F = m\frac{d^2x}{dt^2}$ -----------(2)

Equating (1) and (2)

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \boldsymbol{\frac{k}{m}x = 0}$$

This equation is called as differential equation of SHM.

Solution of this equation is $x = a\sin wt$. where, 'a' is the amplitude and 'ω' is the angular frequency and 't' is the time elapsed. 'ω' is also called as natural frequency of vibration and is given as $\omega = \sqrt{\frac{k}{m}}$

The Time period & Frequency of oscillation: The time period of oscillation of a spring is dependent on the spring constant of the spring and the mass of the system. It is independent of the force of gravity. It is given by

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{where } \omega^2 = \frac{k}{m}$$

And **frequency** is given by, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

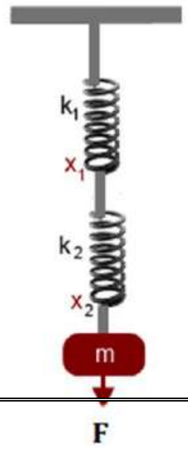
Sprigs

Stiffness Factor and its Physical Significance

Physically, force constant is a measure of stiffness. In the case of springs, it represents how much force it takes to stretch the spring over a unit length. If the spring is strong or stiff, spring constant **k** will be large, and **k** will be small for a weak spring.

Expression for spring Constant for series combination of springs:

Consider two idealized springs with force constants k_1 and k_2 connected in series as shown in the figure. Let a body of load F = mg is suspended at the free end of these two springs in series combination. When the body is pulled downwards through a little distance 'x' , the two springs suffer different extensions say 'x_1' and 'x_2'. But the restoring force is same in each spring.



Following Hook's law, for the spring 1,

$F = -k_1x_1$

$mg = -k_1x_1$

or, $x_1 = -\frac{mg}{k_1}$ -----(1)

Similarly, for he spring 2,

$F = -k_2x_2$

$mg = -k_2x_2$

or, $x_2 = -\frac{mg}{k_2}$ -----(2)

For the combination,

$F = -k_sx$

$mg = -k_sx$

or, $x = -\frac{mg}{k_s}$-----(3)

But, the total extension, $x = (x_1 + x_2)$

Or, $-\frac{mg}{k_s} = -\frac{mg}{k_1} - \frac{mg}{k_2}$

$$\frac{1}{k_s} = \frac{1}{\mathrm{k}_1} + \frac{1}{\mathrm{k}_2}$$

Or, $\boldsymbol{k_s = \frac{k_1k_2}{k_1+k_2}}$

The time period is given as $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k_s}}}$

Expression for spring Constant for parallel combination of springs:

Consider two idealized springs with force constants k_1 and k_2 connected in parallel as shown in figure. Let a body of load F = mg is suspended by these two springs in parallel combination. Let the body be pulled downwards through a small distance ‘x’. Each spring gets stretched by a length ‘x’. If F_1 and F_2 are the two restoring forces set up due to extension of springs, then

from Hook’s law, for the spring 1,

$F_1 = -k_1x$-----(1)

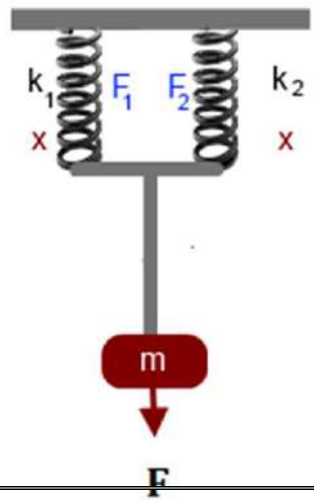
Similarly, for the spring 2,

$F_2 = -k_2x$-----(2)

∴For the combination,

$F_p = -k_px$ -----(3)

The restoring force F_p is shared by the two springs.



Therefore,

$F_p = \mathrm{F}_1 + \mathrm{F}_2$

$-k_p x = -\mathrm{k}_1\mathrm{x} - \mathrm{k}_2\mathrm{x}$

Or $\boldsymbol{k_p = \mathrm{k}_1 + \mathrm{k}_2}$

The time period is given as $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k_p}}}$

Different types of springs

- Springs are defined as an elastic body, whose function is to distort when loaded and to recover its original shapes when the load is removed.
- Springs are widely used in machines, instruments, devices and installations for various purposes

Applications

- It is used in spring balance and engine indicator to measure force.
- To absorb shocks and vibrations in automobile, railway shock absorber for suspension
- To apply a force in clutch, brake and spring loaded valve.
- To stored energy in clocks, toys, etc.

Common Types of Springs

1) Compression Spring
2) Tension Spring
3) Torsion Spring
4) Leaf Spring

Compression Spring

It is designed to operate with a compression load, so the spring gets shorter as the load is applied to it

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Tension Spring

The spring is designed to operate with a tension load, so the spring stretches as the load is applied to it.

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Torsion Spring

In which the load is an axial force, the load applied to a torsion spring is a torque or twisting force, and the end of the spring rotates through an angle as the load is applied.

Leaf spring

The leaf spring consists of a number of flat plates of varying lengths held ogether by means of clamps and bolts. These are mostly used in truck suspension system. The material used for leaf springs is usually plain carbon steel

Different cases of SHM: Under SHM, we have the following 3 cases

1. Free oscillations
2. Damped oscillations
3. Forced oscillations

Free Oscillation: ‘If an oscillating body oscillates with constant amplitude at its own natural frequency without the help of any external force is called free oscillation.

Natural Frequency: The natural frequency is the rate at which an object vibrates when it is not disturbed by an outside force.

Examples for free oscillations:

1. The vertical oscillations of a loaded spring suspended from a rigid support.

2. Motion of needle of sewing machine

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3. Motion of pendulum
4. Vibration of tuning fork

Equation of motion for free oscillations:

The equation of motion of a free oscillation is given by

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Where, *m* is the mass of the oscillating body, *k* is the force constant, *x* is the displacement at the instant *t* of an oscillating body.

Damped Oscillation: The oscillations of a body whose amplitude goes on decreases with time due to the presence of resistive forces is called damped oscillation.

Examples:

1. Mechanical oscillations of a simple pendulum
2. A swing left free to oscillate after being pushed once.

Theory of damped oscillations:

Consider a body of mass 'm' executing oscillations in a resistive medium. The oscillations are damped due to resistance offered by the medium.

The damping force, acts in a direction opposite to the movement of the body and velocity dependent

i.e.,

$F_{dam} \; \alpha \; -v$

$F_{dam} = -bv$, where 'b' is damping constant

$$F_{net} = F_{res} + F_{dam}$$

$$F_{net} = -kx - bv$$

$$F_{net} = -kx - b\frac{dx}{dt}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2} + \left(\frac{b}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = 0 \text{ -------(1)}$$

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Where, $2\beta = \frac{b}{m}$ $\beta = \frac{b}{2m}$, $\omega^2 = \frac{k}{m}$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = 0 \quad \text{----------(2)}$$

The solution of the above equation is

$x = Ae^{\alpha t}$ --------(3) where, 'A' and 'α' are constants

$$\frac{dx}{dt} = A.\alpha.e^{\alpha t} = \alpha.x$$

$$\frac{d^2x}{dt^2} = A.\alpha.\alpha e^{\alpha t} = \alpha^2.x$$

$$\alpha^2 x + 2\beta.\alpha x + \omega^2 x = 0$$

$$x(\alpha^2 + 2\beta.\alpha + \omega^2) = 0$$

$$\Rightarrow (\alpha^2 + 2\beta.\alpha + \omega^2) = 0$$

The solution of this equation can be given using

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega^2}}{2}$$

$$\alpha = \frac{-2\beta \pm 2\sqrt{\beta^2 - \omega^2}}{2}$$

$$\alpha = -\beta \pm \sqrt{\beta^2 - \omega^2} \quad \text{-------(4)}$$

(4) in (3)

$$x = Ae^{\left(-\beta \pm \sqrt{\beta^2 - \omega^2}\right)t}$$

Or, $x = A_1 e^{\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)t} + A_2 e^{\left(-\beta - \sqrt{\beta^2 - \omega^2}\right)t}$ -------(5)

Where, A_1 and $A_2$are constants to be evaluated

Now, at t=0, $x = x_0$ (i.e., maximum displacement)

∴equation (5) becomes $x_0 = A_1 + A_2$ -------(6)

Also, at t=0, $x = x_0$, the velocity is zero. i.e., $\frac{dx}{dt} = 0$

$$\therefore \frac{dx}{dt} = \left(-\beta + \sqrt{\beta^2 - \omega^2}\right)A_1 e^{\left(-\beta+\sqrt{\beta^2-\omega^2}\right)t} + \left(-\beta - \sqrt{\beta^2 - \omega^2}\right)A_2 e^{\left(-\beta-\sqrt{\beta^2-\omega^2}\right)t} = 0$$

Since t=0,

$$\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)A_1 + \left(-\beta - \sqrt{\beta^2 - \omega^2}\right)A_2 = 0$$

$$-\beta(A_1 + A_2) + \sqrt{\beta^2 - \omega^2}\,(A_1 - A_2) = 0$$

$$-\beta x_0 + \sqrt{\beta^2 - \omega^2}\,(A_1 - A_2) = 0$$

$$\frac{\beta x_0}{\sqrt{\beta^2 - \omega^2}} = A_1 - A_2 \quad \text{-------(7)}$$

Adding (6) and (7)

$$2A_1 = x_0 + \frac{\beta x_0}{\sqrt{\beta^2 - \omega^2}}$$

$$2A_1 = x_0 \left(\frac{1+\beta}{\sqrt{\beta^2 - \omega^2}}\right)$$

$$A_1 = \frac{x_0}{2}\left(1 + \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right) \quad \text{----------(8)}$$

Subtracting, (6) - (7)

$$2A_2 = x_0 - \frac{\beta x_0}{\sqrt{\beta^2 - \omega^2}}$$

$$2A_2 = x_0 \left(1 - \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right)$$

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$$A_2 = \frac{x_0}{2}\left(1 - \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right) \quad \text{-------(9)}$$

∴ Equation (5) is given as

$$x = \frac{x_0}{2}\left(1 + \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right).e^{\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)t} + \frac{x_0}{2}\left(1 - \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right).e^{\left(-\beta - \sqrt{\beta^2 - \omega^2}\right)t} \quad \text{-------(10)}$$

This is the expression for decay amplitude.

Depending upon the strength of damping force the quantity $(\beta^2 - \omega^2)$ can be positive /negative /zero giving rise to three different cases.

We have the general equation of damped oscillation as (eqn (5))

$$x = A_1 e^{\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)t} + A_2 e^{\left(-\beta - \sqrt{\beta^2 - \omega^2}\right)t}$$

Or, $x = e^{-\beta t}(A_1 e^{\left(\sqrt{\beta^2 - \omega^2}\right)t} + A_2 e^{\left(-\sqrt{\beta^2 - \omega^2}\right)t}$ ---------**

Case 1: If $\beta^2 > \omega^2$, over damping

When $\beta^2 > \omega^2$, $\sqrt{\beta^2 - \omega^2}$ is positive.

Let $\alpha = \sqrt{\beta^2 - \omega^2}$

Then the eqn ** becomes

$$x = e^{-\beta t}(A_1 e^{\alpha t} + A_2 e^{-\alpha t})$$

That means, there is an exponential decay of the displacement w.r.t time.

i.e., the body, without oscillating, returns from its maximum displacement to the equilibrium position very slowly and rests there. This is referred as over damped motion.

Eg: The motion of simple pendulum in a highly viscous medium.

Case 2 : If $\beta^2 = \omega^2$, critical damping

When $\beta^2 = \omega^2$,

Then the eqn ** will be

$$x = e^{-\beta t}(A_1 + A_2)$$

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i.e., the body doesn't oscillate and returns to its equilibrium position very rapidly. This is referred as critical damping.

Critical damping provides the quickest approach to zero amplitude compared to other two cases.

Eg: The spring of the automobiles, the speedometers of the vehicles.

Case 3: If $\beta^2 < \omega^2$, under damping

When $\beta^2 < \omega^2$, $\sqrt{\beta^2 - \omega^2}$ is negative.

If $\sqrt{\beta^2 - \omega^2} = \omega_1$

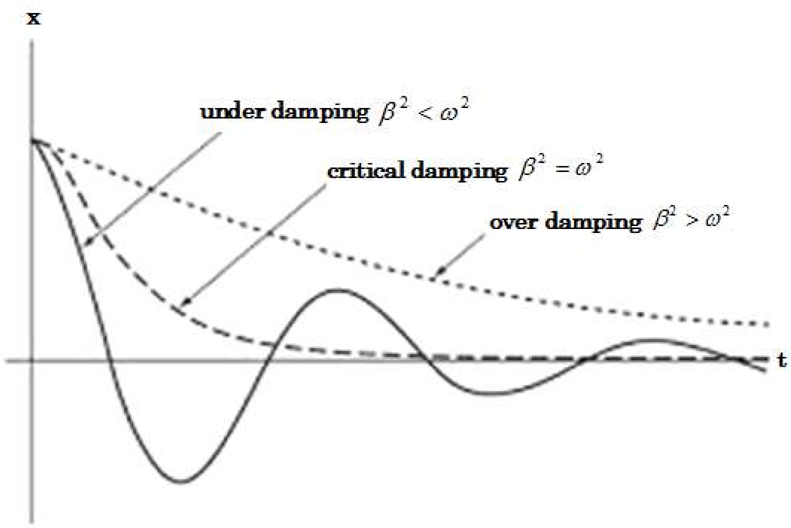
Then, $\sqrt{\omega^2 - \beta^2} = i\omega_1$

∴the eqn ** will be

$$x = e^{-\beta t}(A_1 e^{i\omega_1 t} + A_2 e^{-i\omega_1 t})$$

i.e., the amplitude of the oscillating system will not be constant and decreases exponentially with time, till the oscillation dies out. This is referred as under damping

Eg: The motion of simple pendulum in air.



Quality Factor

It is defined as the ratio of the energy of the oscillator to the energy lost radian.

It is given by

$$Q = 2\pi \frac{Energy\ stored\ in\ the\ oscillator}{Energy\ Loss\ per\ period}$$

$$Q = \frac{2\pi E}{pT}$$

Physical Significance:

Q is a measure of the extent to which oscillator is free from damping. High value of Q means the damping of oscillating system is low. For an undamped oscillator r = 0, so that Q is infinite. It specifies the degree damping.

Forced Oscillation: The oscillation in which a body oscillates under the influence of an external periodic force (also called as driving force) is known as forced oscillation. Here, the amplitude of oscillation, experiences damping but remains constant due to the external energy supplied to the system.

Eg: Oscillations of a swing which is pushed periodically by a person.

Theory of Forced Oscillations: Consider a body of mass 'm' executing oscillations in a damping medium, acted upon by an external periodic sinusoidal force '$F sin\omega_o t$'. Then its equation of motion is

$$F_{net} = F_{res} + F_{dam} + F_{forced}$$

$$F_{net} = -kx - bv + F\sin\omega_o t$$

$$F_{net} = -kx - b\frac{dx}{dt} + F\sin\omega_o t$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F\sin\omega_o t$$

$$\frac{d^2x}{dt^2} + \left(\frac{b}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = \frac{F}{m}\sin\omega_o t$$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = \frac{F}{m}\sin\omega_o t \text{ ------(1)}$$

The solution of this equation is

$$x = a\sin(\omega_o t - \alpha) \text{ --------(2)}$$

Where, 'a' is the amplitude and $'\alpha'$ is the phase of the oscillating body to be determined.

$$\frac{dx}{dt} = a\omega_o \cos(\omega_o t - \alpha)$$

$$\frac{d^2x}{dt^2} = -a\omega_o\omega_o \sin(\omega_o t - \alpha)$$

$$\frac{d^2x}{dt^2} = -a\omega_o{}^2 \sin(\omega_o t - \alpha)$$

$$\therefore (1) \Rightarrow -a\omega_o{}^2 \sin(\omega_o t - \alpha) + 2\beta a\omega_o \cos(\omega_o t - \alpha) + \omega^2 a \sin(\omega_o t - \alpha) = \frac{F}{m}\sin\omega_o t$$

$\frac{F}{m}\sin(\omega_o t)$ can be written as $\frac{F}{m}\sin[(\omega_o t - \alpha) + \alpha] = \frac{F}{m}\left(\sin(\omega_o t - \alpha).\cos\alpha + \cos(\omega_o t - \alpha).\sin\alpha\right)$

$\therefore (1) \Rightarrow$

$$-a\omega_o{}^2 \sin(\omega_o t - \alpha) + 2\beta a\omega_o \cos(\omega_o t - \alpha) + a\omega^2 \sin(\omega_o t - \alpha) = \frac{F}{m}\sin(\omega_o t - \alpha).\cos\alpha + \frac{F}{m}\cos(\omega_o t - \alpha).\sin\alpha$$

By equating the coefficients of $\sin(\omega_o t - \alpha)$ from both the sides, we get

$$-a\omega_o{}^2 + a\omega^2 = \frac{F}{m}.\cos\alpha$$

$$a(\omega^2 - \omega_o^2) = \frac{F}{m}.\cos\alpha \text{ --------(2)}$$

Similarly, by equating the coefficients of $cos(\omega_o t - \alpha)$ from both the sides, we get

$$2\beta a\omega_o = \frac{F}{m}.\sin\alpha \text{ --------(3)}$$

Squaring and adding eqn (2) and (3) we get,

$$(a(\omega^2 - \omega_0^2))^2 + (2\beta a\omega_0)^2 = \left(\frac{F}{m}\right)^2 (\cos^2\alpha + \sin^2\alpha)$$

$$a^2\left[\left(\omega^2 - \omega_0^2\right)^2 + 4\beta^2\omega_0^2\right] = \left(\frac{F}{m}\right)^2$$

$$a^2 = \frac{\left(\frac{F}{m}\right)^2}{\left(\omega^2 - \omega_0^2\right)^2 + 4\beta^2\omega_0^2}$$

$$\boxed{a = \frac{\left(\frac{F}{m}\right)}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\beta^2\omega_0^2}}}$$

This equation represents the amplitude of the forced vibrations.

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Phase of the forced vibrations:

Dividing eqn (3) by eqn (2)

$$\tan\alpha = \frac{2\beta a\omega_0}{a(\omega^2-\omega_0^2)} = \frac{2\beta\omega_0}{(\omega^2-\omega_0^2)}$$

∴ the Phase of the forced vibration is given by

$$\alpha = \tan^{-1}\left(\frac{2\beta\omega_0}{(\omega^2-\omega_0^2)}\right)$$

Dependence of amplitude and phase on the frequency of the applied force:

Case 1: $\omega_o \ll \omega$, when the frequency of the force is low.

For $\omega_o \ll \omega$, ω_o^2 will be very small and damping is small ($\beta \to 0$)

i.e., $\omega^2 - \omega_o^2 \approx \omega^2$ and $2\beta\omega_o \approx 0$

∴The amplitude,

$$a = \frac{\left(\frac{F}{m}\right)}{\omega^2} = \frac{F}{m\omega^2}$$

The Phase α is given as

$$\alpha = \tan^{-1}\left(\frac{2\beta\omega_0}{\omega^2-\omega_0^2}\right)$$

since$\omega_o$is very small and damping (β)is also small

$$\omega^2 - \omega_o^2 \approx \omega^2 \; and \; \frac{2\beta\omega_o}{\omega^2} \approx 0$$

$$\therefore \alpha = \tan^{-1}(0)$$

$$\alpha = 0$$

This shows that the amplitude of vibration is independent of the frequency of force. The amplitude depends on the magnitude of the applied force. The displacement and force are always in phase.

Case 2: $\omega_o = \omega$, the frequency of force is equal to frequency of the body.

For $\omega_0 = \omega, \left(\omega^2 - \omega_0^2\right) = 0$

$\therefore$ amplitude $a = \dfrac{F/m}{2\beta\omega_0} = \dfrac{F/m}{2\left(\dfrac{b}{2m}\right)\omega_0} = \dfrac{F}{b\omega_0}$

$$a = \frac{F}{b\omega_0}$$

The phase, $\alpha = \tan^{-1}\left(\dfrac{2\beta\omega_0}{0}\right)$

$$\alpha = \tan^{-1}(\infty) = \frac{\pi}{2}$$

This shows that the amplitude of vibration is governed by the damping and for small damping forces the amplitude of vibration is quite large. The displacement lags behind the force by $(\frac{\pi}{2})$.

Case 3: $\omega_o \gg \omega$,the frequency of force is greater than frequency of the body.

For $\omega_o \gg \omega$, and damping is small ($\beta \to 0$)

$$(\omega^2 - \omega_0^2)^2 \approx (\omega_o^2)^2$$

$$\therefore a = \frac{F/m}{\sqrt{4\beta^2\omega_0^2 + (\omega_0^4)}}$$

Since β is very small, $4\beta^2\omega_o^2 \langle\langle \omega_o^4$

$$a = \frac{F/m}{\sqrt{\omega_0^4}} = \frac{F/m}{\omega_0^2} = \frac{F}{m\omega_0^2}$$

The phase, $\alpha = \tan^{-1}\left[\dfrac{2\beta\omega_0}{\omega^2 - \omega_0^2}\right] = \tan^{-1}\left[\dfrac{-2\beta}{\omega_0}\right]$

Since β is very small $\dfrac{2\beta}{\omega_o} \approx 0$

$$\alpha = \tan^{-1}(-0) = \pi$$

In this case, the amplitude goes on decreasing and phase difference tends towards π.

Resonance: The amplitude of the forced vibrations is given as

$$a = \frac{\left(\dfrac{F}{m}\right)}{\sqrt{(\omega^2 - \omega_0^2)^2 + 4\beta^2\omega_0^2}} \quad \text{---------(1)}$$

Conditions for resonance: For 'a' to be maximum, the denominator in the above equation must be minimum. It is possible when,

i) $\beta = \frac{b}{2m}$ is minimum, i.e., when the damping caused by the medium is made minimum.

ii) $\omega_o = \omega$, i.e., when frequency of the applied force (ω_o) becomes equal to the natural frequency of vibration of the body (ω).

Therefore eqn (1) reduces to

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4\beta^2\omega^2}} = \frac{\left(\frac{F}{m}\right)}{2\beta\omega} \quad \text{--- (2)}$$

This is the expression for maximum amplitude (resonance).

"When the frequency of periodic force acting on a vibrating body is equal to the natural frequency of vibrations of the body, the body vibrates with maximum amplitude. This phenomenon is called resonance".

At resonance, the energy transfer from the periodic force to the vibrating body is maximum.

Eg: 1. Helmholtz resonator,

2. the vibrations caused by an excited tuning fork in another nearby identical tuning fork.

Sharpness of resonance: -

The amplitude is maximum at resonance frequency which decreases rapidly as the frequency increases or decreases from the resonant frequency.

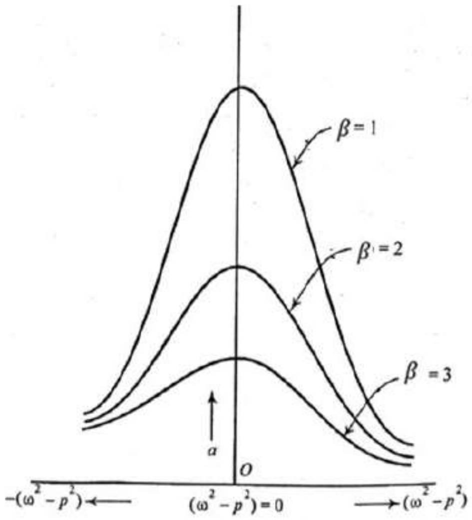
The rate at which the amplitude decreases with the frequency of the applied external force on either side of resonant frequency is termed as "sharpness of resonance".

$$\boldsymbol{Sharpness\ of\ resonance = \frac{change\ in\ amplitude}{change\ in\ frequency}}$$

Note: The sharpness of a resonance is measured by its Q-factor.

Significance of sharpness of resonance: The rate at which the change in amplitude occurs near resonance depends on damping. For small damping, the rate is high and the resonance is said to be sharp. For heavy damping it will be low, and resonance is said to be flat.

Effect of damping: The response of amplitude to various degrees of damping is as shown in the graph. From the graph, it can be observed that, for larger values of damping coefficient 'β', the curve is flat and hence the resonance is flat. On the other hand, for smaller values of damping coefficient β the curve is sharp and it refers to sharp resonance.



SHARPNESS OF RESONANCFE

The sharpness of resonant peak depends on the damping. If the damping is small, the resonant peak is sharp, if the damping is large, it is less sharp.

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Shock waves

In Aerodynamics, the speed of the bodies moving in a fluid medium can be classified into different categories on the basis of following terms

Mach number: it is defined as ratio of the speed of the object to the speed of the sound in a given medium. i.e.,

$$\boldsymbol{Mach\ number = \frac{object\ speed}{speed\ of\ the\ sound\ in\ a\ given\ medium}}$$

$$\boldsymbol{M = \frac{v}{a}}$$

It is denoted as M. If v is the speed of object and 'a' is the speed of sound in the medium , then $\boldsymbol{M = \frac{v}{a}}$

$$a = \sqrt{\gamma RT}$$

γ – ratio of specific heats

R – specific gas constant

T – local temperature in Kelvin

Mach number gives a measure of how fast a body is moving with respect to the speed of sound. It is a very important quantity in compressible flow theory (where density of the fluid changes).

Acoustic, ultrasonic (based on frequency), Subsonic, Supersonic waves

1) **Acoustic wave**

Acoustic wave is simply a sound wave which moves with the speed of 330m/s, in air at STP. they have frequencies between 20 -20,000 Hz. Amplitude of acoustic wave is very small.

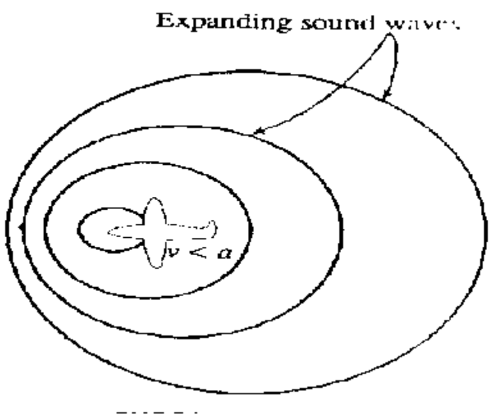
2) Ultrasonic waves

These are pressure waves having frequencies beyond 20,000 Hz. but they travel with the same speed as that of sound. Amplitude of ultrasonic wave is very small.

Mach number regimes

1) **Subsonic waves**

If the speed of mechanical wave or body moving in a fluid is lesser than that of sound. Then, such a speed is referred to as subsonic and the wave is subsonic wave. All the subsonic waves have Mach number less than 1. For a body moving with subsonic speed sound emitted by it manages move ahead and away from the body since it is faster than the body.



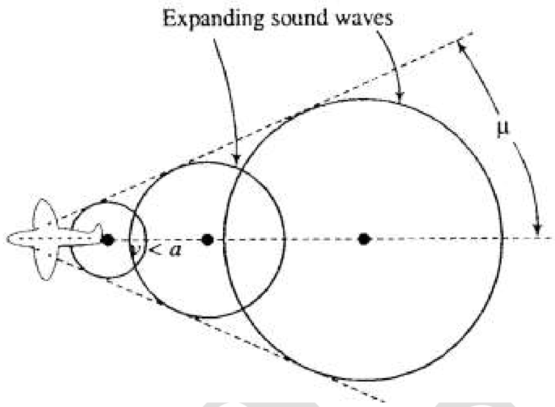
Eg : speed of car and train

2)Super sonic

Supersonic waves are mechanical waves which travel with speeds greater than that of sound i.e., with speeds for which Mach no. >1.

A body with supersonic speed moves ahead leaving behind series of expanding sound waves. Amplitude of supersonic wave will be very high and it effects medium in which it is travelling.

Eg: fighter planes.



3) **Transonic :** As the speed of the object approaches the speed of sound, the flight Mach number is nearly equal to one, **M = 1**, and the flow is said to be transonic. At some places on the object, the local speed exceeds the speed of sound.

4) **Hypersonic:** For speeds greater than five times the speed of sound, **M > 5**, the flow is said to be hypersonic. At these speeds, some of the energy of the object now goes into exciting the chemical bonds which hold together the nitrogen and oxygen molecules of the air.

Mach cone

A number of common tangents drawn to the expanding sound waves emitted from a body at supersonic speed constitute a cone called Mach cone.

Mach angle

The angle made by the tangent with the axis of the Mach cone is called the Mach angle (μ)

$\mu = \sin^{-1}(\frac{1}{M})$where M is the Mach number.

Shock waves

Any fluid that propagates at supersonic speeds gives rise to shock wave.

- Shock waves are produced by a sudden dissipation of mechanical energy in a medium enclosed in a small space. The Shock waves depends on pressure, temperature and density of the medium through which it propagates.

- Ex: Shock waves are produced in nature during earth quakes called Seismic waves (2km/s – 8km/s)

Properties of shock waves:

1) They travel in the medium with Mach number exceeding 1.
2) Shock waves obey the laws of fluid mechanics.
3) The effect caused by the shock waves result in increase of entropy.
4) Across the shock wave supersonic flow is decelerated into subsonic flow.
5) Shock wave exists in a very thin space of thickness not exceeding one micro meter.

There are two kinds of shock waves

1) Strong Shock waves: It is a compressed region possessing very high pressure and temperature having Mach number greater than 1

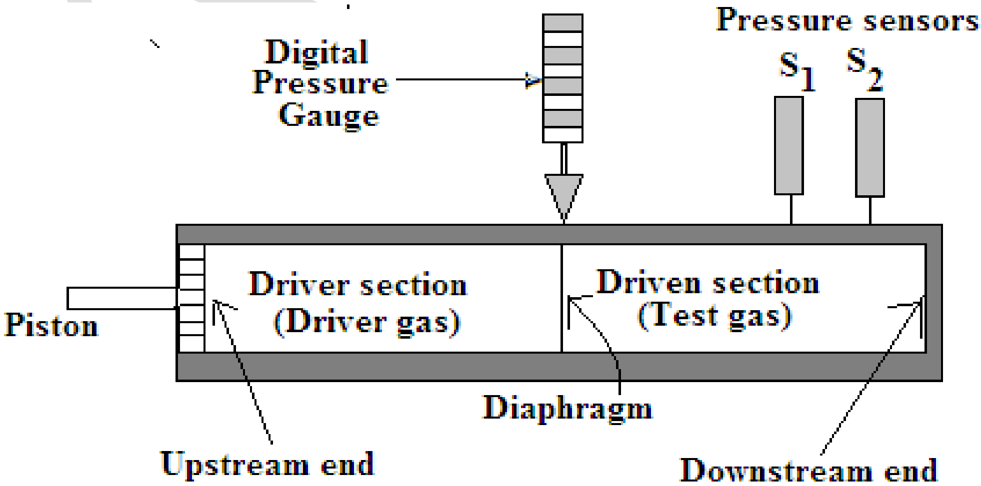
Ex: Lighting thunder or bombing, etc.,

2) Weak Shock waves: It is a compressed region possess Low pressure and temperature having Mach number less than or closer to 1

Ex: Explosion of a Cracker, Bursting of aTyre, etc.,

Reddy tube or Reddy shock tube

Reddy tube is hand operated shock tube capable of producing shock waves by using human energy. It is a long cylindrical tube with two sections separated by a diaphragm. Its one end is fitted with the piston and the other end is closed or open to the surroundings.



REDDY TUBE

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Description

- Reddy tube consists of a cylindrical stainless steel tube of about 30mm diameter. And of length nearly 1 m.
- It is divided into 2 sections each of length about 50 cm. one is the **driver tube** other is **driven tube** separated by a 0.1mm thick aluminum or paper diaphragm.
- It has a piston fitted at the far end of the driver section. Whereas the end of the driven section is closed.
- A digital pressure gauge is mounted in the driver section next to the diaphragm.
- Two piezoelectric sensors S_1 and S_2 are mounted on the driven section.
- The driver section is filled with gas termed as driver gas (high pressure) and the gas in the driven section is termed **driven gas.**

Working:

- The driver gas is compressed by pushing the piston hard into the driver tube until the diaphragm ruptures.
- Following the rupture, the driver gas rushes into the driven section and pushes the driven gas towards the downstream end which generates a moving shock wave that traverses the length of the driven section.
- The shock wave instantaneously raises the temperature and the pressure of the driven gas.

Characteristics of a Reddy's tube.

- The Reddy's tube operates on the principle of free piston driven shock tube(FPST)
- It is a hand operated shock producing device
- It is capable of producing Mach no exceeding 1.5
- The rupture pressure is a function of the thickness of the diaphragm
- Temperatures exceeding 900 K can ben be easily obtained by the reddy tube. By using He as the driver gas and Ar as the driven gas.

Applications of shock waves

1. Cell information

By passing shock wave of suitable strength, DNA can be pushed inside a cell without affecting the functionality of DNA. This has wide biological applications.

2. Wood preservation

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By using shock waves, chemical preservatives in the form of solutions could be pushed into the interior of wood samples which helps in withstanding the microbial attacks. By this method the life of ordinary wood can be increased.

3. Use in Pencil Industry:

In the manufacture of pencils, in the industry, the wood needs to be softened by soaking it in a polymer at 700C for about 3 hours and then dried. It takes days for the wood to dry.
In the modern process, the liquid is passed into the wood almost instantaneously by placing it in a liquid and sending a shock wave. The wood is then taken out and it will not take longer time to dry. The treated wood is ready for the next process without any delay.

4. Kidney stone treatment:

Shock wave is used in a therapy to crush the kidney stones into smaller pieces after which, they are passed out of the body smoothly through the urinary tracts.

5. Gas dynamics studies:

The extreme conditions of pressure and temperature that can be produced in the shock tube, helps us to the study of high temperature gas dynamics. This knowledge is useful in the study of supersonic motion of bodies & hypersonic re-entry of space vehicles into the atmosphere.

6. Shock wave assisted needleless drug delivery:

By using shock waves, drugs can be injected into the body without using needles. The drug is filled into a cartridge which is kept pressed on the skin & the shock wave is sent into the body using high pressure. The drug enters the body directly through pores of the skin. In this process, the patients do not experience any pain.

7. Treatment of dry bore wells:

Water will be available in the bore wells when water accumulates in the bore well through a number of seepage points which are porous. Sometimes, such seepage points are blocked by sand particles. A shockwave sent through such a dry bore well, clears the blockages and makes the bore well into a water source.

Numerical problem

1) The distance between the two pressure sensors in a shock tube is 100mm. The time taken by a shock wave to travel this distance is 200 microseconds. If the velocity of sound under the same conditions is $340ms^{-1}$, find the Mach number of the shock wave. (4 marks)

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QUESTIONS:

1. Define simple harmonic motion, Mention the characteristics of SHM, Derive the equation for simple harmonic motion using Hooke's law.
2. Obtain the expression for period of oscillations of two springs in series and parallel combination.
3. What are damped oscillations? Give the theory of damped oscillations and hence discuss the case of under damping.
4. Give the theory of forced oscillations.
5. Define shock waves. Explain the experimental method of producing shock waves and measuring its Mach number using Reddy Shock tube.
6. Define Mach Number. Distinguish between subsonic, supersonic and hypersonic waves.

NUMERICALS

1) A mass of $25\text{x}10^{-2}$ kg is suspended from the lower end of vertical spring having a force constant 25N/m. What should be the damping constant of the system so that motion is critically damped?

2) A spring undergoes an extension of 5 cm for a load of 50 g. Find its force constant, angular frequency and frequency of oscillation, if it is set for vertical oscillations with a load of 200 g attached to its bottom. Ignore the mass of the spring.

3) A free particle is executing SHM in a straight line with a period of 25 seconds, after 5 seconds it has crossed the equilibrium point, the velocity is found to be 0.7m/s. find the displacement at the end of 10 seconds, and also the amplitude of oscillations.

4) The distance between the two pressure sensors in a shock tube is 100mm. The time taken by a shock wave to travel this distance is 200 micro second. If the velocity of sound under the same conditions is 340ms^{-1}, find the Mach number of the shock wave.

5) A mass of 0.5kg causes an extension of 0.003m in a spring and the system is set for oscillations. Find i) The force constant for the spring ii) Angular frequency and iii) Time period of the resulting oscillation.

6) Calculate the peak amplitude of vibration of system whose natural frequency is 1000Hz when it oscillates in a resistive medium for which the value of damping/unit mass is 0.008rad/s under the action of an external periodic force /unit mass of amplitude 5N/kg, with tuneable frequency

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MODULE 2 - ELASTIC PROPERTIES OF THE MATERIALS

Elasticity Stress-Strain Curve, Stress hardening and softening. Elastic Moduli, Poisson's ratio and its limiting values. relation between Y , n and σ (with derivation), Beams, bending moment and derivation of expression, Cantilever and I section girder and their Engineering Applications, Elastic materials (qualitative). Failures of engineering materials - ductile fracture, brittle fracture, stress concentration, fatigue and factors affecting fatigue (only qualitative explanation), Numerical problems Pre requisites: Elastcity,Stress & Strain Self-learning: Stress-Strain Curve

The study of strength of materials is to provide the means of analysing and designing various machines and load bearing structures.

Elasticity: The property of material body to regain its original shape and size on removal of the deforming forces is called elasticity. Within certain elastic limit steel and quarts show elastic properties. The elastic property is desirable for materials used in tools and machines.

Plasticity: Bodies which does not show any tendency to recover their original condition are said to be Plastic and the property is called plasticity. Eg – polyethylene, Polystyrene etc
This property of the material is necessary for forging, stamping images on coins and ornamental works.

Load: The term load implies the combination of external forces acting on a body and its effect is to change the form or the dimensions of the body. It is essentially a deforming force.

Stress: The restoring force per unit area developed inside the body is called stress.
The magnitude of the restoring force is exactly equal to that of the applied force, **stress is given by the ratio of the applied force to the area of its application**

Stress = $\frac{Restoring\ Force}{cross\ sectiponal\ area} = \frac{F}{A}$, S I unit of stress is N/m^2.

<u>Concept of strain</u>: When a body is subjected to external force, there will be change in dimensions of the body. The change in dimension is called deformation. **The ratio of change in dimension of body or deformation to the original dimension of the body is called known as strain.** Strain $= \frac{change\ in\ dimension\ of\ the\ body}{original\ dimension} = \frac{\delta L}{L}$

Hooke's law Hooke's law states that when a material is loaded within its elastic limit, stress is directly proportional strain. It means that the ratio of stress to strain is constant within the elastic limit. This constant is known as Modulus of elasticity .

$\frac{stress}{strain}$ = constant or modulus of elasticity

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STRESS-STRAIN CURVE

Typical stress strain curve for a metal is shown in Fig. 1.3. This graph is plotted between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced.

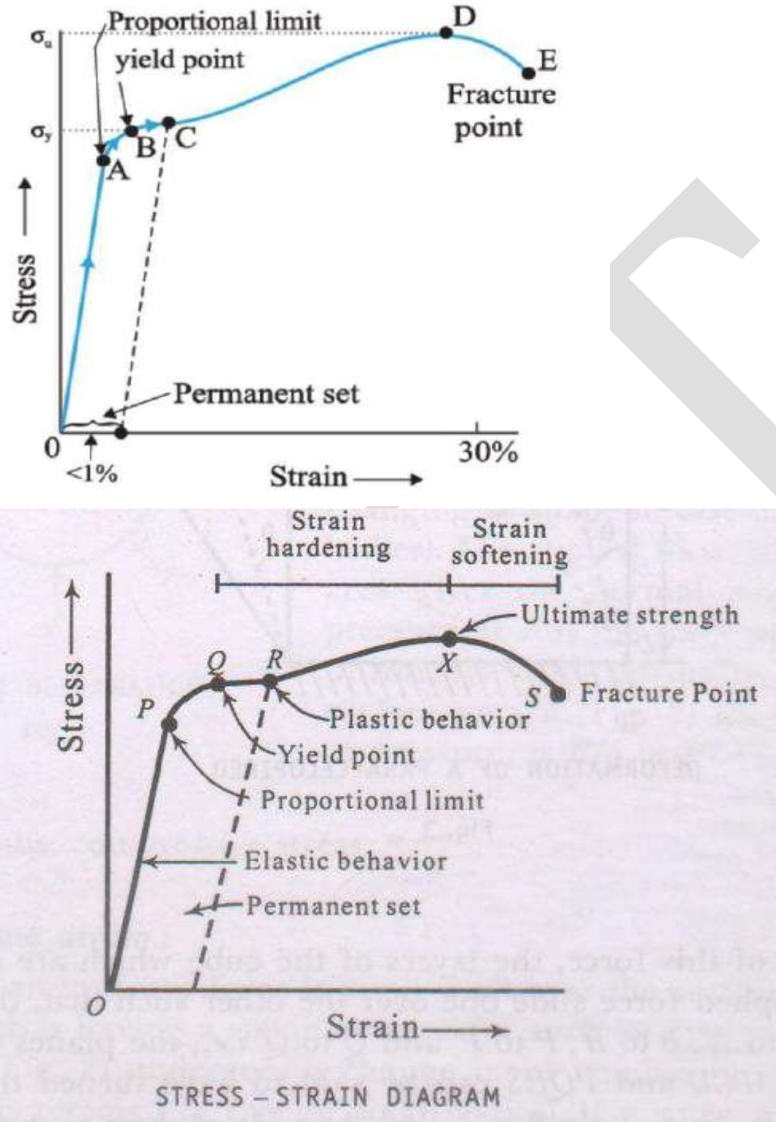


Fig. 1.3 A typical stress-strain curve for a metal

- The curve is linear in the region between O to P (Hooke's law is obeyed in this region and body behaves as elastic). The relationship between stress and strain in this initial region is not only linear but also proportional. Beyond point P, the proportionality between stress and strain no longer exists; hence the stress at P is called the proportional limit.

- With an increase in stress beyond the proportional limit, the strain begins to increase more rapidly for each increment in stress. Consequently, the stress-strain curve has a smaller and smaller slope, until, at point Q, the curve becomes horizontal.

- Point Q is the yield point (also known as elastic limit) and the corresponding stress is yield strength (σ_y) of the material.

- Above point Q the strain increases rapidly even for a small change in the stress (The portion between Q and X). In this region the body does not regain its original dimension and when the stress is made zero, the strain is not zero. Thus the material is said to have a permanent set and the deformation is said to be plastic deformation (See Fig. 1.3).

- After the point X, additional strain is produced even by a small applied force and fracture occurs at point S (See Fig. 1.3).

- The ratio of stress and strain, in the proportional region within the elastic limit of the stress strain curve (region OP in Fig. 1.3) is called modulus of elasticity and is characteristic of the material.

- It is of great importance to know the elastic limit for applications so that we can avoid the region of plastic deformation which may create problems in designing devices.

Strain hardening and strain softening

Strain hardening (also called work-hardening or cold-working) is the process of making a metal harder and stronger through plastic deformation. When a metal is plastically deformed, dislocations move and additional dislocations are generated. The more dislocations within a material, the more they will interact and become pinned or tangled. This will result in a decrease in the mobility of the dislocations and a strengthening of the material.

Strain softening: When the material is loaded either in tension or in compression, strain will build up with the applied stress. When it reaches the peak stress (for your case - yielding in tension) the material drops its shear resistance due to the continuous plastic deformation. This behaviour is called **strain softening.**

Effect of stress, temperature, annealing and impurities on elasticity of the materials:

1) **Effect of stress:**
2) **Effect of temperature:**
3) **Annealing**
4) **Effect of impurities:**

FACTOR OF SAFETY

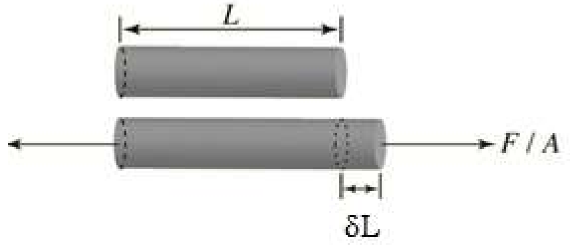
To avoid permanent deformation due to maximum stress, the engineering tools are to be used within the elastic limit with a working stress.

Factor of safety = Breaking stress / Working stress

Types of Elastic Moduli:

1. **Young's Modulus of Elasticity (Y)**

When a wire is acted upon by two equal and opposite forces in the direction of its length, the length of the body is changed. The change in length per unit length (δL/L) is called the longitudinal strain and the restoring force (which is equal to the applied force in equilibrium) per unit area of cross-section of wire is called the longitudinal stress.



For small change in the length of the wire, the ratio of the longitudinal stress to the corresponding strain is called the **Young's modulus of elasticity (Y)** of the wire. Thus,

Y = Longitudinal stress / Linear Strain = $\frac{(F|A)}{(\partial l|l)} = \frac{Fl}{A\partial l}$

Let there be a wire of length 'l' and radius 'r'. It's one end is clamped to a rigid support and a mass M is attached at the other end. Then

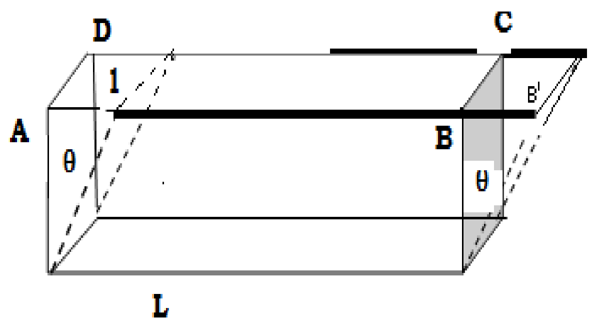
F = Mg and A = πr^2

Substituting in above equation, we have, Y = $\frac{Mgl}{(\pi r^2)\delta l}$.

2. Modulus of Rigidity (n) When a body is acted upon by an external force tangential to a surface of the body, the opposite surfaces being kept fixed, it suffers a change in shape of the body, its volume remains unchanged. Then the body is said to be sheared. The tangential force acting per unit area of the surface is called the **'shearing stress' (F/A).**

The ratio of displacement to perpendicular distance between the two surfaces is known as **shearing strain (θ)**.

Shearing strain $\theta = \frac{l}{L}$ when θ is small.



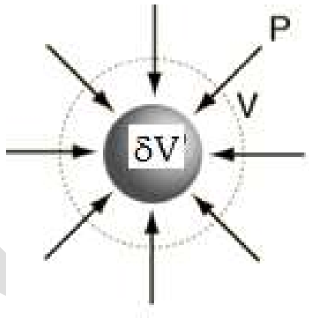
For small strain, the ratio of the shearing stress to the shearing strain is called the **'modulus of rigidity'** of the material of the body. It is denoted by 'n'.

Rigidity modulus (n) = Tangential stress / shear Strain

$$\mathbf{n} = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

3. Bulk Modulus of Elasticity (K)

Describes volumetric elasticity or the tendency of an object to deform in all directions when uniformly loaded in all directions; it is defined as volumetric stress over volumetric strain, and is the inverse of compressibility. K = Volumetric stress / Volume strain $= \frac{FV}{A\delta V}$.



When a uniform pressure (normal force) is applied all over the surface of a body, the volume of the body changes. The change in volume per unit volume of the body is called the '**volume strain'** and the normal force acting per unit area of the surface (pressure) is called the normal stress or **volume stress**. For small strains, the ratio of the volume stress to the volume strain is called the '**Bulk modulus' of the material of the body**. It is denoted by K. Then K = $\frac{-P}{\delta V\,/V}$

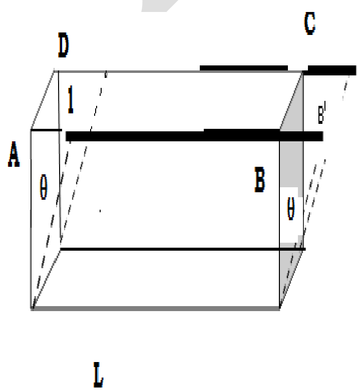
Negative sign in formula implies that when the pressure increases volume decreases and vice-versa. The reciprocal of the Bulk modulus of the material of a body is called the "**compressibility**' of that material. Thus, compressibility = 1/K

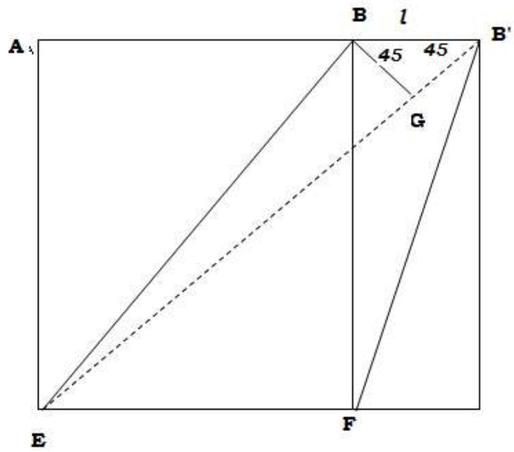
Poisson's ratio (σ)

When a material is stretched, the increase in its length (Linear strain α) is accompanied by decrease in cross section (lateral strain β). Within the elastic limit, the lateral strain is proportional to longitudinal strain and the ratio between them is a constant for a material known as **Poisson ratio** (σ). σ = lateral strain / longitudinal strain, i.e, $\sigma = \frac{\beta}{\alpha}$

The ratio of change in diameter/breadth to original diameter/breadth is called the **lateral strain** (**β**). The ratio of change in length to original length is called the **linear strain (α)**.

Relation between *Y(youngs modulus)* , **Rigidity Modulus (n)** *and* **Poisson ratio** *σ*





Let the face ABCD of a cube of side L be sheared by a Force F through an angle θ.

Then the Shearing stress, $T = \frac{F}{L^2}$

Shearing Strain, $\theta = \frac{l}{L}$

Therefore Rigidity Modulus (n) = $\frac{T}{\theta}$

Shearing stress along AB is equivalent to expansive stress along EB and compressive stress along AF. Let α be the longitudinal expansive strain per unit Stress per unit length and β be the lateral compressive strain per unit stress per unit length respectively.

Elongation along EB= $EB.\alpha.T$.

Compression along AF = $EB..T.\beta$ (Since AF =EB)

Net extension of the body is GB^1= $EB..T(\alpha+\beta))$ = $L.\sqrt{2}.T(\alpha+\beta))$ ---- (1)

From the Triangle AEB, $(EB)^2 = (AB)^2 + (BE)^2$

$(EB)^2 = L^2 + L^2 \rightarrow$ **EB = L.√2**

Also , from right angled triangle BB^{1}G,

Elongation GB^1= BB^1 cos45 (θ is approximately 45 in the Δle B B^{1}G) → **GB^1 =** $\frac{l}{\sqrt{2}}$

Eqn (1) becomes, $L.\sqrt{2}.T(\alpha+\beta)) = \frac{l}{\sqrt{2}}$

$$\frac{T}{\left(\frac{l}{L}\right)} = \frac{1}{2(\alpha+\beta)}$$

$$\boxed{n = \frac{1}{2(\alpha+\beta)}}$$

If the unit stress (F/A equal to 1) acting on a body along longitudinal direction, the strain produced is linear strain (α). Then the Young's modulus, Y = stress/strain,

$\boxed{\mathbf{Y} = \frac{1}{\alpha}}$. This is the relation between young's modulus (Y) and linear strain (α).

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$$n = \frac{1}{2\alpha\left(1+\frac{\beta}{\alpha}\right)} = \frac{1}{2\alpha(1+\sigma)} = \frac{Y}{2(1+\sigma)} \quad \Rightarrow \boxed{Y = 2n(1+\sigma)}$$

Relation between Bulk Modulus (K) -α –β:

$$\boxed{\text{Bulk Modulus (K)} = \frac{P}{3P(\alpha - 2\beta)} = \frac{1}{3(\alpha - 2\beta)}}$$

$$K = \frac{1}{3(\alpha - 2\beta)} = \frac{1}{3\alpha\left(1-\frac{2\beta}{\alpha}\right)}$$

$$\Rightarrow \quad K = \frac{1}{3\alpha(1-2\sigma)} = \frac{Y}{3(1-2\sigma)} \quad \Rightarrow \boxed{Y = 3K(1-2\sigma)}$$

RELATION BETWEEN ELASTIC CONSTANTS

$$\alpha - 2\beta = \frac{1}{3K} \text{......(1)}$$

$$\alpha + \beta = \frac{1}{2n} \text{..........(2)}$$

Equation $(2)-(1)$,

$$3\beta = \frac{1}{2n} - \frac{1}{3K} = \frac{3k-2n}{6nk}$$

$$\beta = \frac{3K-2n}{18nK}$$

Again, multiplying (2) by 2 and adding to (1), We have

$$3\alpha = \frac{1}{n} + \frac{1}{3K} = \frac{3K+n}{3Kn}$$

$$\alpha = \frac{3K+n}{9Kn}$$

$$\frac{1}{Y} = \frac{3K+n}{9Kn} \quad [\,\Theta\ \alpha = \frac{1}{Y}\,]$$

$$\frac{9}{Y} = \frac{3K+n}{Kn} \quad \Rightarrow \quad \boxed{\frac{9}{Y} = \frac{3}{n} + \frac{1}{K}}$$

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Relation between K - n - σ

From the equation, $K = \frac{1}{3(\alpha - 2\beta)} = \frac{1}{3\alpha\left(1 - \frac{2\beta}{\alpha}\right)}$

$$\Rightarrow \quad K = \frac{1}{3\alpha(1-2\sigma)} = \frac{Y}{3(1-2\sigma)} \Rightarrow \boxed{Y = 3K(1-2\sigma)} \text{ ------ (i)}$$

From the equation, $\alpha + \beta = \frac{1}{2n}$. $\Rightarrow \quad n = \frac{1}{2(\alpha+\beta)}$

$$n = \frac{1}{2\alpha\left(1+\frac{\beta}{\alpha}\right)} = \frac{1}{2\alpha(1+\sigma)} = \frac{Y}{2(1+\sigma)} \Rightarrow \boxed{Y = 2n(1+\sigma)} \text{ ------- (ii)}$$

From relations (i) & (ii), We have, $3K(1-2\sigma) = 2n(1+\sigma)$

$$3K - 2n = \sigma(2n + 6K)$$

$$\boxed{\sigma = \frac{3K - 2n}{6K + 2n}}$$

LIMITS OF σ : Generally the Poissions ratio of materials varies from -1 to 0.5

We have the Equation 3K(1-2σ) = 2n(1+σ)

1. If σ be a positive quantity, (1-2σ) should be positive
   2σ<1 = σ < 0.5,

A perfect incompressible material deformed elastically at small strains would have a poissions ratio exactly 0.5.

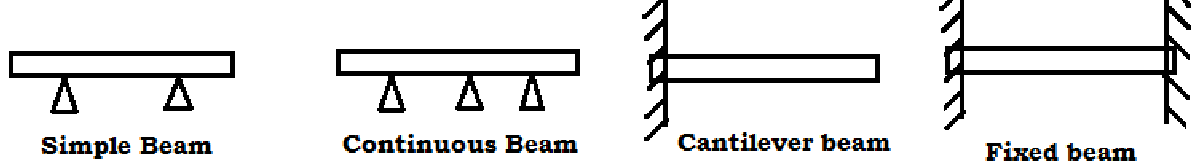
2. If σ be a negative quantity, (1 + σ) should be positive, σ < -1.

Resilience: Capacity to resist a heavy stress without acquiring permanent elongation.

Types of Beam

Beam is a bar or rod of uniform cross section whose length is very much larger than thickness.

Depending on the support, beams are classified as following four types



1. **Simple beam**: It is bar resting upon supports at its ends and is the kind most commonly in use.
2. **Continuous beam**: It is a bar resting upon more than two supports.
3. **Cantilever beam**: It is a beam whose one end is fixed and the other end is free.
4. **Fixed beam**: A beam fixed at its both ends is called a fixed beam.

Applications of beam: Beams are used

i) In the fabrication of trolley ways.

ii) In the Chassis/ frame as truck beds.

iii) In the elevators.

iv) In the construction of flatform and bridges.

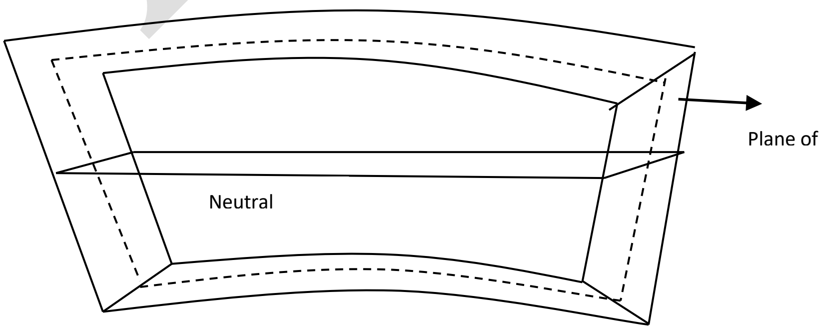
v) Beams are an integral part of Civil engineering structural elements (bridges, dams, multistoreid buildings).

vi) In the measuring devices (Tunneling microscopes)

BENDING OF BEAM

Beam is a bar or rod of uniform cross section whose length is very much larger than thickness. When such a beam is fixed at one end and loaded at the other, the beam is bent under the action of couple produced by the load. Upper surface of the beam gets stretched and lower surface gets compressed. The extension is maximum in the upper most filaments and compression, maximum in the lowermost ones. The surface which does not get affected is known as neutral surface.

If the bending is uniform, the longitudinal filaments get bent into circular arcs in planes parallel to the plane of symmetry (plane of bending).The line of intersection of plane of bending with neutral surface is called neutral axis.



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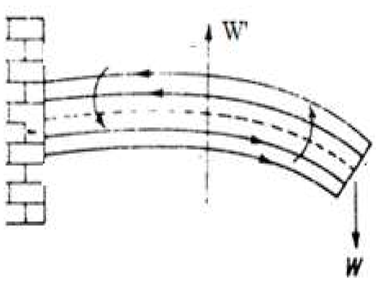
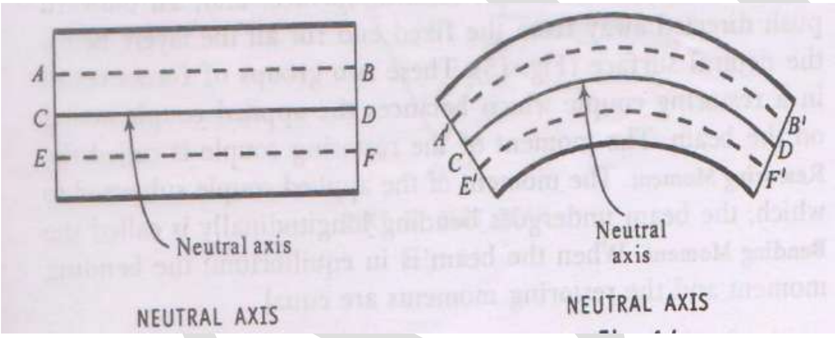
BENDING MOMENT The load is attached to the beam at the other end the beam bends. The successive layers now strained.

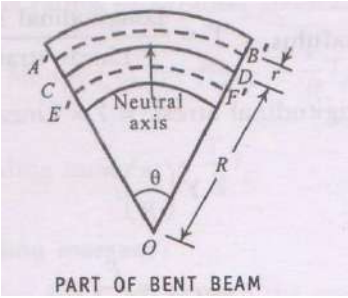
In the figure, ABCD is a beam fixed at one end and loaded at another end. EF is neutral axis. The applied load (W) tends to bend the beam, an equal and opposite reactional force W^1 will be acting upwards. These two forces constitute a couple and the moment of this couple is called **bending moment**. When the beam is in equilibrium position the bending moment and restoring moment should be equal. In order to find the expression for moment of restoring couple, consider a fiber A'B' at a distance r from neutral axis CD shown in Fig. Let the beam be bent in the form of circular arc subtending angle θ at the centre of curvature O.





Consider a long uniform beam whose one end is fixed. The beam can be thought of made up of a number of parallel layers like AB, CD, EF etc. If now a load is attached to the other end the beam bends. The successive layers now strained. The layer AB which is above the neutral axis will be elongated to A'B' and the one like EF below the neutral surface will be contracted to E'F'. CD is the neutral surface which does not change.

The shape of the different layers of the bent beam can be imagined to form part of concentric circles of varying radii as shown in figure

.

Let R be the radius of curvature of the circle to which the neutral surface forms a part.

CD = Rθ

where θ is the angle subtended by the layers at the centre of curvature O.

Then, the length of fibre in the unstrained position AB= CD = Rθ

The length of fibre in the strained position A'B' = (R + r)θ

Change in length = (R + r)θ -Rθ = rθ

Original length = Rθ

Liner strain $= \frac{r\theta}{R\theta} = \frac{r}{R}$

Youngs modulus Y=$\frac{Longitudinal\ stress}{Linear\ strain}$

Longitudinal stress=Y x Linear strain

$=Y\frac{r}{R}$

Stress $=\frac{F}{a}$ where F is is the force acting on the beam and a is the area of the layer AB

$\frac{F}{a} = Y\frac{r}{R}$

$F = \frac{Yar}{R}$

Moment of this force about the neutral axis

=F x its distance from the neutral axis.

M = F×r $=\frac{Yar^2}{R}$

Total moment of forces acting on the entire beam $M = \sum \frac{Yar^2}{R} = \frac{Y}{R}\sum ar^2 = \frac{Y}{R}I$, here I = $\sum ar^2$ = aK^2 & I is Geometrical Moment of Inertia.

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Bending moment of the beam, $\mathbf{M} = \frac{Y}{R} I$

For rectangular cross section, area = b X d, $k^2 = \frac{d^2}{12}$

$$I = ak^2 = \frac{bd^3}{12}$$

$$M = \frac{Ybd^3}{12R}$$

For Circular cross section, area = πr^2, $k^2 = \frac{r^2}{4}$

$$I = ak^2 = \frac{\pi r^4}{4}$$

$$M = \frac{Y\pi r^4}{4R}$$

Where I is the geometrical moment of inertia and k is **Radius of gyration**, It is the distance of a point from the axis of rotation where whole mass of the body is assumed to be concentrated.

Concept of cantilever and I girders Cantilever:

A cantilever is a rigid structural element that extends horizontally and is supported at only one end. Typically, it extends from a flat vertical surface such as a wall, to which it must be firmly attached. Like other structural elements, a cantilever can be formed as a beam, plate, truss, or slab. When subjected to a structural load at its far, unsupported end, the cantilever carries the load to the support where it applies a shear stress and a bending moment. Cantilever construction allows overhanging structures without additional support.

A cantilever beam moves downwards when it is loaded to vertical loads. A cantilever beam can be put through a point load, uniform load, or varying load. Ignoring the type of load, it moves downwards by making a convexity upwards. This movement makes tension in the upper fiber and compression in the lower fibers. Hence major reinforcement is given to the upper fiber of the concrete beam, as there is high tensile stress.

The bending moment of a cantilever beam differs from zero at the free end to the highest value at the fixed end support. Hence during the preparation of cantilever beams, the major reinforcement is given to the upper fiber of the concrete beam to resist the tensile stress safely.

Cantilever beam structures are used in the following applications:

- Construction of cantilever beams and balconies
- Temporary cantilever support structures
- Freestanding radio towers without guy wires
- Construction of cantilever beam for pergolas
- Lintel construction in buildings.

I Girders:

Girder a horizontal main structural member (as in a building or bridge) that supports vertical loads and that consists of a single piece or of more than one piece bound together. A girder is a support beam used in construction. It is the main horizontal support of a structure which supports smaller beams. Girders often have an I-beam cross section composed of two load-bearing flanges separated by a stabilizing web, but may also have a box shape, Z shape, or other forms.

Advantages of I girders

- to minimize the depression in a beam, it is designed as I shape girder
- The I-shape girder have large load bearing surface, which decreases stress.
- In a plate girder bearing stiffeners are designed for bearing forces and they must also be checked for safety against compressive forces.

Applications: I shaped girders made to bear the high load of things. They are used in building of pulls, fly overs etc. They are also used to give main support to the roofs of houses. Sometimes they are also used in construction of public shades and public places.

Failures (Fracture/Fatigue)

Mechanical failure is defined as any change in the size, shape or material properties of a structure, machine or machine parts that renders it incapable of satisfactory performance its intended function.

A Machine is meant for repeated use. Machine has many structural components/parts [example parts of aircraft, automobile, pumps etc] that are subjected to repeated loading while in use it gets stressed. Due to their continuous use, they are stressed repeatedly and cracks begin to form. Formation of a crack which results in a complete destruction of continuity constitutes fracture.

Types of Failure :

- Simple fracture :-

- Ductile fracture
- Brittle fracture

– Fatigue Failure

- Creep

Funadmentals of Fracture Simple fracture is the separation of a body into two or more pieces in response to an imposed stress .The applied stress may be tensile, compressive, shear, or torsional. For engineering materials, two fracture modes are possible**: ductile and brittle** based on the ability of the material to deform plastically.

Ductile materials typically exhibit substantial plastic deformation with high energy absorption before fracture. In brittle materials little or no plastic deformation with low energy absorption before fracture

Any fracture process involves two steps in response to the imposed stress

a) crack formation

b). crack propagation

The mode of fracture is highly dependent on the mechanism of crack propagation.

Ductile fracture is charecterized by extensive plastic deformation in the vicinity of an advancing crack. The process proceeds relatively slowly as crack length is extended . Such a crack is often said to be stable. That is, it resists any further extension unless there is an increase in the applied stress.

In **Brittle fracture**, cracks may spread extremely rapidly, with very little accompanying plastic deformation. Such cracks may be said to be unstable, and crack propagation , once started, will continue spontaneously without an increase in magnitude of the applied stress.

Ductile fractur**e** is almost always preferred for two reasons. First, brittle fracture occurs suddenly and catastrophically without any warning; this is a consequence of the spontaneous and rapid crack propagation. On the other hand, for ductile fracture, the presence of plastic deformation gives warning that fracture is imminent, allowing preventive measures to be taken. Second, more strain energy is required to induce ductile fracture in as much as ductile materials are generally tougher.

Under the action of an applied tensile stress, most metal alloys are ductile, whereas ceramics are notably brittle, and polymers may exhibit both types of fracture.

Ductile Fracture : Ductile fracture surfaces will have their own distinctive features on both macroscopic and microscopic levels. Figure (3.1) shows schematic representations for two characteristic macroscopic fracture profiles. The configuration shown in Figure (3.1a) is found for extremely soft metals, such as pure gold and lead at room temperature, and other metals, polymers,

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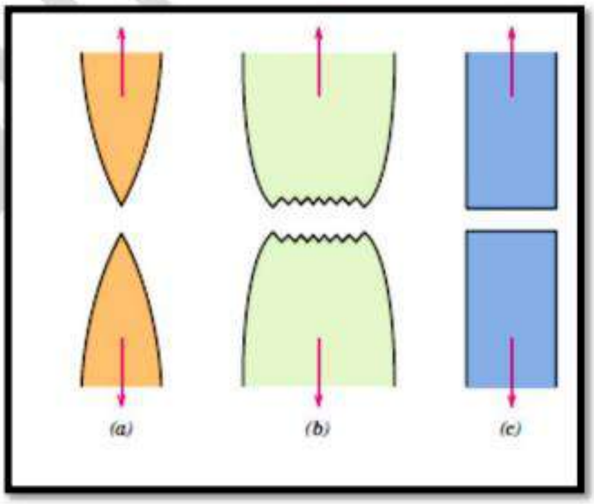
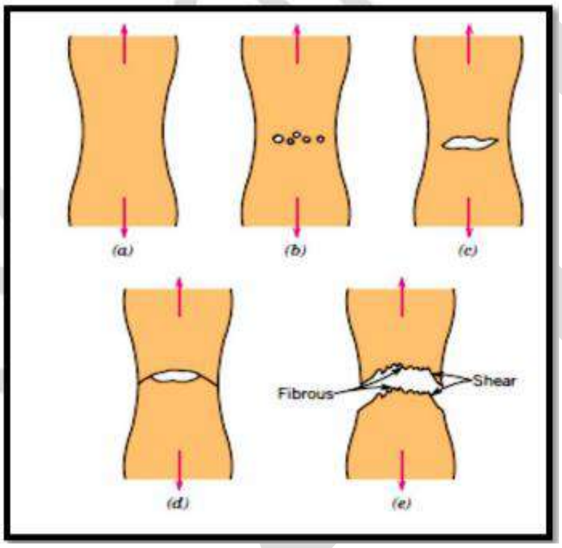
and inorganic glasses at elevated temperatures. These highly ductile materials neck down to a point fracture, showing virtually 100% reduction in area.



Figure(3.1) : (a) Highly ductile fracture in which the specimen necks down to a point. (b) Moderately ductile fracture after some necking. (c) Brittle fracture without any plastic deformation.

The most common type of tensile fracture profile for ductile metals is that represented in Figure 3.1b, where fracture is preceded by only a moderate amount of necking. The fracture process normally occurs in several stages



Figure(3.2): Stages in the cup-andcone fracture. (a) Initial necking. (b) Small cavity formation. (c) Coalescence of cavities to form a crack. (d) Crack propagation. (e) Final shear fracture at a °45 angle relative to the tensile direction.

First, after necking begins, small cavities, or micro voids, form in the interior of the cross section, as indicated in Figure (3.2b). Next, as deformation continues, these micro voids enlarge, come together, and coalesce to form an elliptical crack, which has its long axis perpendicular to the stress direction. The crack continues to grow in a direction parallel to its major axis by this micro void coalescence process (Figure 3.2c). Finally, fracture ensues by the rapid propagation of a crack around the outer perimeter of the neck (Figure 3.2d), by shear deformation at an angle of about °45 with the tensile axis—this is the angle at which the shear stress is a maximum. Sometimes a fracture having this characteristic surface contour is termed a cup-and-cone fracture because one of the mating surfaces is in the form of a cup, the other like a cone. In this type of fractured specimen (Figure 3.3a), the central interior region of the surface has an irregular and fibrous appearance, which is indicative of plastic deformation.

Ductility is strongly dependent on the inclusion content of the material. With increasing numbers of inclusions , the distance between the voids decreases, so it easier for them to link together and lower the ductility .

Brittle Fracture : Brittle fracture takes place without any appreciable deformation, and by rapid crack propagation. The direction of crack motion is very nearly perpendicular to the direction of the applied tensile stress and yields a relatively flat fracture surface, as indicated in Figure (3.1c.). **Characterization of brittle fracture :** 1. Rapid rate of crack propagation with no gross deformation and very little micro deformation .

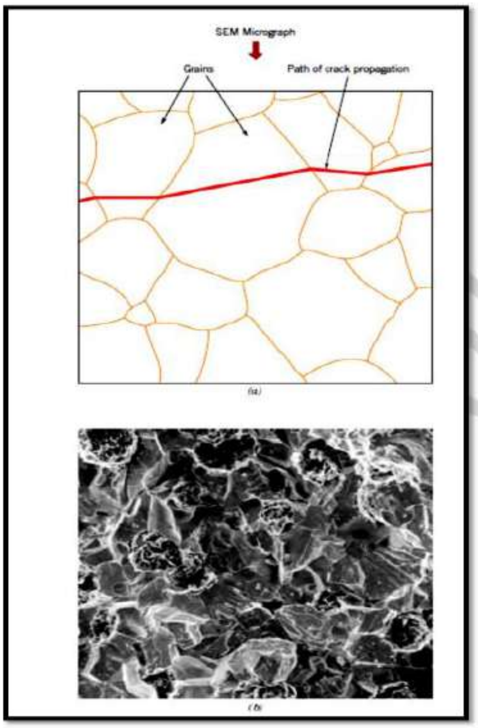
2. It is a kin to cleavage in ionic.

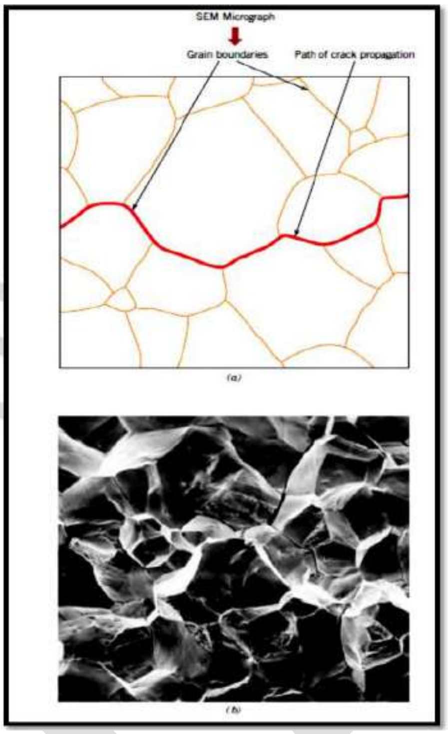
3. A brittle fracture surface typically appears shiny with flat facets.

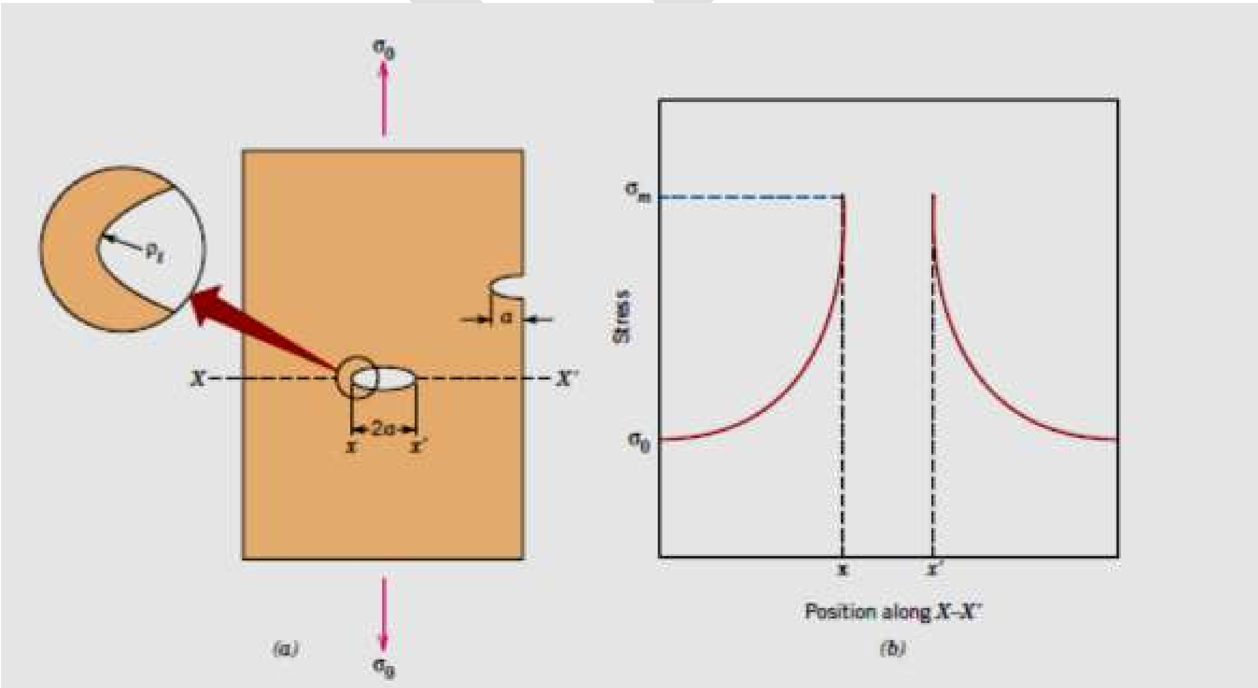
Brittle fracture in amorphous materials , such as ceramic glasses , are relatively shiny and smooth surfaces .

Transgranular & Intergranular Fracture : For most brittle crystalline materials, crack propagation corresponds to the successive and repeated breaking of atomic bonds along specific crystallographic planes (Figure 3.4a). This type of fracture is said to be transgranular (or transcrystalline), because the fracture cracks pass through the grains. Macroscopically, the fracture surface may have a grainy or faceted texture (Figure 3.3b), as a result of changes in orientation of the cleavage planes from grain to grain. This cleavage feature is shown at a higher magnification in the scanning electron micrograph of Figure 3.4b. In some alloys, crack propagation is along grain boundaries (Figure 3.5a); this fracture is termed intergranular. Figure 3.5b is a scanning electron micrograph showing a typical intergranular fracture

Stress Concentration The measured fracture strengths for most brittle materials are significantly lower than those predicted by theoretical calculations based on atomic bonding energies. This discrepancy is explained by the presence of very small, microscopic flaws or cracks that always exist under normal conditions at the surface and within the interior of a body of material. These flaws are a detriment to the fracture strength because an applied stress may be amplified or concentrated at the tip, the magnitude of this amplification depending on crack orientation and geometry. This phenomenon is demonstrated in Figure 3.6, a stress profile across a cross section containing an internal crack. As indicated by this profile, the magnitude of this localized stress diminishes with distance away from the crack tip. At positions far removed, the stress is just the nominal stress or the applied load divided by the specimen cross-sectional area (perpendicular to this load). Due to their ability to amplify an applied stress in their locale, these flaws are sometimes called stress raisers.







Figure(3.6) : (a) The geometry of surface and internal cracks. (b) Schematic stress profile along the line X–X in (a), demonstrating stress amplification at crack tip positions.

If it is assumed that a crack is similar to an elliptical hole through a plate, and is oriented perpendicular to the applied stress, the maximum stress, occurs at the crack tip and may be approximated by

$$\sigma_m = 2\sigma_0\left(\frac{a}{\rho_t}\right)^{1/2}$$

where : σ_0 is the magnitude of the nominal applied tensile stress, ρ_t : is the radius of curvature of the crack tip (Figure 3.6a), and a : represents the length of a surface crack, or half of the length of an internal crack. Sometimes the ratio is denoted as the stress concentration factor (K_t) which is simply a measure of the degree to which an external stress is amplified at the tip of a crack.

$$K_t = \frac{\sigma_m}{\sigma_0} = 2\left(\frac{a}{\rho_t}\right)^{1/2}$$

Sometimes the ratio is denoted as the stress concentration factor (Kt) which is simply a measure of the degree to which an external stress is amplified at the tip of a crack.

Fatigue Failure : Fatigue is a form of failure that occurs in structures subjected to dynamic and fluctuating stresses (e.g., bridges, aircraft, and machine components). The term ―fatigue‖ is used because this type of failure normally occurs after a lengthy period of repeated stress or strain cycling. Fatigue is important in as much as it is the single largest cause of failure in metals, estimated to comprise approximately 90% of all metallic failures; polymers and ceramics (except for glasses) are also susceptible to this type of failure. Furthermore, fatigue is catastrophic and insidious, occurring very suddenly and without warning.

Fatigue failure is brittle like in nature even in normally ductile metals, in that there is very little, if any, gross plastic deformation associated with failure. The process occurs by the initiation and propagation of cracks, and ordinarily the fracture surface is perpendicular to the direction of an applied tensile stress.

FACTORS THAT AFFECT FATIGUE LIFE

The fatigue behavior of engineering materials is highly sensitive to a number of variables. Some of these factors include mean stress level, geometrical design, surface effects, and metallurgical variables, as well as the environment.

Mean Stress

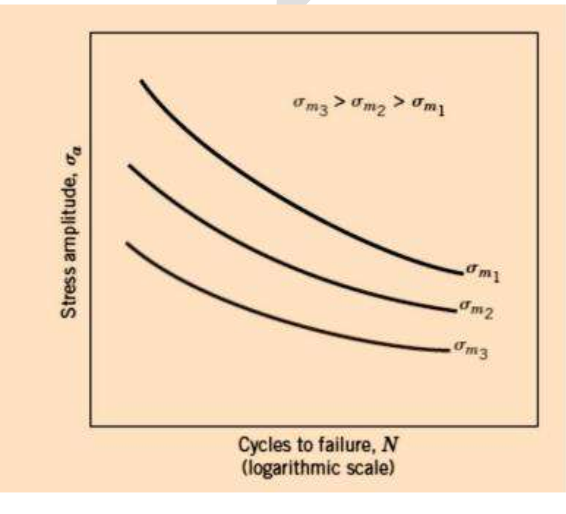
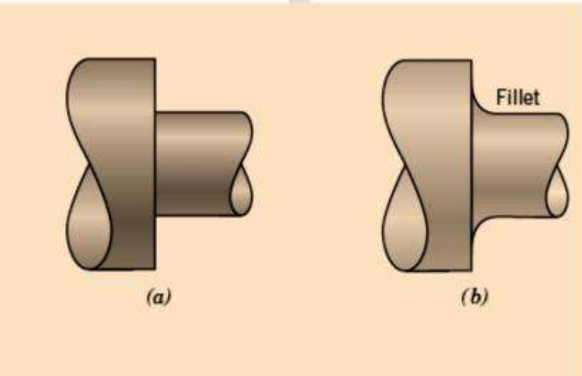


Figure 3.16: Demonstration of the influence of mean stress on S–N fatigue behavior.

The dependence of fatigue life on stress amplitude is represented on the S– N plot. Such data are taken for a constant mean stress σ_m, often for the reversed cycle situation ($\sigma_m = 0$). Mean stress, however, will also affect fatigue life; this influence may be represented by a series of S–N curves, each measured at a different as depicted schematically in Figure 3.16 As may be noted, increasing the mean stress level leads to a decrease in fatigue life.

Surface Effects: For many common loading situations, the maximum stress within a component or structure occurs at its surface. Consequently, most cracks leading to fatigue failure originate at surface positions, specifically at stress amplification sites. Therefore, it has been observed that fatigue life is especially sensitive to the condition and configuration of the component surface. Numerous factors influence fatigue resistance, the proper management of which will lead to an improvement in fatigue life. These include design criteria as well as various surface treatments

Design Factors The design of a component can have a significant influence on its fatigue characteristics. Any notch or geometrical discontinuity can act as a stress raiser and fatigue crack initiation site; these design features include grooves, holes, keyways, threads, and so on. The sharper the discontinuity (i.e., the smaller the radius of curvature), the more severe the stress concentration. The probability of fatigue failure may be reduced by avoiding (when possible) these structural irregularities, or by making design modifications whereby sudden contour changes leading to sharp corners are eliminated—for example, calling for rounded fillets with large radii of curvature at the point where there is a change in diameter for a rotating shaft (Figure 3.17).



Figure(3.17): Demonstration of how design can reduce stress amplification. (a) Poor design: sharp corner. (b) Good design: fatigue lifetime improved by incorpo rating rounded fillet into a rotating shaft at the point where there is a change in diameter

Surface Treatments During machining operations, small scratches and grooves are invariably introduced into the work piece surface by cutting tool action. These surface markings can limit the fatigue life. It has been observed that improving the surface finish by polishing will enhance fatigue life significantly.

Case hardening : is a technique by which both surface hardness and fatigue life are enhanced for steel alloys. This is accomplished by a carburizing or nitriding process whereby a component is exposed to a carbonaceous or nitrogenous atmosphere at an elevated temperature. A carbon- or nitrogen-rich outer surface layer (or ―case‖) is introduced by atomic diffusion from the gaseous phase. The case is normally on the order of 1 mm deep and is harder than the inner core of material.

ENVIRONMENTAL EFFECTS Environmental factors may also affect the fatigue behavior of materials. A few brief comments will be given relative to two types of environmentassisted fatigue failure: thermal fatigue and corrosion fatigue

Thermal **fatigue** : is normally induced at elevated temperatures by fluctuating thermal stresses; mechanical stresses from an external source need not be present

Corrosion Fatigue : Failure that occurs by the simultaneous action of a cyclic stress and chemical attack is termed corrosion fatigue. Corrosive environments have a deleterious influence and produce shorter fatigue lives. Even the normal ambient atmosphere will affect the fatigue behavior of some materials.

Questions

1.Define elasticity and the factors affecting elasticity

2.Write a note on strain hardening and strain softening .

3 Derive the expression for couple per unit twist of a solid cylinder.

4. Derive the relation between Bulk modulus (K), Young's modulus (Y) and Poisson's Ratio (σ).

5. Derive the expression for bending moment and depression at a loaded end of single cantilever.

6. Derive the expression of rigidity modulus (n) in terms of linear strain (α) and lateral strain (β).

8. Explain different types of beams.

Problems :

1)Calculate the force required to produce an extension of 1mm in steel wire of length 2 meters and diameter 1mm. (Youngs modulus for steel = $2x10^{11}N/m^2$) (Ans: $F = \frac{YA\partial l}{l} = \mathbf{78.5N}$)

2. Calculate the extension produced in a wire length of 2m and radius $0.013 \text{ x}10^{-2m}$ due to a force of 14.7Newton applied along its length. Given Young's modulus for steel = $2x10^{11}N/m^2$

   Ans: **Y= 98.18 x10^9 N/m2,**

3)Calculate the extension produced in a wire length of 2m and radius $0.013 \text{ x}10^{-2m}$ due to a force of 14.7 N applied along its length. Given Young's modulus for steel = $2x10^{11}N/m^2$

4. A rod of cross section of area 1cmx1cm is rigidly planted into the earth vertically . A string which can withstand a maximum of tension of 2 Kg is tied to the upper end of the rod from the ground . If the length of the rod from the ground level is 2meter, calculate the distance through which its upper end is displaced just before the string snaps .(Given Youngs modulus = $2x10^{10}$ N/m^2) (Ans: $= \frac{4mgl^3}{Ybd^3} =$ 0.314.)

5. Calculate the Possion's ratio for the material given that Y=12.25 x 1010 N/m2 & η=4.55 x 1010 N/m2
6. Calculate the extension produced in a wire of length 2 m and radius 0.13 cm due to a force of 15 N applied along its length. (Given: Young's modulus of wire Y= 2.1 x 10 11 N/m2).
7. A rod of cross sectional area 15 mm x 15 mm and 1 m long is subjected to compressive load of 22.5 kN. Calculate the stress and decrease in length if Young's modulus is 200 GN/m2
8. **The** Young's modulus for a material is 100 x 109 N/m2 and its modulus of rigidity is 40 x 109 N/m2 .Determine its bulk modulus for the given material.

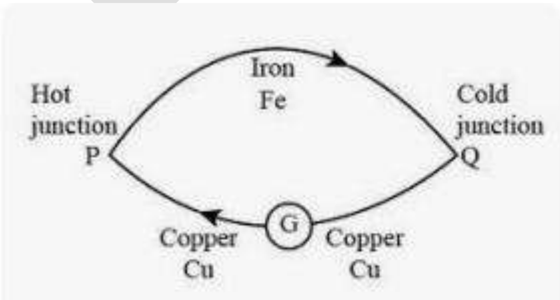
MODULE 3-THERMOELECTRIC MATERIALS AND DEVICES

SYLLABUS: Thermoelectric materials and devices: Thermo emf and thermo current, Seeback effect, Peltier effect, Seeback and Peltier coefficients, figure of merit (Mention Expression), laws of thermoelectricity. Expression for thermo emf in terms of T1 and T2, Thermo couples, thermopile, Construction and Working of Thermoelectric generators (TEG) and Thermoelectric coolers (TEC), low, mid and high temperature thermoelectric materials, Applications: Exhaust of Automobiles, Refrigerator, Space Program (RTG), Numerical Problems

Pre requisites: Basics of Electrical conductivity Self-learning:

Thermoemf and thermocurrent

When two dissimilar metals are joined at their ends to form two junctions and if these two junctions are maintained at two different temperatures a current is found to flow in the closed circuit and hence emf is produced. The emf is called as thermo emf and current is known as thermo current. Thermo emf is an electromotive force which is generated due to the thermal gradient.



In figure we have 2 thermally and electrically conducting Copper and Iron wires. The 2 wires are attached terminal to terminal to become a closed circuit. A galvanometer is attached to the circuit to detect any current flow through the circuit. A junction P is hot terminal where the temperature is comparatively higher as compared to the cold terminal which is Q.

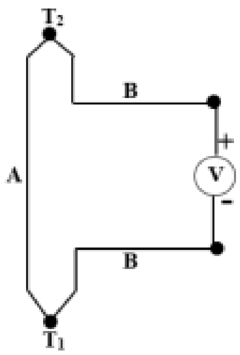
We know that electrons flow from a denser electron metal to a lower electron denser metal, thus creating a gradient. So, if temperature of the junction is maintained at 2 different values, electron diffusion happens, which generates larger potential at the hotter junction when compare to the colder junction. This variation in potential creates a flow of current, which is detected by the galvanometer. The existence of current flow indicates that an emf exists in the circuit.

Seeback effect

Definition: The production of electromotive force (emf) and hence current by maintaining the junctions of two dissimilar metals at different temperatures is called Seebeck effect

.

In 1821 Thomas Johann Seebeck discovered this phenomenon. The emf is known as thermoelectric emf. The thermoelectric emf causes a continuous current in the conductors, if they form a complete loop and the current is known as thermo electric current.



The voltage (thermo electric emf) created is of the order of several micro volts per kelvin difference. The thermo electric emf will exist and the current will flow in the circuit as long as the 2 junctions, known as the "hot" junction and "cold" junction, are at different temperatures. Thus, the Seebeck effect is the conversion of temperature differences directly into electricity. The magnitude and direction of thermoelectric current depends on the types of metals used and the temperature between the hot and cold ends. It does not depend on the temperature distribution along the conductors.

The voltage developed in the circuit, is proportional to the temperature difference between the 2 junctions.

$$V = (T_2 - T_1) \text{ Where } \alpha = \alpha_B - \alpha_A$$

α_A *and* α_B are known as the Seebeck coefficients of the metals A and B, and T_1 and T_2 are the temperatures of the two junctions.

Seebeck effect is observed not only in metals but as well in semiconductors also. It is not necessarily a junction phenomenon, but arises in a single conductor also. If temperature gradient (difference) is caused in conductor, electrons diffuse from the hot side to the cold side. Electrons migrating to the cold side leave behind their oppositely charge and immobile nuclei on the hot side and thus give rise to a thermoelectric voltage.

Seeback coefficient:

Seebeck coefficient or thermo power of a material measures the magnitude of an induced thermoelectric voltage in response to a temperature difference across that material. It is defined as the open circuit voltage produced between two points on a conductor, where a uniform temperature difference of 1K exists between those points.

If the temperature difference ΔT between the two ends of a material is small, then the thermo power or Seebeck coefficient of a material may be written as $\alpha = \frac{\Delta V}{\Delta T}$

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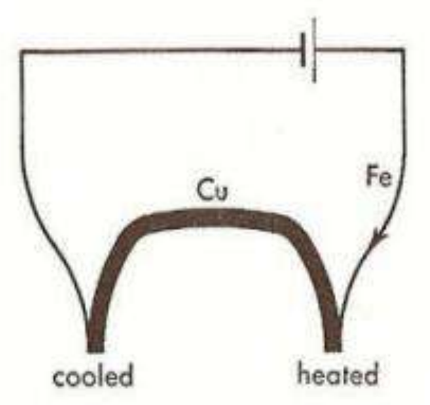
This can also be expressed in terms of the electric field E and the temperature gradient∇T,

as $\alpha = \frac{E}{\Delta T}$

The thermo power is an important material parameter the efficiency of a thermoelectric material. A larger induced thermoelectric voltage for a given temperature gradient will lead to a larger efficiency.

Peltier effect:

In 1834 Peltier discovered that when electric current passed in a circuit consisting of two dissimilar metals, heat is evolved at one junction and absorbed at the other junction. This is known as peltier effect.It is the inverse of the Seebeck effect. The peltier effect is junction phenomenon.



There is heat absorption or generation at the junctions depending on the direction of current flow. Heat generated by current flowing in one direction was absorbed if the current was reversed.

As an example, consider the circuit as shown in the figure. Under these conditions it is observed, as indicated in the diagram, that the right-hand junction is heated, showing that electrical energy is being transformed into heat energy. Meanwhile, heat energy is transformed into electrical energy at the left junction, there by causing it to be cooled. When the current is reversed, heat is absorbed at the right junction and produced at the left one

Peltier coefficient:

The peltier coefficient is defined as the amount of heat energy absorbed or evolved at the junction of two dissimilar metals when one amper of current flows through it for one second.

It is denoted by π and expressed in volts. Its value different for different materials. For the same pair of metals its value depends upon the temperature of junction.

If a current of I ampere flows for t sec. through the junction having a Peltier coefficient π then heat energy absorbed or evolved = πIt Joules ---------------(1)

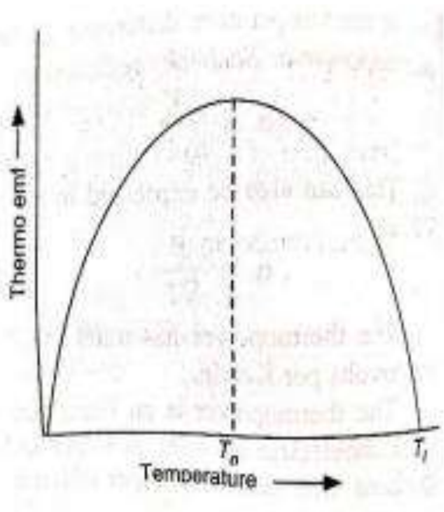
If V is the contact potential difference at the junction in volts , then heat energy absorbed or evolved = VIt Joules ---------------(2)

Equating (1)& (2), we get $\pi = V$

Thus, the Peltier coefficient is numerically equal to the applied potential difference expressed in volts

Variation of Thermoelectric emf with temperature:

If the temperature of the cold junction of a thermo couple is kept at 0°C and the thermo electric e.m.f. 'e' is plotted against the temperature T of the hot junction, we obtain a parabolic curve, as shown in Fig. It is seen that the thermo e.m.f. increases with the temperature of the hot junction and becomes a maximum at a particular temperature, Tn. Tn is known as the neutral temperature which is a constant for the given pair of metals forming the thermocouple. The temperature of the hot junction at which maximum thermo e.m.f. flows is a constant for a given couple and is known as neutral temperature Tn for that couple. If the temperature of the hot junction is increased beyond the neutral temperature, the e.m.f .decreases and becomes zero at a temperature Ti, known as the inversion temperature. The temperature at which the thermo e.m.f. is zero, is known as inversion temperature. Beyond the temperature of inversion, the e.m.f. again increases but in the reverse direction.



The thermo e.m.f. varies with temperature according to the following relation.

e = at +1/2bt² -------(1)

where a and b are Seebeck constants for the thermo couple, Eqn.1 is known as Seebeck equation, and t = Ti-Tn

Differentiation of eqn.(1) gives $\frac{de}{dT} = a + bT$ -----------(2)

At T = T_n, e is maximum and hence $\frac{de}{dT} = 0$ Therefore,

$$0 = a + bT_n$$

$$T_n = -\frac{a}{b} \quad \text{---------(3)}$$

At T = Ti , e = 0. Therefore, it follows from equation (1) that

$$\text{OR} \quad 0 = aT_i + \frac{1}{2}bT_i^2$$

$$0 = T_i(a + \frac{1}{2}bT_i)$$

$$T_i = -\frac{2a}{b} \quad \text{----------(4)}$$

From equation (3) & (4) we get

$$T_i = 2T_n$$

Thermoelectric power:

The rate of change of emf with temperature is called thermoelectric power and is denoted by P. Thus $p = \frac{de}{dt}$

Relation between Peltier coefficient and thermoelectric power is given by π = T P where T is the temperature of the junction and P is the thermoelectric power at that temperature.

Figure of -Merit, Z:

The efficiency of conversion of thermal energy into electrical energy is denoted by the parameter called the figure-of-merit of a thermoelectric material. It is denoted as

$$Z = Z = \frac{\alpha^2 \sigma}{K}$$

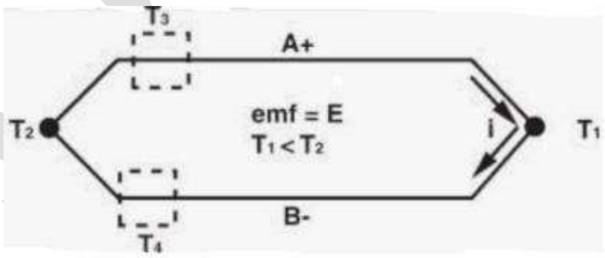
Where α is the Seebeck coefficient of the material (measured in microvolts/K), σ is the electrical conductivity of the material and K is the total thermal conductivity of the material.

Laws of thermoelectricity:

Law of homogeneous circuit

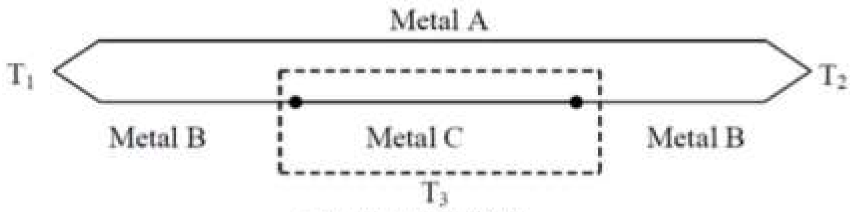
Statement: A thermoelectric current cannot be sustained in a circuit of single homogenous material by the application of heat alone.

Practical significance: Two different materials are required for any thermocouple circuit to produce thermo emf.



Law of intermediate metals

Statement: A third metal may be inserted into a thermocouple system without effecting the emf generated, if and only if, the junctions with the third metal are kept at the same



Law of Intermediate Metals

temperature.

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Practical significance:

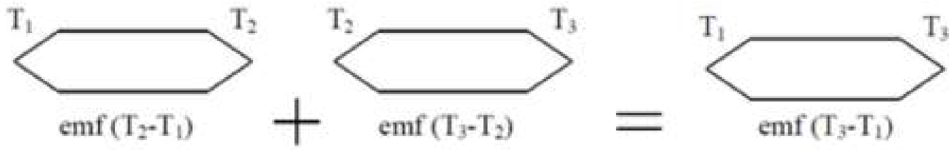
1. It allows the use of extension wires of metal, different from the metal used to form thermocouples

2. It allows the use of measuring instruments into the circuit without disturbing the emf generated by the thermocouple.

3. It permits the use of joining materials (like soldering) to form thermocouple junctions without effecting the performance of the junction.

Law of intermediate temperature

Statement: the sum of the emf developed by a thermocouple with its junctions at temperatures T_1 and T_2, and with nits junctions at temperatures T_2 and T_3, will be the same as the emf developed if the thermocouple junctions are at temperatures T_1 and T_3

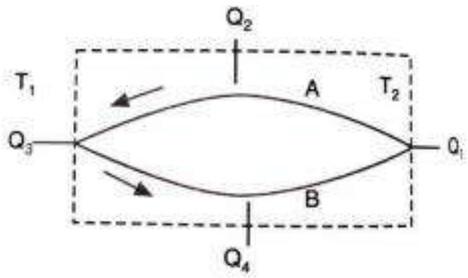


Law of Intermediate Temperatures

Practical significance: This law, illustrated in figure, is useful in practice because it helps in giving a suitable correction in case of a reference junction temperature other than 0^oC is employed. For example, if a thermocouple is calibrated with reference junction at 0^oC and is with a junction temperature of 20^oC, then correction required for the observation would be the emf produced by the thermocouple between 0^oC and 20^oC.

Expression for thermo emf in terms of T_1 and T_2:

Consider a circuit consisting of two metals A and B as shown in Fig. The hot junction is at a temperature *T2ºK* and the cold junction is at a temperature *T1K*. Due to the Seebeck effect, i.e., due to temperature difference between the junctions, thermoelectric current flows through the circuit. As the current flows through the hot and cold junctions, heat is absorbed at the hot junction and evolved at the cold junction due to Peltier effect.



Let π_1 and π_2 be the Peltier coefficients at T_1 and T_2. During the passage of current an amount of heat energy equal to $\pi 2q$ is absorbed at hot junction and heat energy π1q is evolved at the cold junction. Then, the energy (π2-π1)q is used in driving the current through the circuit. As π2 and π1are equal to the potential differences at hot and cold junctions respectively, the thermo e.m.f. developed is given by e= (π_2-π_1) The current in the circuit is small, and the joules heating effect is negligible. As Peltier effect is reversible, a thermocouple may be regarded as a reversible heat engine taking heat from the source at the hot junction at temperature T2, does work in driving the current through the circuit, and rejecting heat to the sink, the cold junction at temperature T_1. By the Carnot's engine we have $Q_2 / T_2 = Q_1 / T_1$

Now during the flow of current in the thermocouple, heat absorbed at the hot junction is $Q_2 = \pi_{2q}$ joule while the energy given out to the sink is $Q_1 = \pi_{1q}$ joule

$$\frac{\pi_2 q}{T_2} = \frac{\pi_2 q}{T_1} \qquad \therefore \frac{\pi_2}{\pi_1} = \frac{T_2}{T_1} \qquad \therefore \frac{\pi_2}{\pi_1} - 1 = \frac{T_2}{T_1} - 1$$

Or $$\frac{\pi_2 - \pi_1}{\pi_1} = \frac{T_2 - T_1}{T_1}$$

Or $$\pi_2 - \pi_1 = \pi_1 \left(\frac{T_2 - T_1}{T_1} \right)$$

But $\pi_2 - \pi_1 = e$ $\qquad \therefore e = \frac{\pi_1}{T_1}(T_2 - T_1)$

If the cold junction temperature is held constant, the Peltier coefficient π_1 will also be constant. Then

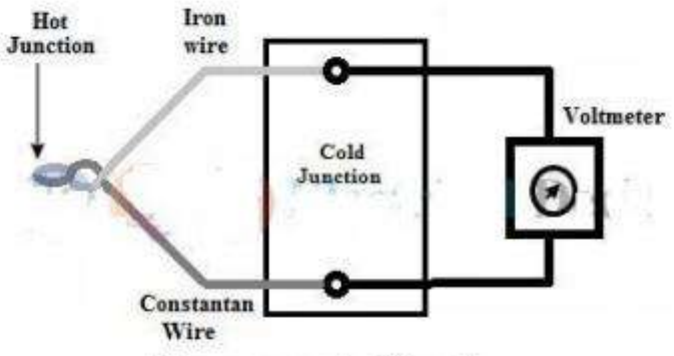
$$e \propto (T_2 - T_1)$$

Thermo couple

Description:

A thermocouple is a transducer that converts thermal energy into electrical energy and is constructed by joining wires made from dissimilar metals to form a junction. Voltage is produced when the temperature at the junction changes. Principle: The concept of the thermocouple is based on the Seebeck Effect, which states that if dissimilar metals are joined at a point, they will generate a small measurable voltage when the temperature of the point of connection changes. The amount of voltage depends on the amount of temperature change and the characteristics of the metals. Construction:

Thermocouples are constructed by two different metals that exist in the form of wires. The two ends are joined by twisting the two wires and welded them together. The figure shows the thermocouple formed by two dissimilar metalsi.e, Iron andConstanton.A protective sealing is provided around the junction and a portion of extension leads. Generally, a diameter of wire ranging from 1.5 to 3mm is used for base metals and a diameter of 0.5mm wire is used for noble metals.



Thermocouple Circuit

Working:

In a thermocouple transducer, out of two junctions, one junction is referred to as hot junction or measuring junction, which is placed at the process media where the temperature is to be measured. Another junction is referred to as a cold junction or reference junction is maintained at a constant reference temperature. When there exists a temperature difference between hot and cold junctions an emf will be set up at the free ends due to temperature gradient and is measured by millivoltmeter. The amount of induced emf depends upon the difference in temperature between two junctions and the material used to build the thermocouple. The temperature is determined by calibrating the millivoltmeter. Since the cold junction is at 0°C, the induced emf measured by the voltmeter is the function of the temperature of the hot junction. It is essential to keep the reference junction at 0°C to avoid errors due to change in room temperature.

Advantages of Thermocouple:

• It is an active transducer i.e., it operates without any external power source.

• Measurement of wide ranges of temperature from -200°C to 2800°C. • The response time is fast, which can measure fast-changing temperatures.

• The cost of thermocouples is low compared to thermistors.

• Able to measure temperatures at desired points. Disadvantages of Thermocouple:

• The output voltage produced is low.

• The stray magnetic field can introduce errors in output voltage.

• Accuracy is low.

Thermopile:

Description: A thermopile is an electronic device that converts thermal energy into electrical energy. It is composed of several thermocouples connected usually in series or, less commonly, in parallel.

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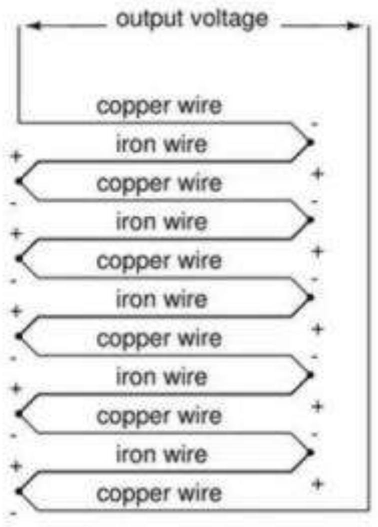
Principle: Principle is thermoelectric effect, i.e., generating a voltage when its dissimilar metals (thermocouples) are exposed to a temperature difference i.e Seeback effect

Construction: The structure of the thermopile is shown in figure. The output voltage of a single thermoelectric cell is extremely small. So a number of these cells is connected in series/parallel to get a larger signal output. The arrangement of this thermocouple stack is called "thermopile". To make a thermopile, we need to connect more thermocouple pairs in series, so that it increases the output voltage. Thermopiles are designed with a set of thermocouples which includes dual thermocouple junctions otherwise various thermocouple pairs. A thermopile includes a series of thermocouples where each includes two special materials with large thermoelectric power & reverse polarities which are interconnected in series.

Working: These thermocouples are arranged throughout the cold & hot areas of the arrangement where the hot junctions are isolated thermally from the cold junctions. In reply to the temperature variation across the material, the output voltage of the thermopile is called a Seebeck coefficient or thermoelectric coefficient. So it is measured per kelvin (V/K) otherwise mV/K in volts.



Thermopile Structure

Thermopile Advantages

- It doesn't need an external power supply.
- It gives a stable response to radiation which is gone from temperature-measuring bodies.
- It has stable response characteristics.
- Thermopile is a non-contact temperature-detecting device that uses IR radiation to transfer heat.
- These are available in small sizes.
- It is less costly.
- It generates larger o/p voltage because of the usage of several thermocouple devices.

Thermopile disadvantages

- These are static, so not used ones should be stored within conductive material to defend them from static discharges & static fields.
- These can be damaged due to stress and reverse the polarity of the supply.

• These should not be directly exposed to moisture or sunlight because this may harm or will have corrosion on the device's performance.

• This device should not be operated with dirty or oily fingers because this dust will affect the device's performance. For superior performance, we need to clean with cotton swabs or alcohol.

• For precise temperature measurement, an object should fill the field of view completely of the thermopile device.

Thermoelectric generators (TEG):

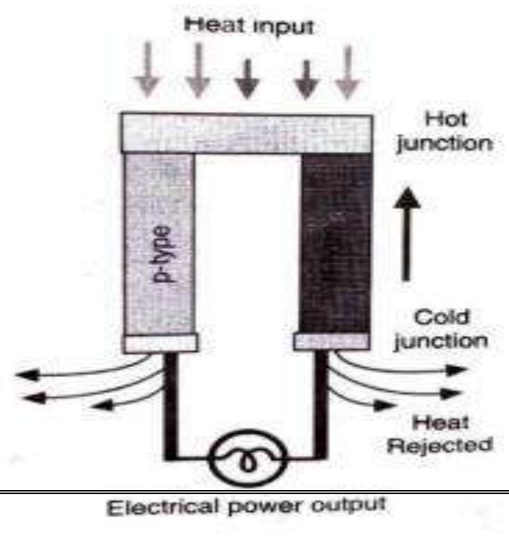
Thermoelectric is the name which is the combination of words electric and thermo. So, the name signifies that thermal corresponds to heat energy and electricity corresponds to electrical energy. And thermoelectric generators are the devices that are implemented in the conversion of the temperature difference that is generated between the two sections into the electrical form of energy. This is the basic thermoelectric generator definition.

These devices are dependent on the thermoelectric effects which involve interface that happens between heat flow and the electricity through solid components.

Principle: The Seebeck effect forms the basis for power generation. Thermoelectric generators convert heat energy to electricity. When a temperature gradient is created across the thermoelectric device, a DC voltage develops across the terminals. When a load is properly connected, electrical current flows. Typical applications for this technology include providing power for remote telecommunication, navigations, and petroleum installations.

As early as 1929, A. F. Iofe (1880-1960) showed that a thermoelectric generator utilizing semiconductors could achieve a conversion efficiency of 4%, with further possible improvement in its performance.

Construction: The simplest thermoelectric generator consists of a thermocouple, comprising a p-type and n-type thermo-element connected electrically in series and thermally in parallel (Fig).The P-type and N-type semiconductors are interconnected through a metal. Load is connected to free end of P and N type semiconductors. To design such thermoelectric generators, semiconductors are used which have high electrical conductivity and low thermal conductivity.



Working: Heat is pumped into one side of the couple and rejected from the opposite side. The electrons present at the hot end would be

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at a high energy level as compared to electrons present at the cool end side. This means that the hot electrons will tend to move towards the cool end due to the temperature gradient. When a temperature gradient is produced between two ends, the electrons start flowing from one end to another end and create a potential difference. An electrical current is produced, proportional to the temperature gradient between the hot and cold junctions. Of the great number of materials studied, semiconductors based on bismuth telluride, lead telluride and silicon-germanium alloys are found to be the best.

Thermoelectric Generator Applications:

- For enhancing the fuel performance of cars, the TEG device is mostly employed. These generators make use of heat that is generated at the time of vehicle operation
- Seebeck Power Generation is utilized to provide power for the spacecraft.
- Thermoelectric generators to implemented provide power for the remote stations such as weather systems, relay networks, and others

THERMOELECTRIC COOLERS

Thermoelectric coolers are solid state heat pump used in applications where temperature stabilization, temperature cycling, or cooling below ambient are required.

Principle: The principle used in this is Peltier effect. i.e: ‘when electric current passed in a circuit consisting of two dissimilar metals, heat is evolved at one junction and absorbed at the other junction.’

Construction: A thermoelectric cooling arrangement is a shown in figure. It consists of a thermoelectric module, a heat sink and the object to be cooled. A typical thermoelectric module consists of an array of **bismuth telluride** semiconductor pellets that have been “doped” so that one type of charge carrier-either positive or negative carriers the majority of current. The pairs of P/N pellets are configured so that they are connected electrically in series, but thermally in parallel. **Metalized ceramic substrates** provide the platform for the pellets and the small conductive tabs that connect them. The ceramic material on both sides of the thermoelectric adds rigidity and the necessary electrical insulation. The pellets, tabs and substrates thus form a layered configuration. Module size varies from less than 0.25” by 0.25” to approximately 2.0” by 2.0”.Thermoelectric modules can function singularly or in groups with either series, parallel, or series/parallel electrical connections. Some applications use stacked multi-stage modules

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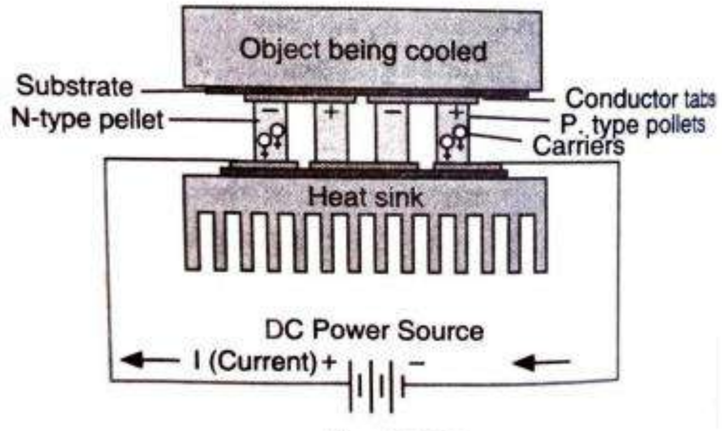

Working: When DC voltage is applied to the module, the positive and negative charge carriers in the pellet array absorb heat energy from one substrate surface and release it to the substrate at the opposite side. The surface where heat energy is absorbed becomes cold the opposite surface where heat energy is released, becomes hot.

These devices cannot only pump appreciable amount of heat, but with their series electrical connection, are suitable to be used as DC power supplies. Thus, the most common thermoelectric devices now in use connecting 254 alternating P and N-type pellets can run from a 12 to 16 VDC supply and draw only 4 to 5 amps. A means to mechanically hold everything together is to mount the conductive tabs to thin ceramic substrates (Fig) the outer faces of the ceramics are then used as the thermal interface between Peltier device and the ‘outside world’. Ceramic materials represent the best compromise between mechanical strength, electrical resistivity, and thermal conductivity

Thermoelectric materials:

Thermoelectric (TE) materials have the capability of converting heat into electricity, which can improve fuel efficiency as well as provide a robust alternative energy supply in multiple applications by collecting wasted heat, and therefore assist in finding new energy solutions.

Classification: The thermoelectric materials can be divided into the following three categories according to their operating temperature. They are low, mid and high temperature thermoelectric materials.

1.Low temperature thermoelectric materials: Bismuth telluride and its alloys. This is a material widely used in thermoelectric coolers, and its optimal operating temperature is<450°C.

Bismuth (Bi), Antimony (Sb), and the Bi–Sb alloys form a complete class of thermoelectric semiconductors that are particularly suited to thermoelectric applications below room temperature. Thermoelectric materials allow direct conversion of waste heat energy into electrical energy, thus contributing to solving energy related issues. Polymer-based materials have been considered for use in heat conversion in the temperature range from 20 to 200 °C, within which conventional materials are not efficient enough, whereas polymers due to their good electronic transport properties, easy processability, non-toxicity, flexibility, abundance, and simplicity of adjustment, are considered as promising materials.

2. Mid temperature thermoelectric materials: Lead telluride and its alloys. This is a material widely used in thermoelectric generators, and its optimal operating temperature is about 1000°C. Mg_2Sn is a potential mid-temperature thermoelectric material.

3.High temperature thermoelectric materials: Silicon-germanium alloy. This type of material is also commonly used in thermoelectric generators, and its optimal operating temperature is about 1300°C. Thermoelectric materials, which can be applied in highly efficient cooling and refrigeration, energy scavenging, sensing, and thermo power systems, can make significant contributions to solve the global energy crisis by providing a sustainable energy solution. Metal oxides have become important thermoelectric materials due to their high-temperature stability, tunable electronic and phonon transport properties, and well-established synthesis techniques. In this chapter, Na_xCoO_2, $Ca_3Co_4O_9$, $SrTiO_3$, $CaMnO_3$, and ZnO are reviewed as promising metal oxide-based thermoelectric materials.

Applications: Exhaust of Automobiles, Refrigerator, Space Program (RTG):

Application in exhaust of Automobiles: Automotive thermoelectric generator (ATEG) technology involves converting the waste heat available in the exhaust gas in internal combustion engine into electricity using Seebeck effect. That electricity can be stored and utilized for various electrical inputs of a vehicle so that the fuel efficiency can be improved. ATEG is gaining significant importance since a direct conversion of exhaust waste heat into electricity allows for a reduction in fuel consumption. The role of exhaust flow rate, temperature and heat exchanger will affect on the performance of ATEG.

A typical ATEG consists of four main elements A hot-side heat exchanger,a cold side heat exchanger, thermoelectric materials and a compression assembly system. In ATEG, thermoelectric materials are packed between the hot-side and cold-side heat exchangers. The thermoelectric materials are made up of p-type and n-type semiconductors. The heat exchangers are metal plates with high thermal conductivity. The temperature difference between the two surfaces of the thermoelectric modules generates electricity using Seebeck effect. When hot exhaust from the engine passes through an exhaust ATEG, the charge carriers of the semiconductors within the generator diffuse from the hot-side heat exchanger to cold-side exchanger. The build-up of charge carriers results in net charge, producing an electrostatic potential while the heat transfer drives a current. With exhaust temperature of 7000 c or more, the temperature difference between exhaust gas on the hot side and coolant on the

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cold side is several hundred degrees. This temperature difference is capable of generating 500-750 W of electricity

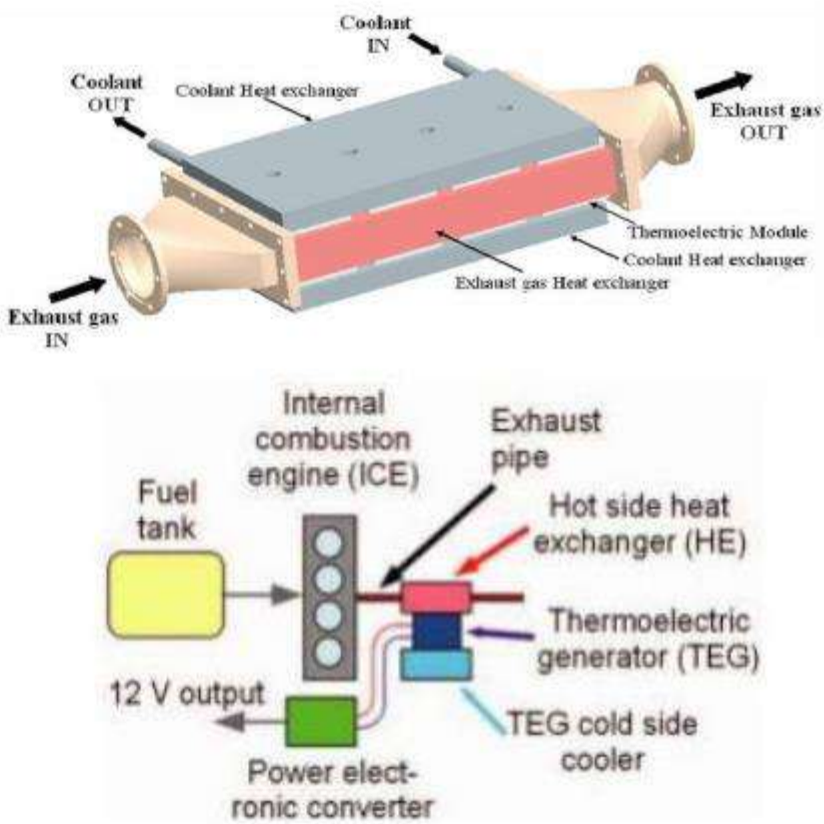


Fig: Layout of the main components of Exhaust gas Heat recovery system with TEG

Refrigerator: Thermoelectric cooling is a way to remove thermal energy from a medium, device or component by applying a voltage of constant polarity to a function between two dissimilar semiconductors or electrical conductors. Thermoelectric cooling uses the Peltier effect to create a heat flux between the junction of two different types of materials. This effect is commonly used in camping and portable coolers and for cooling electronic components and small instruments. The device has two sides, and when a DC electric current flows through the device, it brings heat from one side to the other, so that one side gets cooler while the other gets hotter. The major components of the refrigerator are given below.

The thermoelectric devices used in thermoelectric refrigeration (or thermoelectric coolers) are based on the Peltier effect to convert electrical energy into a temperature gradient [1]. A conventional thermoelectric cooler is composed of a number of N-type and P-type semiconductor junctions connected electrically in series by metallic interconnects (conducting strips, in general made of copper) and thermally in parallel, forming a single-stage cooler. If a low-voltage DC power source is applied to a thermoelectric cooler, heat is transferred from one side of the thermoelectric cooler to the other side. Therefore, one face of the thermoelectric cooler is cooled and the opposite face is heated.

Fig. 1 depicts a thermoelectric cooling module considered as a thermoelectric refrigerator, in which the electrical current flows from the N-type element to the P-type element. The temperature Tc of the cold junction decreases and the heat is transferred from the environment to the cold junction at a lower temperature. This process happens when the transport electrons pass from a low energy level inside the P-type element to a high energy level inside the Ntype element through the cold junction. At the same time, the transport electrons carry the absorbed heat to the hot junction which is at temperature . This heat is dissipated in the heat sink, whilst the electrons return at a lower energy level in the P-type semiconductor (the Peltier effect). If there is a temperature difference between the cold junction and hot junction of N-type and P-type thermoelements, a voltage (called Seebeck voltage) directly proportional to the temperature difference is generated.

The quality of a thermoelectric cooler depends on parameters such as the electric current applied at the couple of N-type and P-type thermoelements, the temperatures of the hot and cold sides, the electrical contact resistance between the cold side and the surface of the device, the thermal and electrical conductivities of the thermoelement, and the thermal resistance of the heat sink on the hot side of the thermoelectric cooler. The number of thermoelements in a thermoelectric module mainly depends on the required cooling capacity and the maximum electric current. The characteristics and performance of a thermoelectric refrigerator are described by parameters like the figure of merit, the cooling capacity, and the coefficient of performance. This review is specifically focused on these parameters, addressing the concepts in a different way with respect to various review papers appearing on thermoelectric cooling in the past years. Specific aspects such as thermoelectric cooling system design, experimental assessment, numerical analysis and simulation are outside the scope of this review

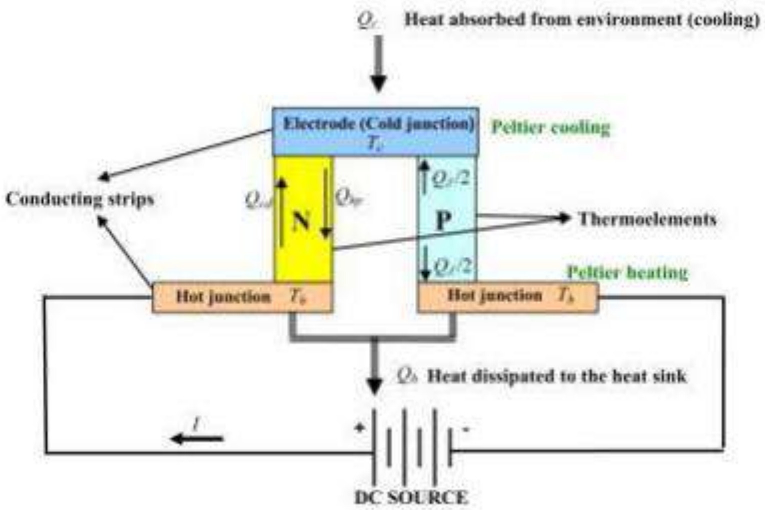
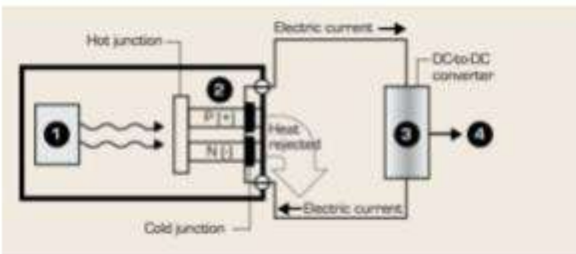


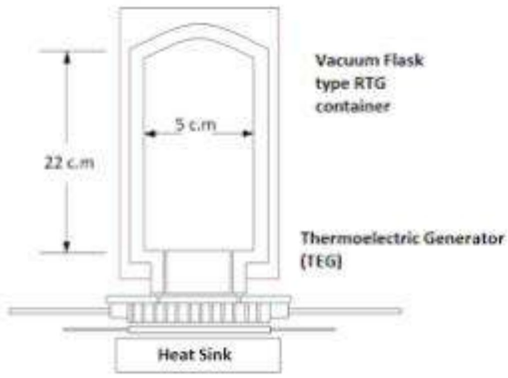
Fig. 1. Scheme of a thermoelectric refrigerator.

Space Program (Radioisotope Thermoelectric Generators)

A major practical application of thermoelectric generators has been in the space program, where the need is for an electric generator with no moving parts and which will supply power for the long duration of the space missions. A radioisotope thermoelectric generator (RTG, RITEG), sometimes referred to as a radioisotope power system (RPS), is a type of nuclear battery that uses an array of thermocouples to convert the heat released by the decay of a suitable radioactive material into electricity by the Seebeck effect. RTGs provide electrical power using heat from the natural radioactive decay of plutonium-238, in the form of plutonium dioxide. In the isotopic decay process, alpha particles are released which bombard the inner surface of the container. The energy released is converted to heat and is the source of heat to the thermoelectric converter. The large difference in temperature between this hot fuel and the cold environment of space is applied across special solid-state metallic junctions called thermocouples, which generates an electrical current. This type of generator has no moving parts.

RTGs have a long operating life, are reasonably lightweight, and require little or no maintenance once assembled and tested. However, because RTGs contain significant quantities of radioactive materials, normally plutonium238 and its decay products, they must be transported in packages.RTG is made up of a radioisotope heat source, a thermoelectric converter, a gas pressure venting system, temperature transducers, connectors, a heat rejecting cylindrical container, and bracketery.





Questions:

1) Explain variation of Variation of Thermoelectric emf with temperature and obtain the relation between inversion temperature and neutral temperature.
2) State and explain Laws of thermoelectricity.
3) Describe the Seebeck effect and and Seebeck coefficient.
4) Describe Peltier effect and Peltier coefficient.
5) Explain the construction working of thermocouple. Also discuss on its advantages and limitations

6) Explain the construction working of thermopiles. Also discuss on its advantages and limitations. (8)

7) Describe the construction and working thermoelectric generator (TEG).

8) Describe the construction and working thermoelectric cooler.

9) Define thermo emf, thermo current, Neutral temperature, thermo electric power, thermocouple and thermopile.

10) Explain thermoelectric power and figure of merit.

11) Derive expression for thermo emf in terms of T1 and T2.

12) What are thermoelectric materials? Explain Low, mid and high temperature thermoelectric materials

13) Explain applications of thermoelectricity on exhaust of automobiles.

14) Explain applications of thermoelectricity on refrigerator. (7)

15) Explain applications of thermoelectricity on Space Program (RTG) (6)

Numerical problems

1. EMF of a thermocouple is 1200μV, when working between 0 ^{o}C and 100^{o}C . Its neutral temperature is 300^{o}C. Find the values of a and b for it.
2. The thermo emf of a Cu-Fe thermocouple is 2160μVwhen the cold junction is at 0 ^{o}C and hot junction at 250^{o}C. Calculate the constants a and b if the neutral temperature is 330oC.
3. For Fe-Cu thermocouple it is observed that the thermo emf is zero when one of the junctions is at 20^{o}C and the other one is at some higher temperature. If the neutral temperature is 285^{o}C, calculate the higher temperature. Hence find out the temperature of inversion, if the cold junction temperature is at -20^{o}C.
4. The e.m.f. in micro volts (e) of a thermocouple, one junction of which is at 0 ^{o}Ce is given by e = 1600 T – 4T^{2} where T ^{o}Cis the temperature of hot junction. Find (i) neutral temperature (ii) Peltier Coefficient.
5. . The e. m. f. in lead – iron thermocouple, one junction of which is at 0 ^{o}C, is given by E = 1784 t – 2.4 t^{2} ($in\ \mu\ volts$) where t is temperature in ^{o} C. Find the neutral temperature and π
6. In a thermocouple, the neutral temperature is 270^{o}C and the temperature of inversion is 525^{o}C. Calculate the temperature of cold junction

MODULE 4- CRYOGENICS

LOW TEMPERATURES IN SCIENCE AND TECHNOLOGY

Cryogenics is defined as that branch of physics which deals with the production of very low temperatures and their effect on matter, a formulation which addresses both aspects of attaining low temperatures which do not naturally occur on Earth, and of using them for the study of nature or the human industry. In a more operational way, it is also defined as the science and technology of temperatures below 120 K. The limit temperature of 120 K comprehensively includes the normal boiling points of the main atmospheric gases, as well as of methane which constitutes the principal component of natural gas. Today, liquid natural gas (LNG) represents one of the largest – and fast-growing – industrial domains of application of cryogenics, together with the liquefaction and separation of air gases.

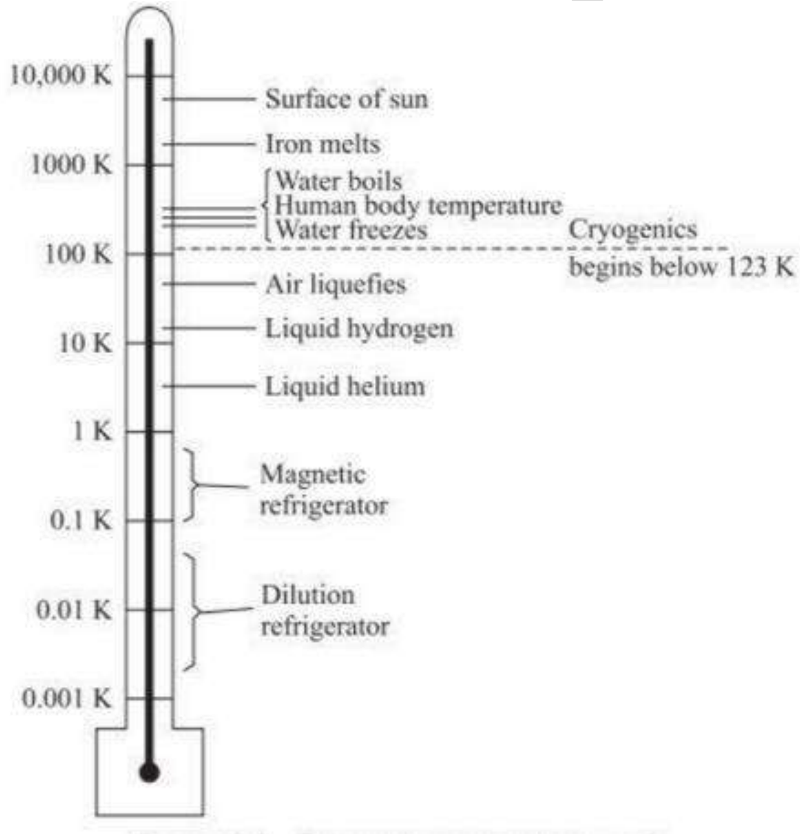


Fig: Cryogenic temperature scale

Joule – Thomson's effect

Statement: If a gas initially at constant high pressure is allowed to suffer throttle expansion through the porous plug of silk, wool or cotton wool having a number of fine pores, to a region of constant low pressure adiabatically, a change in temperature of gas (either cooling or heating) is observed. This effect is called as Joule –Thomson or Joule-Kelvin effect.

Theory of Joule Thomson effect:

Suppose that one mole of gas is allowed to expand through a porous plug from a pressure P_1 and volume V_1 to a pressure P_2 and volume V_2. Let the temperature change from $T_1$to

T_2 due to Joule-Thomson effect.

Net external work done by the gas = $P_2V_2 - P_1V_1$, -------------------------------- (1)

Now, an internal work is also done by the gas in overcoming the forces of molecular attraction. For a van der Waals gas, the attractive forces between the molecules are equivalent to an internal

Pressure= $\frac{a}{V^2}$, a is a constant

Internal work done by the gas when the gas expands from a volume V_1 to V_2 is $= \frac{a}{V^2}$

Internal work done by the gas when the gas expands from a volume V_1 to V_2 is

$$\int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{a}{V^2} dV = \left[-\frac{a}{V}\right]_{V_1}^{V_2} = \frac{a}{V_1} - \frac{a}{V_2} \ \ldots\ldots\ldots (2)$$

Total work done by the gas = external work +internal work

$$W = P_2V_2 - P_1V_1, \ +\frac{a}{V_1} - \frac{a}{V_2} \text{-------}(3)$$

Now, van der Waals equation of state for a gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

or $PV = RT + Pb - \frac{a}{V}$ (neglecting $\frac{ab}{V^2}$)

$$P_1V_1 = RT_1 + bP_1 - \frac{a}{V_1} \quad \text{and} \quad P_2V_2 = RT_2 + bP_2 - \frac{a}{V_2}$$

Substituting these values for P_1V_1 and P_2V_2 -in eqn (3) we get,

$$w = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2a}{V_1} - \frac{2a}{V_2} \ \ldots\ldots (4)$$

Since a and b are very small, PV=RT or V=RT/P.

Hence, $V_1 = RT_1/P_1$ and $V_2 = RT_2/P_2$.

As T_1 and T_2 are nearly equal, we may write T_1=T_2=T and hence $V_1 = \frac{RT}{P_1}$ and $V_2 = \frac{RT}{P_2}$

Substituting in eq(4) we have, $w = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2a}{RT}[P_1 - P_2]$

Let $T_1 - T_2 = \partial T$. Then

$$W = -R\,\partial T - b(P_1 - P_2) + \frac{2a}{RT}(P_1 - P_2)$$

$$W = (P_1 - P_2)\left(\frac{2a}{RT} - b\right) - R\,\partial T. \text{ \_-------}(5)$$

Since the gas is thermally insulated, the energy necessary for doing this work is drawn from the K.E. of the molecules. Hence, the K.E. decreases resulting in a fall of temperature by ∂T.

Heat lost by the gas = $C_v\,\partial T$

$$C_v\,\partial T = (P_1 - P_2)\left(\frac{2a}{RT} - b\right) - R\,\partial T$$

$$\text{Or} \qquad \partial T(C_v + R) = (P_1 - P_2)\left(\frac{2a}{RT} - b\right)$$

$$\text{i.e,} \qquad \partial T C_p = (P_1 - P_2)\left(\frac{2a}{RT} - b\right)$$

$$\delta T = \frac{(P_1 - P_2)}{C_p}\left(\frac{2a}{RT} - b\right) \qquad \ldots\ldots(6)$$

$$\text{Joule THomsan coeffecient,} \qquad \mu_{JT} = \frac{\delta T}{\delta P} = \frac{1}{C_p}\left(\frac{2a}{RT} - b\right)$$

Eqn (6) gives the fall in temperature or the cooling produced in a van der Waals gas whensubjected to throttling process.

i) If $\frac{2a}{RT} > b$ then δT is positive. Hence there will be a cooling effect.

ii) If $\frac{2a}{RT} < b$ then δT is negative. Hence there will be a heating effect.

iii) If $\frac{2a}{RT} = b$ then δT =0. Hence there will be neither a heating nor a cooling effect.

Note: For a gas temperature that is above the inversion temperature, the μ_{JT} would be negative. The ∂P shall be always negative in this case, which means that the ∂P must be positive. Consequently, the warming of the gas will take place.

Inversion Temperature:

The temperature at which the Joule –Thomson effect changes sign is called the temperature of Inversion. (T_i). At this temperature $\frac{2a}{RT_i} = b$ or $T_i = \frac{2a}{Rb}$. Thus, above the temperature of inversion, Joule- Thomson effect will be a heating effect and below it a cooling effect.

Porous Plug experiment

Construction:

Joule in collaboration with Thomson [Lord Kelvin] devised a very sensitive technique known as Porous Plug experiment. The experiment set up of porous plug experiment to study the Joule-Thomson effect is shown in Fig.2.1. It consists of the following main parts:

(a) A Porous plug having two perforated -brass discs D & D_1.

(b) The space between D & D_1 is placed with cotton wool or silk fibers.

(c) The porous plug is fitted in a cylindrical box-wood W which is surrounded by a vessel containing cotton wool. This is to avoid loss or gain of heat from the surroundings.

(d) T_1 &T_2 are two sensitive platinum resistance thermometers and they measure the temperatures of the incoming and outgoing gas.

(e) The gas is compressed to a high pressure with the help of piston P and it is placed through a spiral tube immersed in water bath maintained at a constant temperature. If there is any heating of the gas due to compression, this heat is absorbed by the circulating water in the water bath.

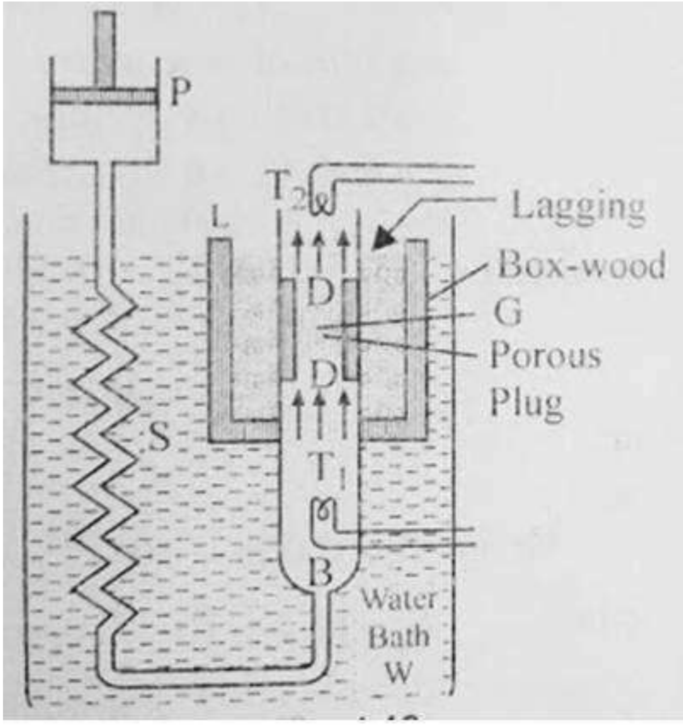


Fig.2.1

Experimental Procedure

The experimental gas is compressed by Pump P and is passed slowly and uniformly through the porous plug keeping the high pressure constant read by pressure gauge. During the passage through the porous plug, the gas is throttled. The separation between the molecules increases. By passing through the porous plug, the volume of the gas increases against the atmospheric pressure. As there is no loss or gain of heat during the whole process, the expansion of the gas takes place adiabatically. The initial and final temperatures are noted by platinum resistance thermometers T_1 & T_2.

Experimental Results

A simple arrangement of porous plug experiment is shown in Fig.2.2 .The behavior of large number of gases was studied at various inlet temperatures of the gas and the results are as follows:

(1) At sufficiently low temperatures, all gases show a cooling effect.

(2) At ordinary temperatures, all gases except hydrogen and helium show cooling effect. Hydrogen and Helium show heating effect.

(3) The fall in temperature is directly proportional to the difference in pressure on the two sides of porous plug.

(4) The fall in temperature for a given difference with rise in the initial temperature of the gas. It was found that the cooling effect decreased with the increase of initial temperature and becomes zero at a certain temperature and at a temperature higher than the temperature instead of cooling heating was observed. This particular temperature at which the Joule – Thomson effect changes sign is called temperature of inversion.

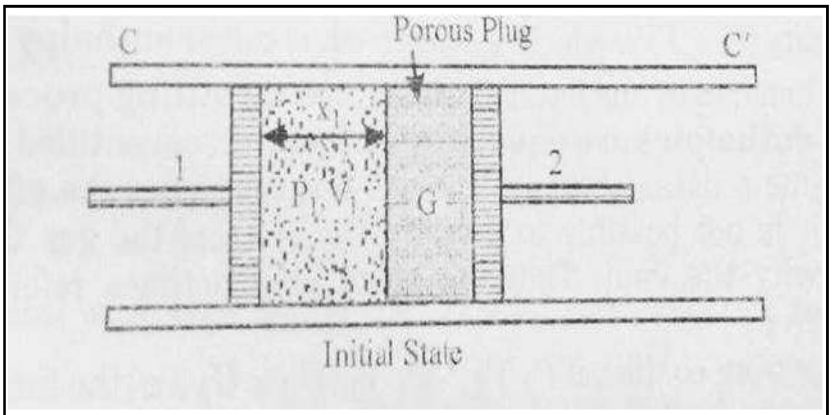


Fig.2.2

Thermo dynamical analysis of Joule Thomson effect:

The arrangement of the porous plug experiment is shown in Fig. The gas passes through the porous plug from the high pressure side to the low pressure side. Consider one mole ofthe gas. Let P_1,V_1 and P_2,V_2 represent the pressure and volume of the two sides of the porous plug. Let dx be the distance through which each piston moves to the right.

The work done on the gas by the piston 1 = $P_1A_1dx = P_1V_1$
The work done by the gas on the piston 2 = $P_2A_2dx = P_2V_2$
Net external work done by the gas = $P_2V_2 - P_1V_1$

Let w be the work done by the gas in separating the molecules against their inter-molecular attraction.

Total amount of work done by the gas = $(P_2V_2 - P_1V_1) + w$

There are three cases depending upon the initial temperature of gas.

(i) Below the Boyle temperature: $P_2V_2 < P_1V_1$. Then $P_2V_2 - P_1V_1$ is +ve. Hence **w** must be positive.Thus, a net +ve work is done by the gas. Hence, there must be a cooling effect.
(ii) At the Boyle temperature: $P_2V_2 = P_1V_1$, Then $P_2V_2 - P_1V_1 =0$. The total work done by the gas in this case is w. Therefore, cooling effect at this temperature is only due to the work done by the gas in overcoming inter-molecular attraction.
(iii) Above the Boyle temperature: $P_2V_2 > P_1V_1$. Then $P_2V_2 - P_1V_1$ is –ve. Hence w must be negative. Thus, a net -ve work is done by the gas. Hence, there must be a heating effect.

Thus, the observed effect will depend upon whether $(P_2V_2 - P_1V_1)$ is greater than or less than w.

If $w > (P_2V_2 - P_1V_{1,})$, cooling will be observed.

If $w < (P_2V_2 - P_1V_{1,})$, heating will be observed.

Thus, the cooling or heating of a gas depends on

(i) The deviation from Boyle's law

(ii) Work done in overcoming inter-molecular attraction.

Note: Boyle temperature can be defined as **the point in the temperature range in which a real gas starts to behave like an ideal gas at a pressure range**.

Liquefaction of gases

Definition: **Liquefaction of gases means the process into which the gas substances are converted from gases to a liquid state.**

Principles of Liquefaction of gases

Principle1: In which when a gas is compressed by a sufficient amount of pressure below its critical temperature, as a result liquefaction starts.

Principle2: When we reduce the pressure, and the gas or the liquid is allowed to evaporate, then due to evaporations, it causes cooling.

Principle3: On the basis of the **Joule Thomson effect** (Porous plug experiment).

Liquefaction of Oxygen gas by cascade process (Pictet process):

This process was first used by **Pictet** in 1878. He successfully obtained a small quantity of Liquid Oxygen with the help of pressure applied, and with other liquefied gases.

Cascade system or Process: A process is called the Cascade process, When a single stage is not enough to produce the desired result, therefore the process takes place in a number of stages in a sequence.

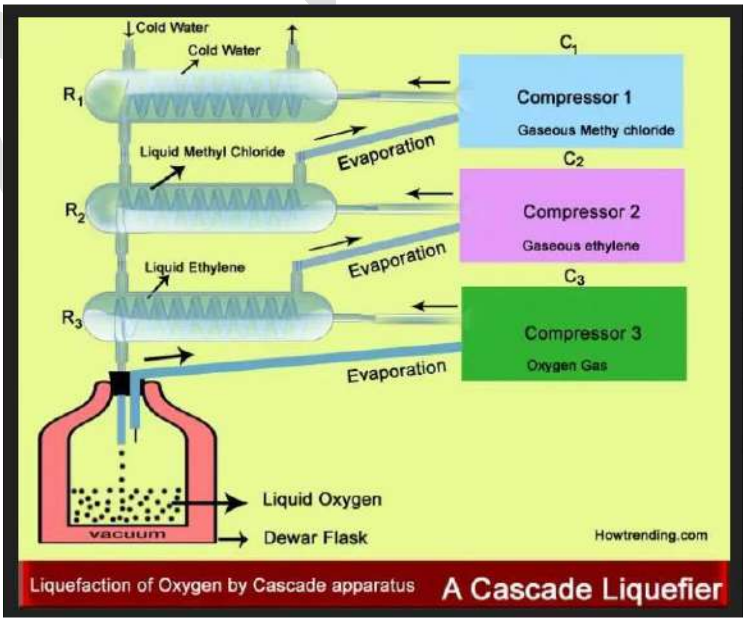


Fig: A Cascade Liquefier

Construction:

1. In this apparatus, three compressors C_1, C_2, C_3 are used to fulfill the requirement of sufficient pressure. Also, the C_1, C_2, and C_3 have a suction side which is used during the process.

2. Three condensers R_1, R_2, R_3 are used, into which three refrigerants cold water, Methyl chloride, and ethylene are used to get the desired result.

3. The Liquid oxygen is collected in the last, into a Dewar flask.

Principle: This apparatus work on two principles.

1. The first, Principle, compression of gases below its critical temperature resulting in a change to liquid.

2. The second is, producing cooling by the principle of evaporation of liquids.

Working:

- The gaseous methyl chloride (CH_3Cl) is pumped by the compressor C_1 into the spiral tube. The refrigerant in condenser R_1 surrounding this tube starts liquefying the methyl chloride.

- This is because the critical temperature of methyl chloride is 143°C, which is more than room temperature as well.

- Now the liquid methyl chloride comes in Condenser R_2 through the tube. Here one portion of condenser R_2 is connected with the suction side of compressor C_1.

- Here due to the evaporation of liquid methyl chloride in reduced pressure, more cooling as a result produced, and the temperature of condenser R_2 decreases more.

- The evaporated methyl chloride return back to the compressor C_1 through the suction side of the compressor.

- Now the gaseous ethylene (C_2H_4) is pumped by the compressor C_2 into the next spiral tube.

- Here the refrigerant, liquid methyl chloride which is achieved in the previous stage, surrounding the tube which contains gaseous ethylene, starts to convert this gas into liquid ethylene.

- This is because the critical temperature of ethylene is around 9.2°C.

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- Now, this liquid ethylene comes in Condensor R_3, and one portion of R_3 condenser is connected with the suction side of compressor C_2.

- Here evaporation of liquid ethylene takes place in reduced pressure like in the previous stage, and the evaporated ethylene return back to the compressor C_2 through the suction side of the compressor.

- Therefore, due to the evaporation process more cooling is produced into the condenser R_3, which is more than the cooling that we achieved in Condenser R_2.

- This cooling has a temperature of around −160°C.

- Now, the oxygen (which is in gaseous form) is pumped by the compressor C_3 into the next spiral tube.

- Here, due to the very low temperature inside the Condenser R_3 the oxygen gas into the spiral tube starts converting into liquid and later collected into a Dewar flask.

- This is because the critical temperature of oxygen gas is around −118°C.

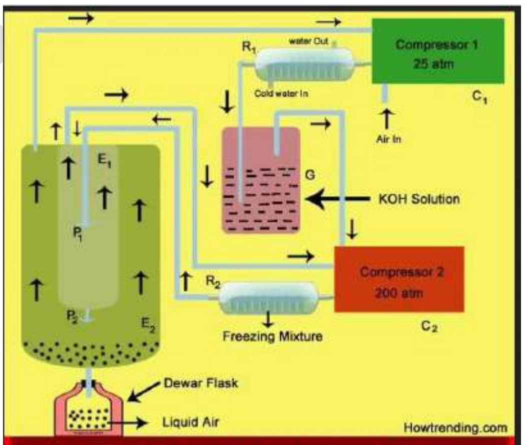
- Here, likewise the previous stages, the evaporated oxygen return back to the compressor C_3 through the suction side of the compressor.

- If we continue this cascade system, we can liquefy air and other gases like Nitrogen, etc.

Limitation: By this system, we cannot liquefy the gases that have very low critical temperatures, such as Hydrogen (T_c around −240 °C) and Helium (T_c around −267.8 °C).

Linde's Method of liquefaction of gases.

The Hampson-Linde cycle or the Linde's liquefaction process is used by coupled with regenerative cooling and the Joule Thomson effect.
By this method, we can easily liquefy air, and many other gases too.



Linde's Method of Liquefaction of Air

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Linde's Method of Liquefaction of Gases

By this figure, you can understand that liquefaction of air or those gases that have a low value of critical temperatures is hard, as compared to those that have high critical temperature values.

Construction:

1. In this method, two compressors C_1 at (25 atm pressure) and C_2 (200 atm pressure) are used.
2. Heat exchangers R_1 and R_2 are used into which cold water and a freezing mixture is used as a refrigerant.
3. A Liquid solution of KOH (Potassium Hydroxide) that is required to get pure air.
4. Two chambers E_1 and E_2, and P_1 and P_2 are the two small nozzles.
5. At last, the liquid air is collected into a Dewar flask.

Principle: Linde's process of liquefaction is work on the principle of the **Joule Thomson effect** coupled with **regenerative cooling**.

Working:

1. The air is pumped at a pressure of 25 atm into the spiral tube. The air gets cooled after passing through the R_1 heat exchangers. Here the gas becomes cool because of cool water inside the R1 heat exchangers. This cooled air then passes through a liquid solution of Potassium hydroxide (KOH).

2. The reason for the use of the KOH solution is that air contains many gases and water vapours too. To separate air from water vapours this solution is used, and also this solution absorbs CO_2 gas from the air. After this, the air further moves in the second compressor C_2.

3. In the C_2 compressor, the air is pumped at a pressure of 200 atm into the next spiral tube. Now the gas becomes cool again, after passing through the second heat exchanger R_2. Here the gas-cooled because of the Freezing mixture inside the R_2 heat exchangers.

4. Now the temperature of this air decreases to around −20°C (253K). Then this pre-cooled air is allowed to expand through nozzle P_1 in a chamber E_1 and suffers the Joule Thomson effect. Due to this effect, more cooling is produced into the chamber E_1, and pressure reduces to about 50 atm.

5. This cooled air then returns back to the compressor C_2 and where it is again pumped at a pressure of 200 atm into the spiral tube. This air again suffers the Joule Thomson effect, and more cooling is produced in chamber E_1.

6. Repeating some cycles of this process, more and more cooling is produced in chamber E_1. After getting sufficient temperature, the cooled air is allowed to expand through nozzle P_2 in

chamber E_2 and again suffers the Joule Thomson effect, and pressure reduces to about 1 atm.

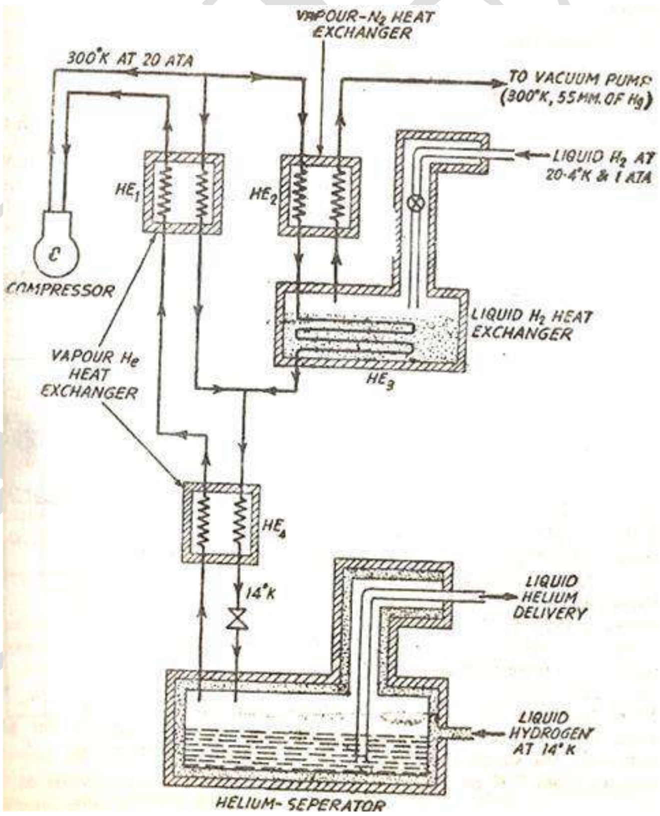
7. Now the temperature decreases to around −188°C(85K) in chamber E_2 and the air gets liquefied. This liquefied air is collected into the Dewar flask.

8. Also, in chamber E_2 the un-liquefied air is returned back to the compressor C_1, this further cooled the air, and where it is again pumped at a pressure of 25 atm into the spiral tube.

The Liquefaction of Helium (Claude's method)

Helium is the only substance which means fluid at temperatures below -259^{o}C (4K) and its inversion point is much lower than that of Hydrogen namely around -233^{o}C(40K). The boiling point of Helium is just around -267^{o}C which is quite close to absolute zero on the Kelvin scale

Principle: Claude's method works on two principles. i.e: **Joule Thomson effect and mechanical expansion** (By, the use of an expansion turbine).



Working:

- Initially Helium is compressed to a pressure of 20 atmospheres which raises its temperature to the region of 300 degrees Kelvin.
- This compressed high temperature Helium is then split into two paths.
- The first part is cooled in the heat exchanger labelled HE1 with the help of Helium

vapours

- The other part passes through the heat exchanger HE2 to be cooled with Hydrogen vapours.
- Both these streams combine to be passed through the liquid Hydrogen heat exchanger HE3
- Then again getting cooled in HE4 by Helium vapours.
- Finally the throttle valve is used to initiate the Joule Thomson effect and Helium is collected in the liquid state in the Helium separator.

Properties and Uses

Though the process described in the above section is generic in nature, in actual practice there are two main isotopes of Helium used for liquefaction namely H4 and H3 and there is a slight difference between the properties of the two in terms of their boiling point, critical temperature and so forth. The properties talked about earlier are those of H4 while the boiling point of H3 is even one degree lower.

Liquid Helium is used extensively for use in superconducting magnets which need to be cooled to extremely low temperatures during their use, that in turn are used in several fields such as say for Magnetic Resonance Imaging and Nuclear Magnetic Resonance.

Platinum Resistance Thermometer (PRT)

The Platinum Resistance Thermometer uses platinum for determining the temperature. It works on the principle of positive temperature coefficient of resistance that is the resistance of platinum increases with increase in temperature. The platinum is a chemically inert metal and can easily be drawn into fine wires. Because of these properties of platinum, it is used as a sensing element in thermometer.

Construction of platinum resistance Thermometer:

The PRT consists of pure platinum wire wound on hollow pipe made up of insulating mica or ceramic, which is placed in porcelain sheath. Free ends of platinum wire are attached to long leads of low resistance copper wires (Fig.1). To measure change in resistance, Wheatstone bridge is used. Two long extension leads form one arm of Wheatstone bridge (fig.2).

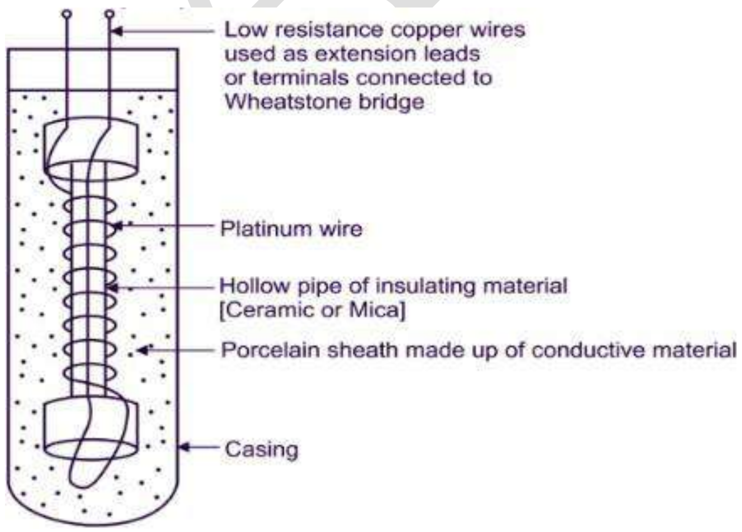


Fig.1: Platinum Resistance Thermometer

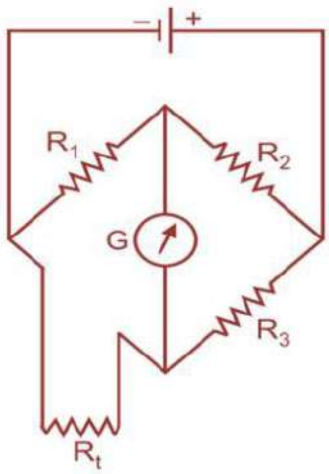


Fig2: PRT Wheatstone bridge

Working of platinum resistance Thermometer:

When platinum resistance Thermometer subjected to temperature variation, the wheatstone bridge gets unbalanced due to change in resistance R_t of platinum resistance. This makes the pointer move over circular scale of galvanometer, which is directly calibrated to give measured value of temperature.

Advantages of Platinum Resistance Thermometer

- Linear relationship of resistance with temperature
- The meter gives the precise reading of temperature.
- The thermometer has a wide range from 200°C to 1200°C.
- The thermometer is quite sensitive.
- **high accuracy (0.01 °C),**
- The platinum gives stable value of resistance at the given temperature.
- Chemically inert

Disadvantages of Platinum Resistance Thermometer

- The thermometer gives the slow response.
- The melting point of the thermometer is 1800°C. But when platinum measures the temperature higher than 1200°C they start evaporating.

Questions

1. What is joule Thomson effect ? Derive the expression for Joule Thomson coefficient, using the theory of Joule Thomson effect.
2. Explain briefly the application of cryogenics.
3. Explain the working of porous plug experiment with neat diagram.
4. Explain the process of liquefaction of Helium
5. Explain the process of liquefaction of oxygen
6. Explain the process of liquefaction of air
7. Explain the construction and working of platinum resistance thermometer.

Numerical problems

1. In Joule Thomson experiment Temperature changes from 100^oC to 150^oC for pressure change of 20 MPa to 170 MPa. Calculate Joule Thomson Coefficient.

Given:
T1=100+273=373 K T2=150+273=423 K , P1=20Pa P2=170 pa
Change in temperature from T2- T1 (K) = 50 K
Change in pressure from P1 to P2 (Pa) = 150MPa

JTC = dT / dP = 50/150M =1/3 μK/Pa

2, Calculate the inversion temperature of gas. Given a=0.244 atomL2/mol2. B=0.027L/mol and R=0.08211 atom/K mol

Ans : Ti=220K

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Module-5

Material Characterization Techniques and Instrumentation

Syllabus :

Introduction to nano materials: Nanomaterial and nanocomposites. Principle, construction and working of X-ray Diffractometer, Crystallite size determination by Scherrer equation, Atomic Force Microscopy (AFM): Principle, construction, working and applications, X-ray photoelectron spectroscopy(XPS), Scanning electron microscopy (SEM), Transmission electron microscopy (TEM), Numerical Problems.

Pre requisites: Quantum Mechanics

Self-learning: Crystallites Course outcome (Course Skill Set

Introduction to Nano science and Nanotechnology

Nanoscience is the study of atoms and molecular structures whose size, at least in one dimension is between 1-100 nm. **Nanotechnology** is the design and fabrication of devices using such nanostructures.

Nanomaterials

Size of a nanomaterial varies from few nanometres to a few hundred nanometres. Both surface effects and size effects plays crucial role in the properties of nanomaterial. The properties exhibited by nonmaterial strikingly different from those of bulk materials.

Mesoscopic State

It is the state of matter the physical properties of a material undergo changes when reaching the limiting size (nanometers) is called Mesoscopic State.

Ex: Carbon in bulk is bad conductor to electricity whereas at nano size it is a very good conductor of electricity

Density of states in 1D, 2D, 3D structures:

The various quantum structures are

1. **Quantum Wells:** In a three dimensional structure, if one dimension is of nano size, then it is called as a "quantum well or quantum film".
2. **Quantum wires:** In a three dimensional structure, if two dimensions are of nano size, then it is called as a "quantum wire".
3. **Quantum Dot:** In a three dimensional structure, if all the three dimensions are of nano size, then it is called as a "quantum dot".

Nanocomposites are materials that incorporate Nano sized particles into a matrix of standard material. The result of the addition of nanoparticles is a drastic improvement in properties that can

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include mechanical strength, toughness and electrical or thermal conductivity. The effectiveness of the nanoparticles is such that the amount of material added is normally only between 0.5 and 5% by weight.

Nanocomposites are currently being used in a number of fields and new applications are being continuously developed. Applications for nano composites include: Thin-film capacitors for computer chips, Solid polymer electrolyes for batteries, Automotive engine parts and fuel tanks, Impellers and blades, Oxygen and gas barriers, Food packaging etc.

X-Ray diffraction

X-ray crystallography was developed by physicists William_Lawrence Bragg and his father William Henry Bragg. In 1912-1913, the younger Bragg developed Bragg's law, which connects the observed scattering with reflections from evenly spaced planes within the crystal.

X-ray crystallography, also called X-ray diffraction, is used to determine crystal structures by interpreting the diffraction patterns formed when X-rays are scattered by the electrons of atoms in crystalline solids. X-rays are sent through a crystal to reveal the pattern in which the molecules and atoms contained within the crystal are arranged.

Bragg's Law

The fundamental equation which gives a simple relation between the wavelength(λ) of X-rays, the interplanar distance (d), and the glancing angle (θ) , is known as Bragg's law. It is given as

$$2d \sin \theta = n \lambda$$

Where n=1, 2, 3, etc represents the order of the spectrum.

PRINCIPLE, CONSTRUCTION AND WORKIG OF X-RAY DIFFRACTOMETER:

X-ray diffraction is now a common technique for the study of crystal structures and atomic spacing.

Principle: Based on the constructive interference of monochromatic x-rays and a crystalline sample in which the crystalline structure causes a beam of incident x-rays to diffract into many specific directions.

These X-rays are generated by a cathode ray tube, filtered to produce monochromatic radiation, collimated to concentrate, and directed toward the sample. The interaction of the incident rays with the sample produces constructive interference (and a diffracted ray) when conditions satisfy Bragg's Law ($n\lambda=2d \sin \theta$). This law relates the wavelength of electromagnetic radiation to the diffraction angle and the lattice spacing in a crystalline sample. These diffracted X-rays are then detected, processed and counted.

Working:

X-ray diffractometer consists of three basic elements: an X-ray tube, a sample holder, and an X-ray detector.

X-rays are generated in a cathode ray tube by heating a filament to produce electrons, accelerating the electrons toward a target by applying a voltage, and bombarding the target material with electrons. These X-rays are collimated and directed onto the sample.

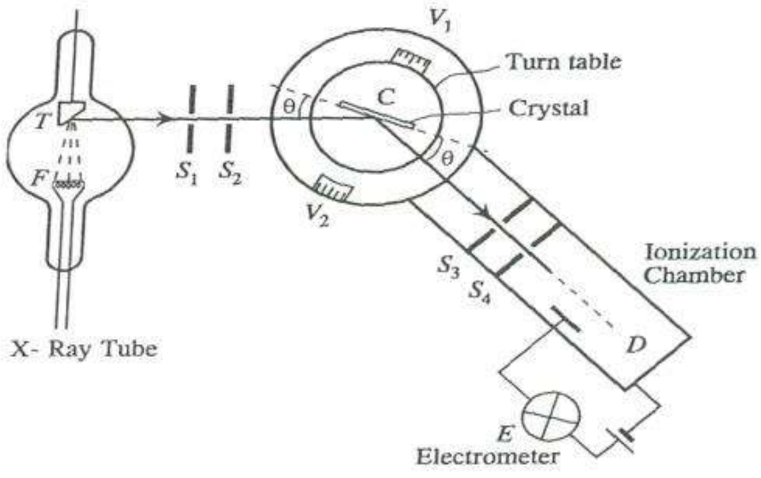
As the sample and detector are rotated, the intensity of the reflected X-rays is recorded. When the geometry of the incident X-rays impinging the sample satisfies the Bragg Equation, constructive interference occurs and a peak in intensity occurs.

A detector records and processes this X-ray signal and converts the signal to a count rate which is then output to a device such as a printer or computer monitor. These X-rays are collimated and directed onto the sample. As the sample and detector is rotated, the intensity of the reflected X-rays is recorded.

When the geometry of the incident X-rays impinging the sample satisfies the Bragg Equation, constructive interference occurs and a peak in intensity occurs. A detector records and processes this X-ray signal and converts the signal to a count rate which is then output to a device such as a printer or computer monitor

The geometry of an X-ray diffractometer is such that the sample rotates in the path of the collimated X-ray beam at an angle θ while the X-ray detector is mounted on an arm to collect the diffracted X-rays and rotates at an angle of 2θ. The instrument used to maintain the angle and rotate the sample is termed a *goniometer*. For typical powder patterns, data is collected at 2θ from ~5° to 70°, angles that are present in the X-ray scan.

Bragg's X-ray spectrometer



Applications

- X-ray powder diffraction is most widely used for the identification of unknown crystalline materials
- Determination of unknown solids is critical to studies in geology, environmental science, material science, engineering and biology.
- characterization of crystalline materials
- identification of fine-grained minerals such as clays and mixed layer clays

  that are difficult to determine optically
- determination of unit cell dimensions

- measurement of sample purity

Scherrer equation

The **Scherrer equation**, in X-ray diffraction and crystallography, is a formula that relates the size of sub-micrometre crystallites in a solid to the broadening of a peak in a diffraction pattern. It is used in the determination of size of crystals in the form of powder.

The Scherrer equation can be written as:

$$D = \frac{K\lambda}{\beta cos\theta}$$

D is the mean size of the ordered (crystalline) domains, which may be smaller or equal to the grain size, which may be smaller or equal to the particle size

K is a dimensionless **shape factor,** with a value close to unity. The shape factor has a typical value of about 0.9 but varies with the actual shape of the crystallite.

λ is the X-ray wavelength;

β is the line broadening at half the maximum intensity (full width at half maximum - FWHM value in radians), after subtracting the instrumental line broadening, in radians.

θ is the peak position Bragg angle

APPLICATION OF XRB:

- XRD is non-destructive technique.
- To identify crystalline phases and orientation.
- To determine structural properties train grain size face composition orientation disorder transformation and thermal expansion extra.
- To measure thickness of thin films multilayers.
- To determine atomic arrangement.
- Structure of crystals.
- Polymer characterization state of anneal of metals.
- Particle size determination.

ADVANTAGES:

- XRD is there least expensive the most convenient and the most widely used method to determine crystal structures.
- XRD techniques gives information about the structures of solids inmates of the atoms that compose the solid.
- XRD permits non-destructive structural analysis.

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DISADVANTAGES:

- XRD has size limitations.
- It is much more accurate for measuring large crystalline structures rather than small ones.
- X rays do not interact very strongly with lighter elements.
- It is relatively low in sensitivity.

ATOMIC FORCE MICROSCOPY (AFM):

INTRODUCTON:

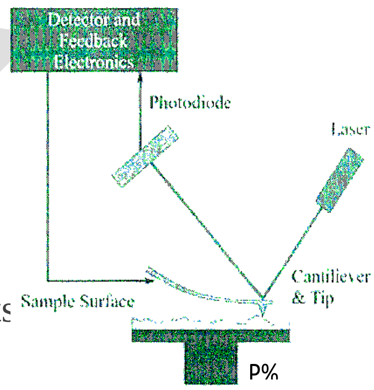
The atomic force microscope (AFM) is a type of scanning probe microscope whose primary roles include measuring properties such as magnetism, height, friction. The resolution is measured in a nanometre, which is much more accurate and effective than the optical diffraction limit. It uses a probe for measuring and collection of data involves touching the surface that has the probe.

PRINCIPLE:

The AFM principle is based on the cantilever/tip assembly that interacts with the sample; this assembly is also commonly referred to as the probe. The AFM probe interacts with the substrate through a raster scanning motion. The up/down and side to side motion of the AFM tip as it scans along the surface is monitored through a laser beam reflected off the cantilever. This reflected laser beam is tracked by a position-sensitive photo-detector (PSPD) that picks up the vertical and lateral motion of the probe. The Atomic Force Microscope works on the principle measuring intermolecular forces and sees atoms by using probed surfaces of the specimen in nanoscale.

CONSTRUCTION:

Its functioning is enabled by three of its major working principles that include Surface sensing, Detection, and Imaging.



AFM consists of following parts

1. LASER
2. Photodiode
3. Cantilever with sharp tip
4. Detector and feedback circuit
5. Piezoelectric sensor

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Working of AFM

1. AFM consists of microscope cantilever with sharp tip(probe) at its end used to scan the specimen surface
2. The cantilever is typically silicon or silicon nitride with the tip radius of curvature of the order of nm. Basically, AFM is modified TEM in which limitations of TEM is overcomed. When the tip is bought close to the sample, force between the tip and the sample leads to the deflection of the cantilever according to the Hookes law. Instead of using the electrical signal, the AFM relies on forces between the atom on the tip and in the sample.
3. The force present in the tip is kept constant and scanning is done. As the scanning continues, the tip will have vertical movements depending upon the topography of the sample. The force present in the tip is kept constant and the scanning is done. As the scanning continues the tip will have vertical movement depending upon the topography of the sample.
4. A LASER beam is used to have a record of vertical movement of the needle. This information is later converted into visible from using photo diode. Depending upon the situation, AFM measures different types of forces like a Vander Waals forces, capillary force, mechanical contact force etc.

APPLICATIONS:

This type of microscopy has been used in various disciplines in naturalscience such as solid-state physics, semiconductor studies, molecular engineering, polymer chemistry, surface chemistry, molecular biology, cell biology, medicine, and physics. Some of these applications include:

- Identifying atoms from samples
- Evaluating force interactions between atoms
- Studying the physical changing properties of atoms
- Studying the structural and mechanical properties of protein complexesand assembly, such as microtubules.
- Used to differentiate cancer cells and normal cells.

Evaluating and differentiating neighbouring cells and their shape andcell wall rigidity

Advantages:

i. Easy to prepare samples for observation

2. It can be used in vacuums, air, and liquids.

3 Measurement of sample sizes is accurate

4 It has a 3D imaging. It can be used to study living and non-living elements

6. It can be used to quantify the roughness of surfaces7- It is used in dynamic environments.

Disadvantages:

i. It can only scan a single Nano sized image at a time of about l_S OXl_S nm.

2. They have a low scanning time, which might cause thermal drift on the sample.

3- The tip and the sample can be damaged during detection.

4 It has a limited magnification and vertical range.

Xray Photoelectron Spectroscopy(XPS),

X-RAY PHOTOELECTRON SPECTROSCOPY (XPS):

INTRODUCTION:

The technique of X-Ray photoelectron Spectroscopy is also known as Electron Spectroscopy for chemical analysis. X-ray photoelectron Spectroscopy is a type of electron Spectroscopy it is an analytical technique to study the electronic structure andit is dynamic in atoms and molecules.

In x-ray photoelectron spectroscopy, primary beam x-ray photon, which are irradiated on sample surface, the secondary beam (electron) obtained is then analysed. The Secondary beam is made up of electrons. The spectrum of X-Ray photoelectron spectroscopy consists plot of number of electrons or power of electron as a function of energy i.e., kinetic energy or binding energy.

PRINCIPLE:

Due to the bombardment of X-Ray Photon on the sample surface K and L electron are ejected which are further analysed by the analyser. Let us consider E_b, E'_b and E_b'' are binding energy of lower energy levels inner core orbitals. Where E_v, $E_v'$and E_v'' are the energies of the valence shell electron.

The monochromatic X-ray Photon incident on the sample surface cell electron abstract the energy from this X-ray Photon and get ejected in terms of electron. Kinetic

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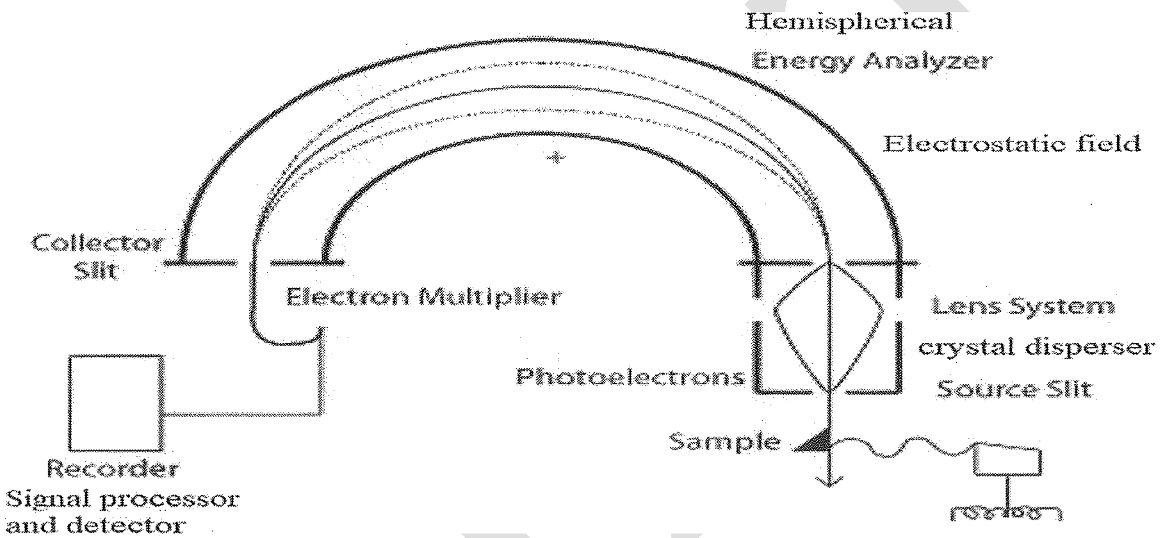
energy of the ejected electron is recorded by spectrometer and is given by

$$E_k = h\nu - E_b - \Phi$$

Where, E_k is kinetic energy of the ejected electron $h\nu$ energy associated with incident

Photon E_b - binding energy ejected electron Φ — Work function

CONSTRUCTION:



The electron spectrometer made up of following components.

- Source
- Sample Holder
- Analyser
- Detector
- Process and read out

SOURCE:

The simple x-ray Photon source for x-ray photoelectron spectra M is X ray tube equipped with magnesium or aluminium metal target. Monochromator crystal can also provide having bandwidth of 0.3 eV. Much smaller spots on a surface to be examined.

SAMPLE HOLDER:

Sample holder is located in between the source and the entrance slit of spectrometer. Crystal disperser selects the photon of known energy from the source and incident on the sample. The area inside the sample holder should be evacuated within 10^{-5} Torr. Pressure to avoid contamination of the surface sample.

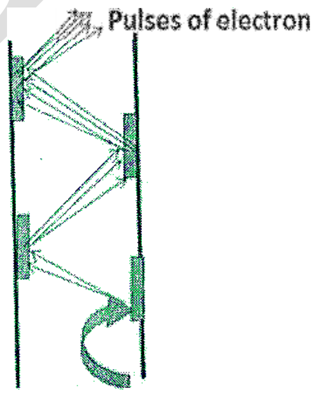
The gaseous sample for introduced into a sample compartment through a slit, to provide a Pressure of 10^{-12} torr. If the pressure is higher than attenuation of electron beam may take place, weaker signal may be obtained.

ANALYSER:

It is hemispherical in shape with very high electrostatic field is applied on analyser. Pressure maintained inside the analyser is 10^{-5} torr. When the electron enters into the hemisphere analyser, it travels in curved path and radius of curvature depends upon magnitude of field and kinetic energy of the electron.

DETECTOR:

The electron channel multiplayer tube or transducer is required of X-Ray photoelectron Spectroscopy. When single electron pass through the electron multiplier tube it is converted into number of electrons are pulses of electrons.



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SIGNAL PROCESSOR AND READ OUT:

The function of signal processor is to amplify the signal and read out deviceconverts signal into spectrum.

APPLICATION OF XPS.

- Identification of active sites
- Determination of surface contamination on semiconductors
- Study of oxide layers on metals
- Analysis of dust on the sample
- Determination of Oxidation State all the elements of periodic table can be determined or identified except hydrogen and Helium, as they don't emit inner core electron.

ADVANTAGES:

- It is surface sensitive technique
- Wide range of solid surface sample can be identified
- Relative non-destructive

DISADVANTAGES:

a. It is very expensive
b. Slow and poor resolution power
c. Required high vacuum.

Scanning electron microscopy (SEM)

Microscope is an instrument which provides a magnified image of an object. Scanning Electron Microscope is the kind of microscope that uses a beam of electrons to create a magnified image of the specimen.

Principle

The principle used in the working of an SEM is the wave nature of electrons. An electron accelerated under a potential difference of V volts behave like a wave of wavelength

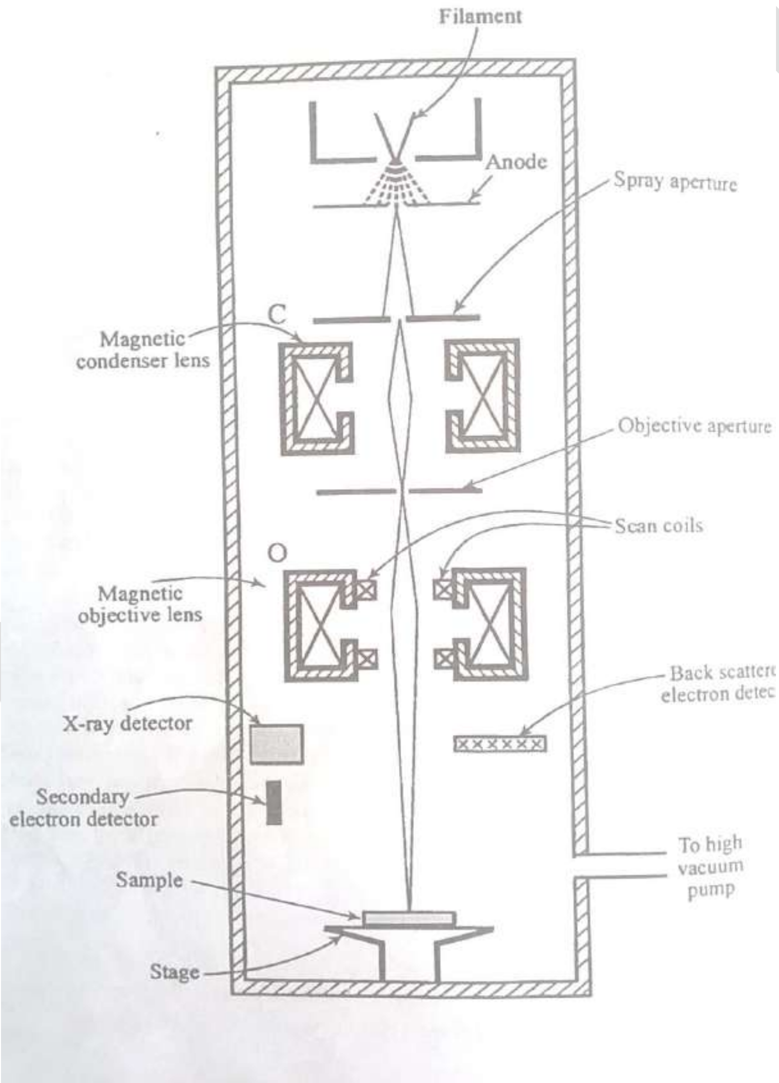
$$\lambda = \frac{h}{\sqrt{meV}} = \frac{1.226}{\sqrt{V}} \text{ (nm).}$$

Construction

- The apparatus consists of an highly evacuated chamber inside which there is an electron gun at the top which comprises of the filament and anode.
- There are two magnetic lenses, one is condensing lens C and the other is objective lens O accompanied by a scan coil.
- There is a spray aperture using which, spherical aberration during focusing will be minimized.
- A flat surface called stage is provided to place the specimen.
- The apparatus has 3 types of detectors namely back scattered electron detector, secondary emission electron detector and x-ray detector.
- The electrons incident on the sample are called primary electrons.
- The electrons scattered by the sample are called back scattered electrons.
- The electrons which are knocked off from the atoms are called secondary electrons.

Working

- The sample is placed on the specimen stage which is evacuated
- Electrons are emitted by the filament by thermionic emission
- A suitable the positive potential accelerates electrons from the electron gun.
- The electron beam falling on the condensing lens C is converged. Then high angle electrons are eliminated.
- The beam passes through the objective aperture where the size of the beam can be controlled.
- The objective lens focuses the thinner beam on to the desired part of the specimen.
- The scan coils enables the beam to scan the specimen in a particular way called raster.
- Upon incidence electrons are knocked out from the specimen.
- Backscattered electrons, secondary electrons and X-rays emitted are detected by the detector. And signals are produced. Which are converted into a micro spot of corresponding brightness on the screen



SCANNING ELECTRON MICROSCOPE (SEM)

Applications.

SEM is used to study

1) External morphology of biological organisms
2) Chemical composition

3) Crystalline structure.
4) Forensic investigations.

TRANSMISSION ELECTRON MICROSCOPE:

INTRODUCTION:

We know in scanning electron microscope the resolution of the image is limited only upto lo to zo nm. This will not be usefiil to view the internal features of an atom or the morphology of a sample of size saw o.mm.

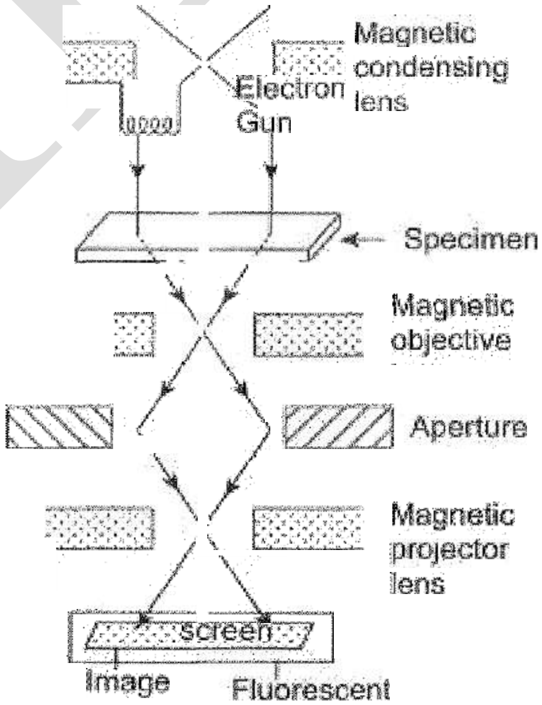
To examine the sample of size of 2 tO NM the transmission electron microscope can be used. In this microscope, the image is obtained by transmitting the electron through the specimen.

PRINCIPLE:

Electrons are made to pass through the specimen and the image is formed in the fluorescent screen there by using transmitted Beam (bright field image) on by using the diffracted beam (dark field image).

CONSTRUCTION:

It consists of an electron gun to produce electron. Magnetic condensing lens is used to condense the electron and is used to adjust the size of the electron that fall ontothe specimen. The specimen is placed in between the condensing lens and objective lens and in turn, it increases the contrast ofthe image.



The magnetic projector lens is placed above the fluorescent screen in order to achieve high magnification. Image is recorded by using a fluorescent screen or CCD charge coupled devices.

as shown in figure.

The magnetic objective lens is used to block the high angle diffracted beam the aperture is used to eliminate the diffracted beam

WORKING:

Stream of electrons are produced by the electron gun is made to fall over the specimen using magnetic condensing lens.

Based on the angle of incidence the Beam is partly transmitted and partly diffracted as shown in figure . Both the transmitted Beam and the diffracted beams are recombined at the E-WALLED SPHERE of reflection, which encloses all possible reflections from the Crystal are specimen satisfying the bragg's law image as shown in figure. The combined image is called the phase contrast image.

In order to increase the intensity and the contrast of the image and amplitude contrast image has to be obtained for stop this can be achieved only by using the transmitting beam and does the diffracted beam has to be eliminated.

Now in order to eliminate the diffracted beam that beam is passed through the magnetic objective lens and the aperture is shown in figure adjusted in such a way that the diffracted image is illuminated. Thus, the final image being alone is passedthrough the projector lens for further magnification. Find image is recorded in thefluorescent screen or CCD this high contrast image is called Bright Field image. Inaddition, it has to be noted that the bright field image obtained is purely due to theelastic scattering non no energy change that is due to the transmitted beam alone

ADVANTAGES:

- It is used to examine the specimen of size of 0.2 to 2 nm
- The magnification is 10 lakh times greater than the size of the object
- It has high resolution
- The resolution power is 1& to 2Å
- We can get high contrast image due to both transmitted be bright field and the diffracted be dark Beam.

DISADVANTAGES:

- The specimen should be very thing.
- Answers for the structural change during sample preparation
- Three-dimensional image cannot be obtained
- In case of biological samples, the electrons may interact with the samples,which may even damage the sample.

APPLICATIONS:

- The main application of transmission electron microscope is in nanoscience nanotubes micro machines used to find the internal structures of Nanomaterials.
- It is used to find the two-dimensional image of very small biological cells virus

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bacteria etc

- It is used in thin film Technology metallurgy biochemistry microbiology etc
- It is used to study the composition of paints papers fibres composite materials alloys etc.

Questions

1. Describe the construction and working of Scanning Electron Microscope (SEM)
2. Define nano-material and nano composite and classify the nano-materials based on the dimensional constraints.
3. Describe the construction and working of X-ray photoelectron spectroscopy.
4. Describe the construction & working of transmission electron microscope (TEM).
5. With neat diagram, explain the principle, construction and working of Atomic Force Microscopy (AFM)
6. Ray diffractometer and how the crystal size is determined using Scherrer equation.

NUMERICALS

1, The spacing between principal planes of NaCl crystal is 2.82Å. it is found that first order Bragg reflection occurs at an angle of 100. Calculate the wavelength of X-rays.

2, Determine the wave length of X-rays for crystal size of 1.19 μm, peak width is 0.5 o and peak position 35o, for a cubic crystal. Given Scherrer's constant k=0.92.

4. X-rays are diffracted in the first order from a crystal with d spacing 2.8 Ȧ at a glancing angle 60°. Calculate the wavelength of X-rays.

5. Determine the crystallite size given the Wavelength of X-rays 10 nm , the Peak Width 0.5 °and peak position 25 ° for a cubic crystal given K = 0.94

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