

**VISVESVARAYA TECHNOLOGICAL UNIVERSITY
BELGAUM**



ENGINEERING MECHANICS

(Subject Code: BCIVC203)

LECTURE NOTES

(MODULE-3)

II-SEMESTER

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A J INSTITUTE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

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ENGINEERING MECHANICS

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Department: Civil Engineering

MODULE - 3

Analysis of Trusses: Introduction, Classification of trusses, analysis of plane perfect trusses by the method of joints and method of sections, Numerical examples.

Friction: Introduction, laws of Coulomb friction, equilibrium of blocks on horizontal plane, equilibrium of blocks on inclined plane, ladder friction, wedge friction Numerical examples.

ANALYSIS OF TRUSSES

The engineering structures are mainly classified in to three types.

1. Trusses
2. Frames
3. Machines

Plane trusses are those structures in which all members are lying in a single plane. Plane trusses are made of several bars or members connected together at the joints by riveting or welding to form a rigid formwork, and also support stationary loads or moving loads. Individual members of a truss is a two fore member subjected to either tension or compression. The forces are acting only on loads.

Classification of trusses

1. **Rigid truss or perfect truss:** is the one in which the number of members are sufficient to resist the external loads and in which deformation is very small. The relationship between the number of members and number of joints is given by

$$m = 2j - 3$$

2. **Non rigid truss or deficient truss:** is the one in which the number of members are less than that required for a perfect truss.

$$m < 2j - 3$$

3. **Over rigid truss or redundant truss:** is the one in which the number of members are more than that required for a perfect truss.

$$m > 2j - 3$$

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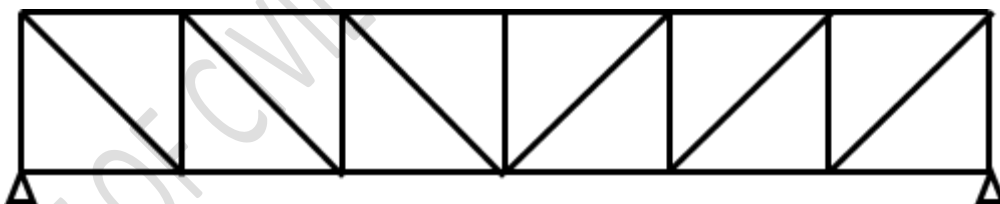
Types of Trusses

1. Warren truss
2. Pratt truss
3. Bowstring truss
4. King post truss
5. Lenticular truss

1. **Warren truss** - Truss members form a series of equilateral triangles, alternating up and down.



2. **Pratt truss** - Pratt trusses are commonly used in long span buildings ranging from 20 to 100 m in span. In a conventional Pratt truss, diagonal members are in tension for gravity loads. This type of truss is used where gravity loads and uplift loads are predominant

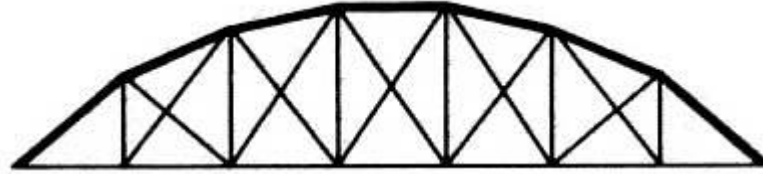


3. **Bowstring truss** - Named for their shape, bowstring trusses were first used for arched truss bridges, often confused with tied-arch bridges. Thousands of bowstring trusses were used during World War II for holding up the curved roofs of aircraft hangars and other military buildings.

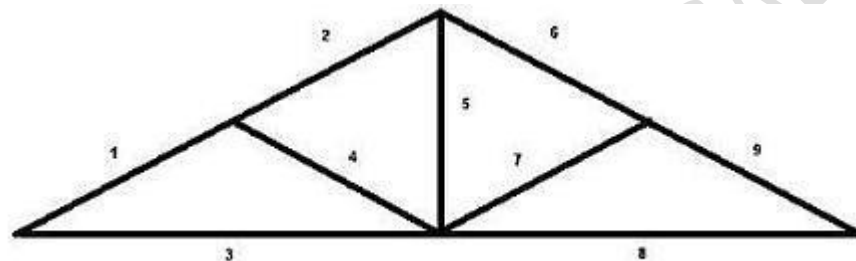
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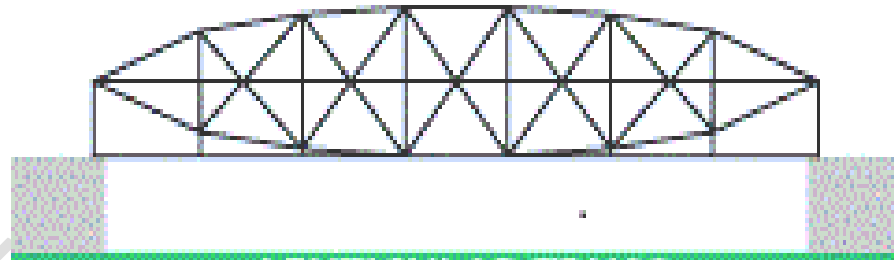
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4. **King post truss** - One of the simplest truss styles to implement, the king post consists of two angled supports leaning into a common vertical support.



5. **Lenticular truss** - have the top and bottom chords of the truss arched, forming a lens shape. A lenticular pony truss bridge is a bridge design that involves a lenticular truss extending above and below the road bed.



Basic assumptions in truss analysis

1. The members cannot develop moments at the ends.
2. The members are subjected to purely axial forces.
3. The connections to other members are perfectly pinned/hinged through frictionless pins.
4. Each member is of uniform cross sectional area.
5. The entire structure is in one plane if it is a plane truss



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6. Loads act at the joints only. Any loads that act between joints are split into equivalent support end reactions and added to the joint loads.
7. Self-weight of the truss can be ignored or at least assumed to be equally distributed as loads at the joints.
8. Even if the members are connected at the ends with gusset plates and welded, no fixity is assumed. A nominal moment that actually develops due to imperfect hinge connection is simply ignored.

METHODS FOR ANALYSIS OF TRUSS

Method of Joints - The various steps involved in the method of joints are as follows:

1. Check whether the given truss is statically determinate or indeterminate.
2. Calculate the support reactions by considering the equilibrium of entire truss.
3. Now, consider a joint with minimum number of unknowns (maximum of 2 unknowns, because only two conditions of equilibrium are available namely $F_x = 0$ and $F_y = 0$).
4. Initially assume that all members under consideration are in tension (arrow head away from the joint is positive).
5. Apply the conditions of equilibrium and by solving the equilibrium, determine the unknown forces in the members.
6. If the result works out to be positive then our assumption is right, if it is negative then our assumption is wrong and it indicates that particular member is under compression, so that reverse the direction of force while considering it in the next joint.
7. Same procedure has to be followed for other joints to determine the internal forces in the remaining members of a truss.
8. Note down the results in a tabular format.

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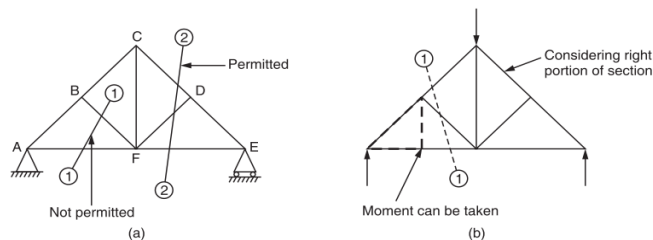
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Method of Sections - Method of section is the method in which a section line has to be passed through the members in which the internal forces need to be calculated. This method is suitable when it is necessary to find the forces induced in a few or selected members of a truss.

Some of the points to be remembered in using the method of section are as follows:

1. The section line should be complete.



2. The section line should pass through the members, but not through the joints.
3. The section line can pass through maximum of three members because only three conditions of equilibrium are available.
4. The section line can pass through the four members in a situation where three members are meeting at a common point.
5. The moment equation of equilibrium can be applied about a point may be beyond the portion under consideration.
6. Consider either left portion or right portion whichever is easy for the analysis, as both portions are under equilibrium.

Problems

1. Analyse the truss given in the Fig. below

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Geometrical Properties

$$\theta = \tan^{-1}(2/4) = 26.57^\circ$$

$$\sum F_y = 0$$

$$-F_{AC} \sin\theta + 9.63 = 0$$

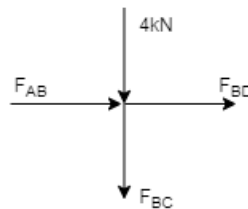
$$F_{AC} = 9.63/\sin 26.57^\circ = 21.53 \text{ kN (Tension)}$$

$$\sum F_x = 0$$

$$F_{AB} + F_{AC} \cos\theta - 3 = 0$$

$$F_{AB} = 3 - (21.53 \cos 26.57) = -16.27 \text{ kN (Compression)}$$

Joint B



$$\sum F_y = 0$$

$$-F_{BC} - 4 = 0$$

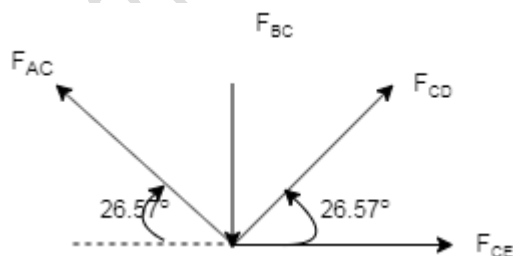
$$F_{BC} = -4\text{kN (Compression)}$$

$$\sum F_x = 0$$

$$F_{AB} + F_{BD} = 0$$

$$F_{AB} = -F_{BD} = -16.27 \text{ kN (Compression)}$$

Joint c



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$$\sum F_x = 0$$

$$-F_{AC} \cos 26.57^\circ + F_{CD} \cos 26.57^\circ + F_{CE} = 0$$

$$-21.53 \cos 26.57^\circ + F_{CD} \cos 26.57^\circ + F_{CE} = 0$$

$$\sum F_y = 0$$

$$F_{AC} \sin 26.57^\circ + F_{CD} \sin 26.57^\circ - F_{BC} = 0$$

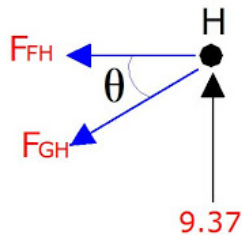
$$21.53 \sin 26.57^\circ + F_{CD} \sin 26.57^\circ - 4 = 0$$

$$F_{CD} \sin 26.57^\circ = -5.63 \text{ N}$$

$$F_{CD} = -12.58 \text{ N (COMPRESSION)}$$

$$-21.53 \cos 26.57^\circ - (12.58) \cos 26.57^\circ + F_{CE} = 0$$

$$F_{CE} = 19.256 + 11.25 = 30.50 \text{ N}$$



Joint H

$$\sum F_y = 0$$

$$-F_{GH} \sin \theta + 9.37 = 0$$

$$F_{GH} = 9.37 / \sin 26.57^\circ = 20.95 \text{ kN (Tension)}$$

$$\sum F_x = 0$$

$$-F_{FH} - F_{GH} \cos \theta = 0$$

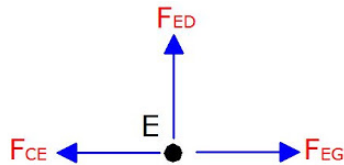
$$F_{FH} = -(20.95 \cos 26.57^\circ) = -18.74 \text{ kN (Compression)}$$

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Joint E



$$\sum F_y = 0$$

$$F_{ED} + 0 = 0$$

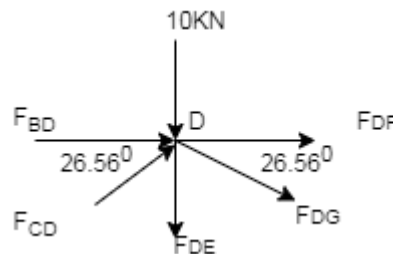
$$F_{ED} = 0 \text{ (No force)}$$

$$\sum F_x = 0$$

$$-F_{CE} + F_{EG} = 0$$

$$F_{CE} = F_{GE} = 30.52 \text{ kN (Tension)}$$

Joint D



$$\sum F_y = 0$$

$$F_{DC} \sin\theta - F_{DG} \sin\theta - 10 = 0$$

$$12.59 \sin 26.56 - F_{DG} \sin\theta - 10 = 0$$

$$F_{DG} = -4.37 / \sin 26.57^\circ = -9.77 \text{ kN (Compression)}$$

$$\sum F_x = 0$$

$$F_{BD} + F_{DF} - F_{DC} \cos 26.57 + F_{DG} \cos 26.57 = 0$$

$$16.27 + F_{DF} - 12.58 \cos 26.57 - 9.77 \cos 26.57 = 0$$

$$F_{DF} = 3.72 \text{ kN (Tension)}$$

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Joint F

$$\sum F_x = 0$$

$$-F_{DF} + F_{FH} = 0$$

$$F_{DF} = F_{FH} = 18.78 \text{ kN (T)}$$

$$\sum F_y = 0$$

$$-F_{FG} - 5 = 0$$

$$F_{FG} = -5 \text{ kN (compression)}$$

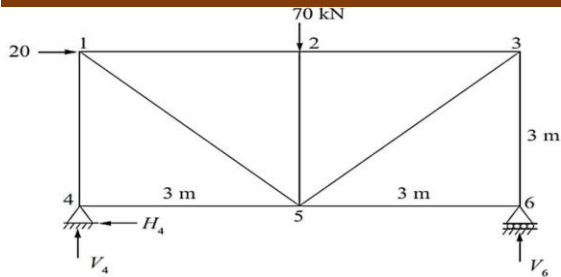
Member	Magnitude	Type
AB	-16.27 kN	(Compression)
AC	21.53 kN	(Tension)
BC	-4kN	(Compression)
CD	-12.58 kN	(Compression)
BD	-16.27 kN	(Compression)
CE	30.52 kN	(Tension)
EG	30.52 kN	(Tension)
FD	18.78 N	(Tension)
DG	-9.77 kN (Compression)	(Compression)
FH	-18.74 kN (Compression)	(Compression)
HG	20.95 kN	(Tension)
ED	0 (No force)	
FG	5kN	(Compression)

2) Analyze the Truss shown in Figure using method of Joints. Indicate the forces in the members pictorially and tabulate the results.

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The reactions at the supports are found out by summing up the forces in horizontal and vertical directions and also by taking moments of applied forces about the hinge support.

$$\sum H = 0; \quad 20 - H_4 = 0$$

$$\boxed{H_4 = 20 \text{ kN}}$$

$$\sum V = 0; \quad V_4 + V_6 = 70 \text{ kN}$$

$$\sum M_4 = 0; \quad 20 \times 3 + 70 \times 3 - 6V_6 = 0$$

$$\boxed{V_6 = 45 \text{ kN}}$$

$$\therefore \boxed{V_4 = 25 \text{ kN}}$$

Joint 4

$$\sum H = 0;$$

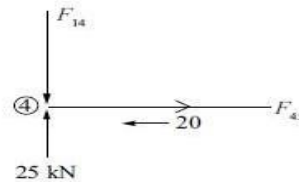
$$F_{45} - 20 = 0$$

$$F_{45} = 20 \text{ kN}$$

$$\sum V = 0;$$

$$-F_{14} + 25 = 0$$

$$F_{14} = 25 \text{ kN}$$



Joint 1

$$\sum V = 0;$$

$$25 - F_{15} \sin 45 = 0$$

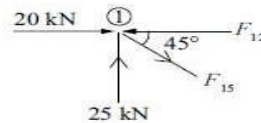
$$F_{15} = 35.4 \text{ kN}$$

$$\sum H = 0;$$

$$20 - F_{12} + F_{15} \cos 45 = 0$$

$$20 - F_{12} + 35.4 \cos 45 = 0$$

$$F_{12} = 45.03 \text{ kN}$$



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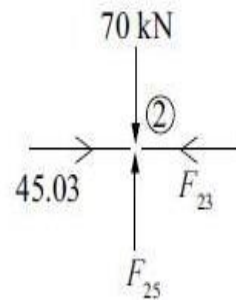
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Joint 2

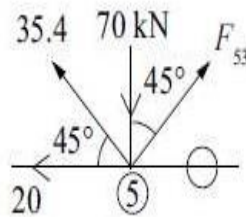
$$45.03 - F_{23} = 0$$

$$F_{23} = 45.03 \text{ kN}$$

$$\sum V = 0; F_{25} = 70.0 \text{ kN}$$



Joint 5



$$\sum V = 0;$$

$$F_{53} \cos 45 + 35.4 \sin 45 - 70 = 0$$

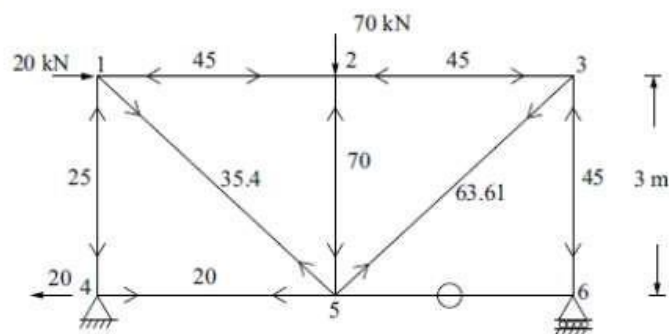
$$F_{53} = 63.61 \text{ kN}$$

Joint 3

$$-F_{36} - F_{53} \cos 45 = 0$$

$$F_{36} = 45 \text{ kN}$$

Hence the final forces are

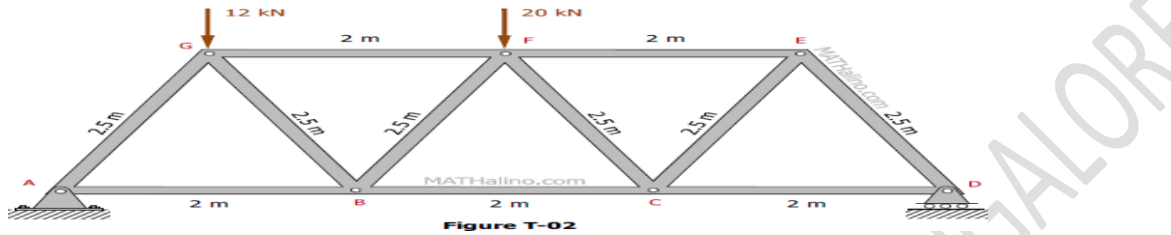


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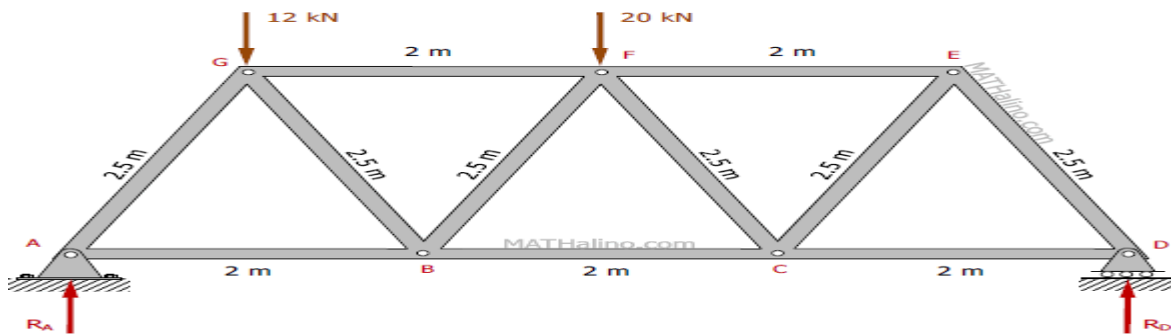
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3.The structure in Fig. T-02 is a truss which is pinned to the floor at point A, and supported by a roller at point D. Determine the force to all members of the truss. using method of joints.



Solution



$$\Sigma M_D = 0$$

$$6R_A = 5(12) + 3(20)$$

$$R_A = 20 \text{ kN}$$

$$\Sigma M_A = 0$$

$$6R_D = 1(12) + 3(20)$$

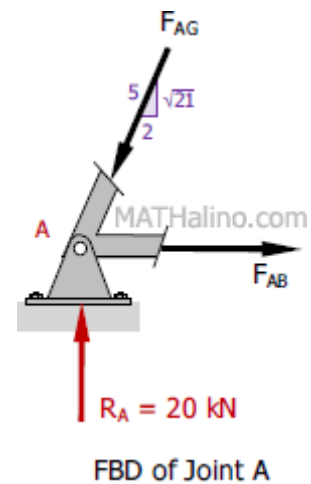
$$R_D = 12 \text{ kN}$$

At joint A

$$\Sigma F_V = 0$$

$$21/\sqrt{5} F_{AG} = R_A$$

$$21/\sqrt{5} F_{AG} = 20$$



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$F_{AG}=21.82$ kN compression

$$\Sigma F_H=0$$

$$F_{AB}=2/5 F_{AG}$$

$$F_{AB}=2/5 (21.82)$$

$$F_{AB}=8.73$$
 kN tension

At joint G

$$\Sigma F_V=0$$

$$21/\sqrt{5} F_{BG}+12=21/\sqrt{5} F_{AG}$$

$$21/\sqrt{5} F_{BG}+12=21/\sqrt{5} (21.82)$$

$$F_{BG}=8.73$$
 kN tension

$$\Sigma F_H=0$$

$$F_{FG}=2/5 F_{AG}+2/5 F_{BG}$$

$$F_{FG}=2/5 (21.82)+2/5 (8.73)$$

$$F_{FG}=12.22$$
 kN compression

At joint B

$$\Sigma F_V=0$$

$$21/\sqrt{5} F_{BF}=21/\sqrt{5} F_{BG}$$

$$F_{BF}=F_{BG}$$

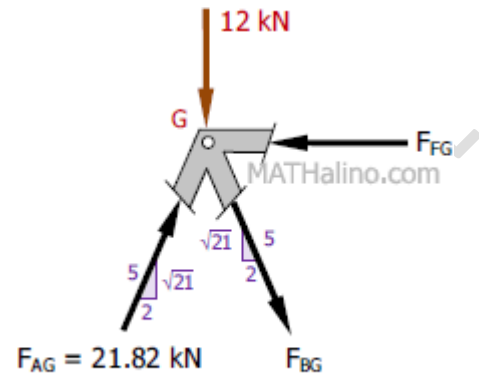
$$F_{BF}=8.73$$
 kN compression

$$\Sigma F_H=0$$

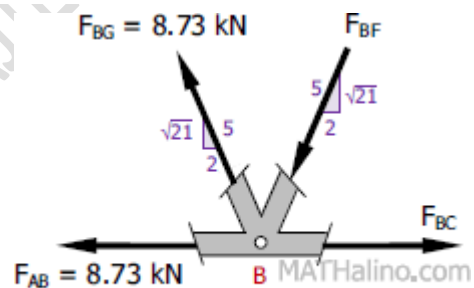
$$F_{BC}=F_{AB}+2/5 F_{BG}+2/5 F_{BF}$$

$$F_{BC}=8.73+2/5 (8.73) +2/5 (8.73)$$

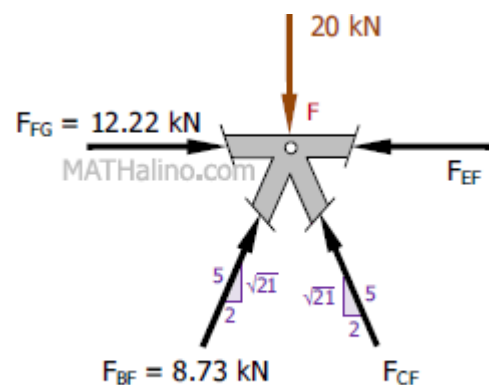
$$F_{BC}=15.71$$
 kN tension



FBD of Joint G



FBD of Joint B



FBD of Joint F

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At joint F

$$\Sigma F_V = 0$$

$$21 / \sqrt{5} F_{CF} + 21 / \sqrt{5} F_{BF} = 20$$

$$21 / \sqrt{5} F_{CF} + 21 / \sqrt{5} (8.73) = 20$$

$$F_{CF} = 13.09 \text{ kN compression}$$

$$\Sigma F_H = 0$$

$$F_{EF} + 2 / 5 F_{CF} = 2 / 5 F_{BF} + F_{FG}$$

$$F_{EF} + 2 / 5 (13.09) = 2 / 5 (8.73) + 12.22$$

$$F_{EF} = 10.48 \text{ kN compression}$$

At joint C

$$\Sigma F_V = 0$$

$$21 / \sqrt{5} F_{CE} = 21 / \sqrt{5} F_{CF}$$

$$F_{CE} = F_{CF}$$

$$F_{CE} = 13.09 \text{ kN tension}$$

$$\Sigma F_H = 0$$

$$F_{CD} + 25 F_{CE} + 25 F_{CF} = F_{BC}$$

$$F_{CD} + 25(13.09) + 25(13.09) = 15.71$$

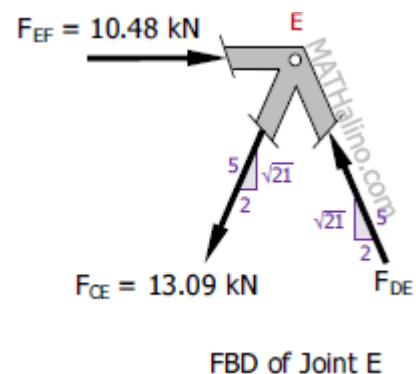
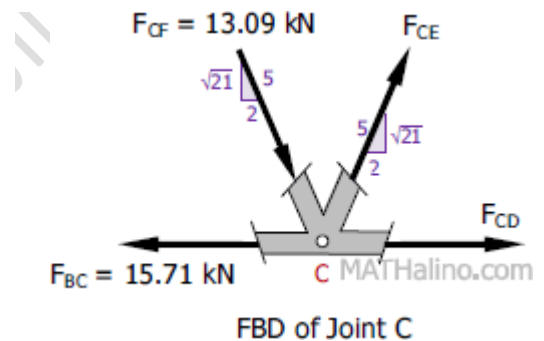
$$F_{CD} = 5.24 \text{ kN tension}$$

At joint E

$$\Sigma F_V = 0$$

$$21 / \sqrt{5} F_{DE} = 21 / \sqrt{5} F_{CE}$$

$$F_{DE} = F_{CE}$$



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$F_{DE} = 13.09$ kN compression

$$\Sigma F_H = 0$$

$$F_{EF} = 25F_{CE} + 25F_{DE}$$

$$10.48 = 25(13.09) + 25(13.09)$$

$$10.5 = 10.5 \quad \text{check}$$

At joint D

$$\Sigma F_V = 0$$

$$R_D = 21 / \sqrt{5} F_{DE}$$

$$12 = 21 / \sqrt{5} (13.09)$$

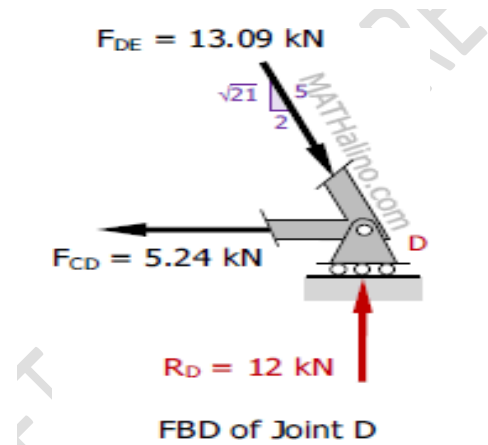
$$12 = 12 \quad \text{check}$$

$$\Sigma F_H = 0$$

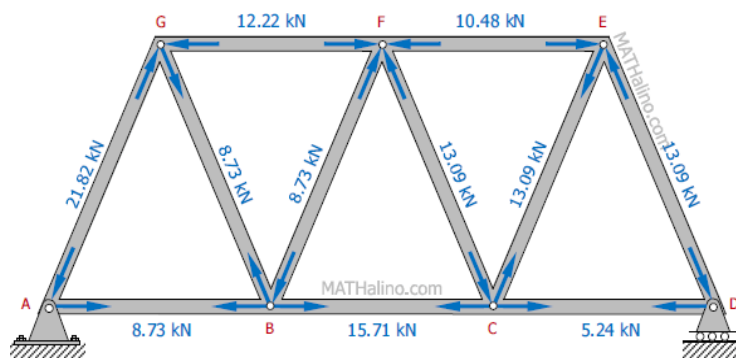
$$F_{CD} = 2 / 5 F_{DE}$$

$$5.24 = 2 / 5 (13.09)$$

$$5.24 = 5.24 \quad \text{check}$$



Summary



$F_{AB} = 8.73$ kN tension

$F_{AG} = 21.82$ kN compression

$F_{BC} = 15.71$ kN tension

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$F_{BF} = 8.73 \text{ kN}$ compression

$F_{BG} = 8.73 \text{ kN}$ tension

$F_{CD} = 5.24 \text{ kN}$ tension

$F_{CE} = 13.09 \text{ kN}$ tension

$F_{CF} = 13.09 \text{ kN}$ compression

$F_{DE} = 13.09 \text{ kN}$ compression

$F_{EF} = 10.48 \text{ kN}$ compression

$F_{FG} = 12.22 \text{ kN}$ compression

4. The roof truss shown in Fig. T-03 is pinned at point A, and supported by a roller at point H. Determine the force in member DG.

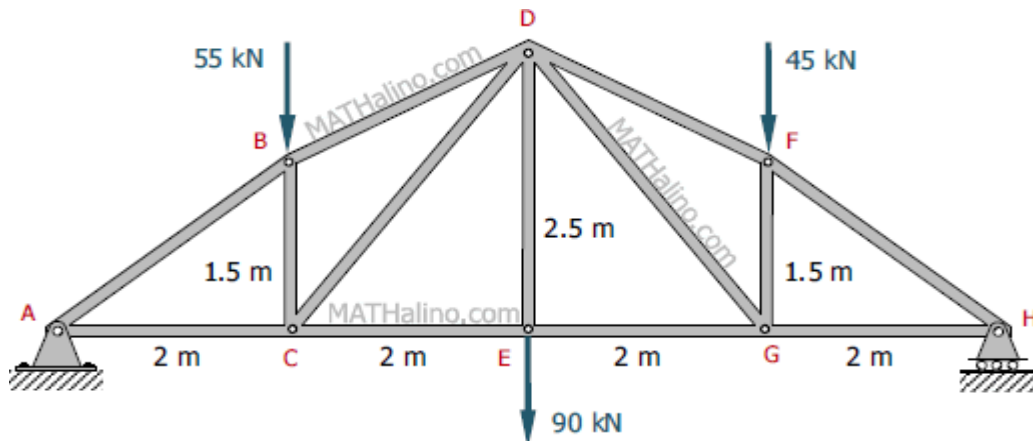


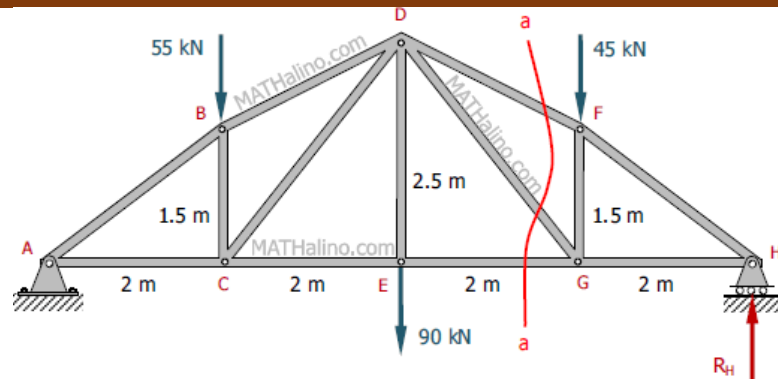
Figure T-03

Solution

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$$\Sigma M_A = 0$$

$$8R_H = 2(55) + 4(90) + 6(45)$$

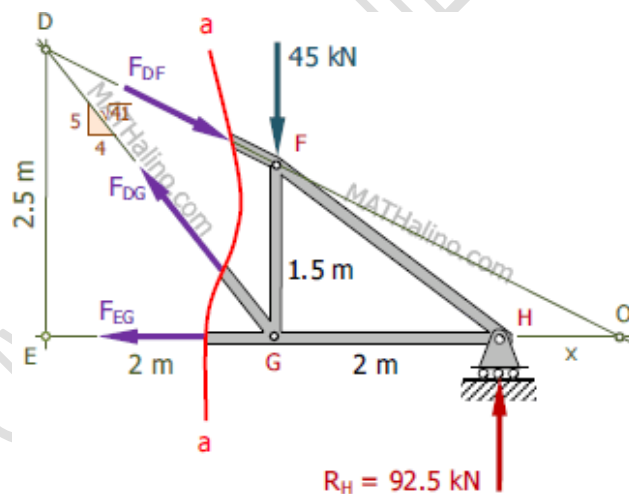
$$R_H = 92.5 \text{ kN}$$

From section to the right of a-a

$$x + 2 / (1.5) = (x + 4) / 2.5$$

$$2.5x + 5 = 1.5x + 6$$

$$x = 1 \text{ m}$$



Section to the right of a-a

$$\Sigma M_O = 0$$

$$(x + 2)(F_{DG} \sin 51.34) + x R_H - (x + 2)(45) = 0$$

$$3 F_{DG} \sin 51.34 + 92.5 - 135 = 0$$

$$F_{DG} = 18.14 \text{ kN tension} \quad \text{answer}$$

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5. From the truss in Fig. T-01, determine the force in members BC, CE, and EF.

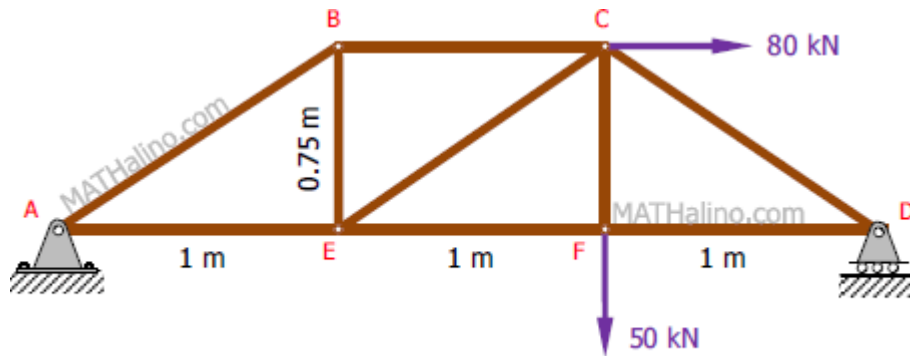
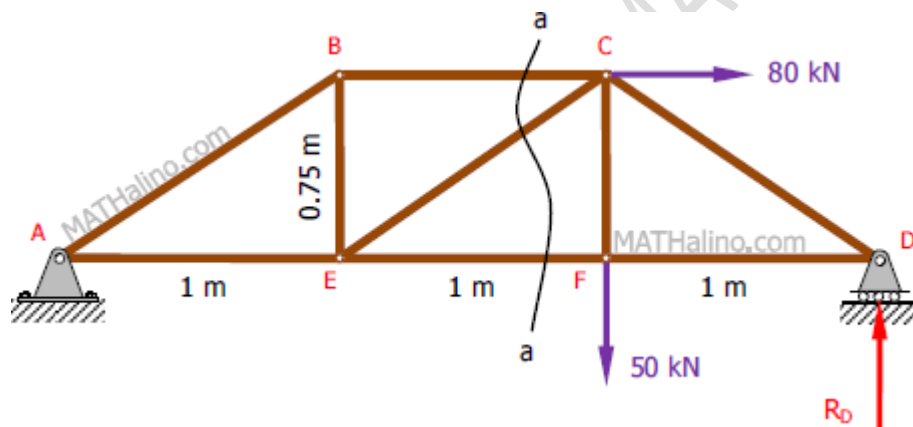


Figure T-01

Solution



$$\Sigma M_A = 0$$

$$3R_D = 50(2) + 80(0.75)$$

$$R_D = 53.33 \text{ kN}$$

From the FBD of the section through a-a

$$\Sigma M_E = 0$$

$$0.75F_{BC} + 2R_D = 0.75(80) + 1(50)$$

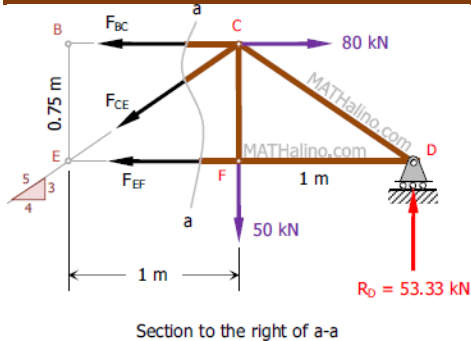
$$0.75F_{BC} + 2(53.33) = 60 + 50$$

$$F_{BC} = 4.45 \text{ kN tension} \quad \text{answer}$$

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Section to the right of a-a

$$\Sigma M_C = 0$$

$$0.75 F_{EF} = 1(R_D)$$

$$0.75 F_{EF} = 53.33$$

$$F_{EF} = 71.11 \text{ kN tension} \quad \text{answer}$$

$$\Sigma F_V = 0$$

$$-F_{CE} \sin 26.56 - 50 = R_D$$

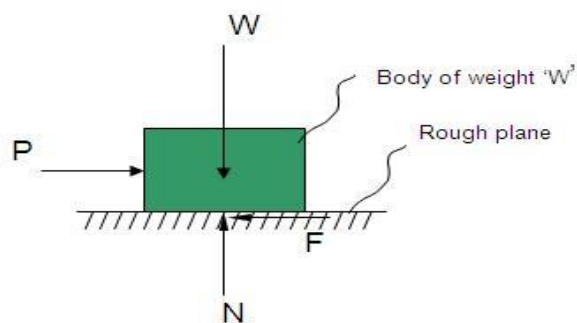
$$F_{CE} = 5.55 \text{ kN tension} \quad \text{answer}$$

FRICITION:

Whenever a body moves or tends to move over another surface or body, a force which opposes the motion of the body is developed tangentially at the surface of contact, such an opposing force developed is called friction or frictional resistance.

The frictional resistance is developed due to the interlocking of the surface irregularities at the contact surface b/w two bodies

Consider a body weighing W resting on a rough plane & subjected to a force P



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Where

P = Applied force

N = Normal reaction from rough surface

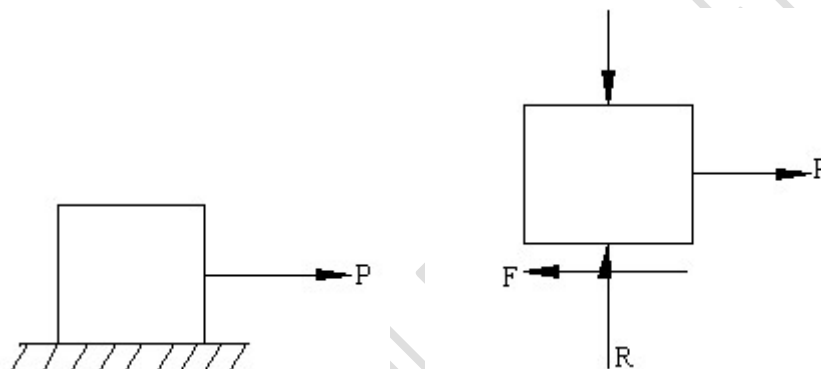
F = Frictional resistance

W = Weight of the body

The body can start moving or slide over the plane if the force „P overcomes the frictional F.

The frictional resistance developed is proportional to the magnitude of the applied force which is responsible for causing motion upto a certain limit.

Limiting friction:



Whenever two surfaces are in contact it is observed that as the applied force increases, friction also increases. But at one stage, the friction attains its maximum value after which the body starts moving even with the application of a small additional force. The maximum value of friction, when the body is in the state of rest is known as Limiting Friction

Co-efficient of friction (μ): It is the ratio between limiting friction and normal reaction.

$$F = \mu R$$

$$\mu = \frac{F}{R}$$



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$\mu=0$, for smooth surface.

Types of friction:

Based on state of rest or motion:

- i) Static friction
- ii) Dynamic friction
- iii) Sliding friction
- iv) Rolling friction

i) Static friction: It is the friction experienced between the two bodies when both bodies are at rest.

ii) Dynamic friction: It is the friction experienced between the two bodies when one body moves over other body.

It is of two types:

- a) Sliding friction: It is the friction between the two bodies when one body slides over the other body.
- b) Rolling friction: It is the friction between the two bodies when one body rolls over the other body.

Based on the surface of contact there are two types of friction namely

- i) Dry friction
- ii) Fluid friction

Dry friction: If the contact surfaces between the two bodies are dry, then the friction between two bodies is known as dry friction.

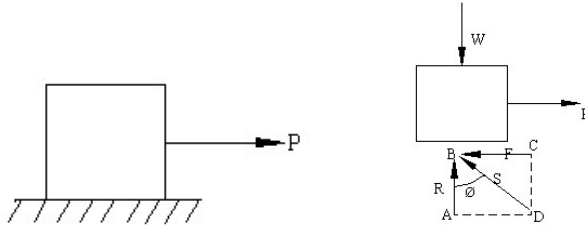
Fluid friction: The friction between two fluid layers or solid and fluid is known as fluid friction.

Angle of friction (ϕ):

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It is the angle made by the resultant(s) of normal reaction and frictional force with normal.

Let us consider a body of mass 'm' which is placed over a rough surface subjected to an external force 'P' as shown in figure.

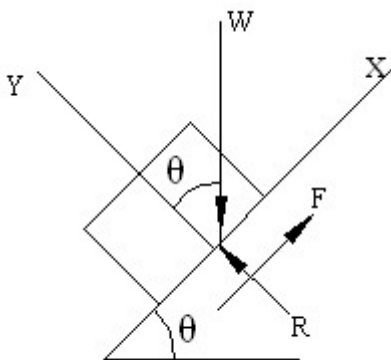
The following forces are acting on a body,

- i) Self-weight, W
- ii) External force, P
- iii) Frictional force, F
- iv) Normal reaction, R

In Triangle ABD

$$\tan \phi = \frac{AD}{AB} = \frac{F}{R} \Rightarrow \tan \phi = \mu$$

Angle of repose (θ):



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It is the maximum angle made by the inclined plane with horizontal, when the body placed on that plane is just at the point of sliding down the plane.

Let us consider a body of mass 'm' which is placed on an inclined plane as shown in figure. The body is just at the point of sliding down the plane when the inclination is θ . The various forces acting on the body are self-weight, normal reaction, frictional force.

Apply the conditions of equilibrium

$$\Sigma F_x=0, \Sigma F_y=0$$

Resolving, along x-axis

$$-F + W \sin \theta=0$$

$$F=W \sin \theta \quad \text{----- (1)}$$

Resolving forces along y-axis

$$R-W \cos \theta =0$$

$$R=W \cos \theta \quad \text{----- (2)}$$

We know that

$$\mu = \frac{F}{R}$$

$$\Rightarrow \mu = \frac{w \sin \theta}{w \cos \theta} = \tan \theta \quad \text{----- (B)}$$

$$\tan \phi = \tan \theta$$

$$\theta = \phi$$

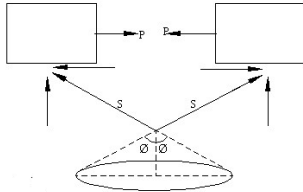
From (A) and (B) we can prove that **Angle of friction=Angle of repose**

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Cone of friction:



The combination of resultant of frictional force and normal reaction obtained by applying forces in opposite direction successively form a right circular cone of angle 2ϕ known as cone of friction.

Laws of friction:

Laws of static friction are:

- i) The maximum value of friction when the body is at rest is known as limiting friction.
- ii) Ratio between the limiting friction and normal reaction is known as coefficient of friction.
- iii) Friction is independent of surface area of contact.
- iv) Friction depends on the nature of the contact surface.

Laws of dynamic friction:

- i) The frictional force acts in a direction opposite to the direction of motion.
- ii) Static friction is greater than dynamic friction.
- iii) Sliding friction is greater rolling friction.

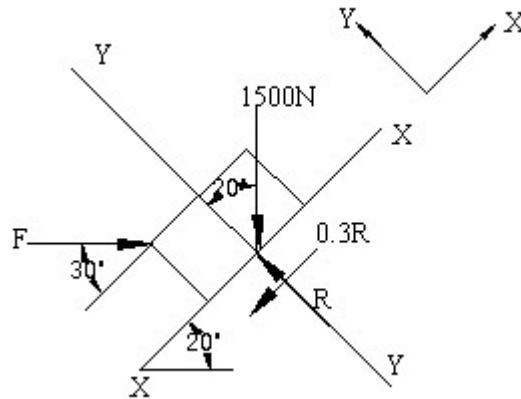
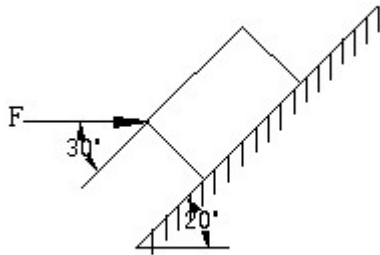
PROBLEMS

1. A block weighing 1500N rests on a plane inclined at 20° to the horizontal. If $\mu = 0.3$, Find the force required to push the block up the plane when the line of action of force makes an angle of 30° with plane.

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$$\sum F_y = 0$$

$$-F \sin 30^\circ - 1500 \cos 20^\circ + R = 0$$

$$R = F \sin 30^\circ + 1500 \cos 20^\circ$$

$$= 0.5F + 1409.539$$

$$\sum F_x = 0$$

$$F \cos 30^\circ - 1500 \sin 20^\circ - 0.3(0.5F + 1409.539) = 0$$

$$0.866F - 513.03 - 0.15F - 422.861 = 0$$

$$0.716F = 935.891$$

$$F = 1307.110 \text{ N}$$

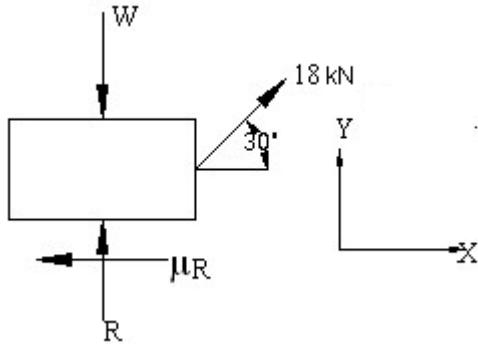
3. A body resting in plane (horizontal) required a pull of 18 kN inclined at 30° to plane just to move it. It was also found that a push of 22 kN inclined at 30° to the plane just moved the body. Determine weight of the body and co-efficient of friction.

When a pull of 18 kN is applied.

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$$\sum F_y = 0$$

$$R - W + 18 \sin 30 = 0$$

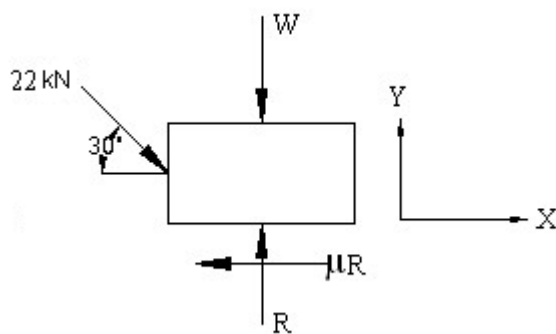
$$R = W - 9 \text{ ----- (1)}$$

$$\sum F_x = 0$$

$$-\mu(W - 9) + 18 \cos 30 = 0$$

$$\mu = \frac{15.588}{(W - 9)} \text{ ----- (2)}$$

When a push of 22 kN is applied.



$$\sum F_y = 0$$

$$R - W - 22 \cos 30 = 0$$



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$$R=W+11$$

$$\Sigma F_x=0$$

$$-\mu (W-11)+22\cos30=0$$

$$\mu = \frac{19.053}{W + 11} \text{ ----- (3)}$$

Equating eqns. (2) and (3), we get

$$\frac{19.053}{W + 11} = \frac{15.588}{W + 9}$$

$$\frac{W - 9}{W + 11} = \frac{15.588}{19.053}$$

$$W-9=0.818W+8.9997$$

$$0.182W=17.9997$$

$$W=98.899\text{KN.}$$

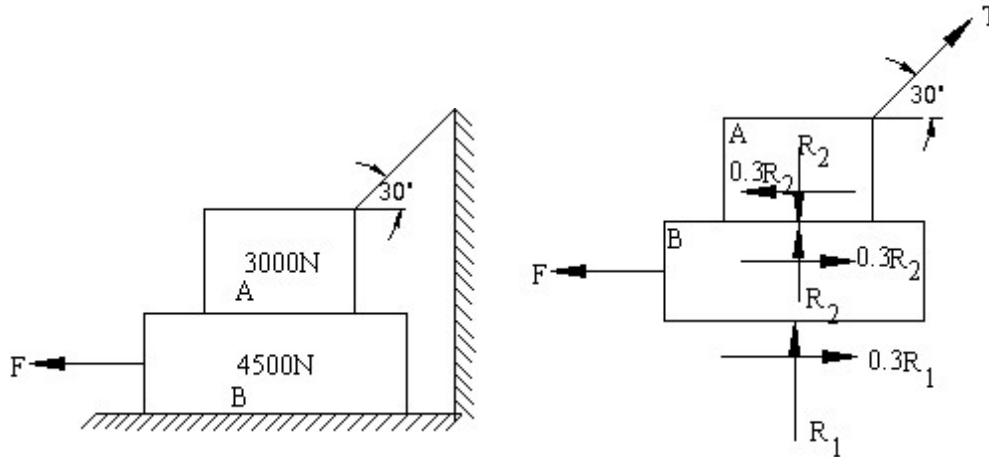
$$\mu = \frac{19.053}{W + 11} = \frac{19.053}{109.899} = 0.173$$

is resting on horizontal surface supports another block of 3000N as shown in figure. Find the horizontal force F just to move the block to the left. Take coefficient of friction for all contact surfaces as 0.3.

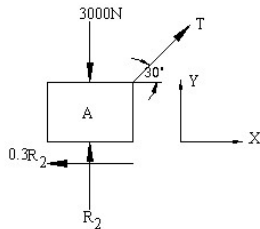
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When one body is placed over the other body, equal and opposite reactions and also equal and opposite frictional force will come into existence between the contact surface of two bodies.



$$\sum F_y = 0$$

$$-3000 + R_2 + T \sin 30^\circ = 0$$

$$R_2 = 3000 - 0.5T$$

$$\sum F_x = 0$$

$$T \cos 30^\circ - 0.3(3000 - 0.5T) = 0$$

$$T \cos 30^\circ - 900 + 0.15T = 0$$

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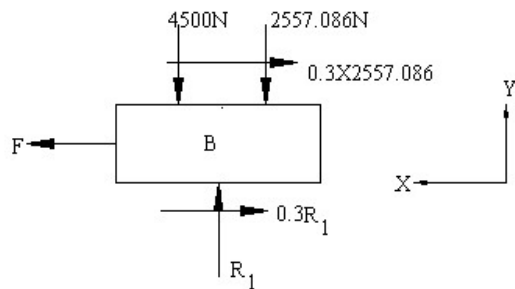
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1.016T=900

T=885.827N.

R₂=2557.086N.



$$\sum F_y = 0$$

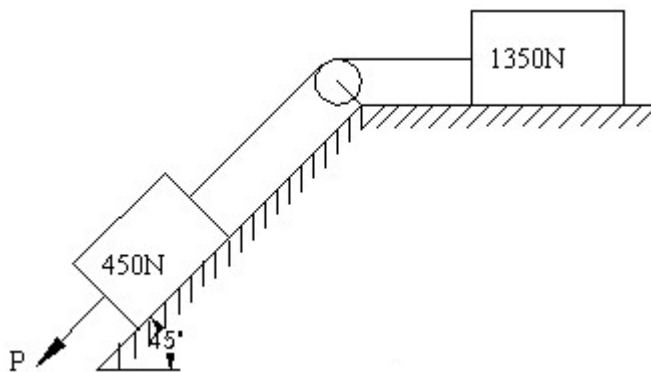
$$R_1 = 4500 + 2557.086 = 7057.086 \text{ N.}$$

$$\sum F_x = 0$$

$$F + 0.3(7057.086) + 0.3 \times 2557.086 = 0$$

$$F = 2884.252 \text{ N.}$$

4. Determine the necessary force P acting parallel to the plane as shown in figure cause motion to impend. Take $\mu = 0.25$

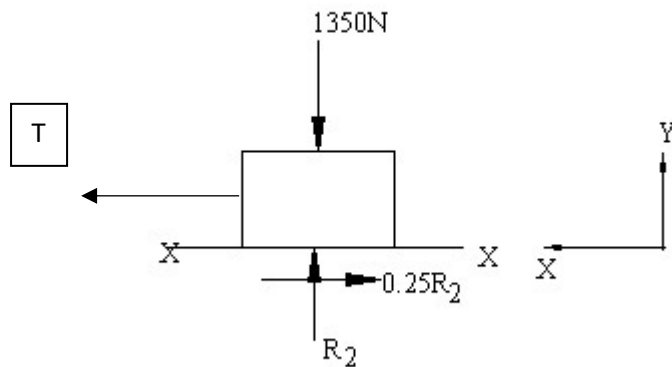


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Consider F.B.D. of 1350N block:



$$\sum F_y = 0$$

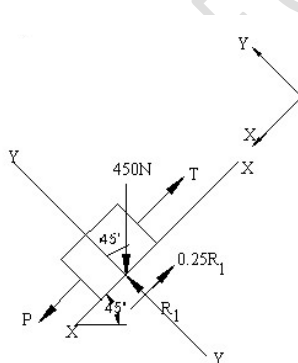
$$R_2 = 1350\text{N}$$

$$\sum F_x = 0$$

$$+\mu R_2 - T = 0$$

$$T = 0.25 \times 1350 = 337.5\text{N}$$

Consider F.B.D. of 450N block:



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$$\sum F_y = 0$$

$$-450 \cos 45^\circ + R_1 = 0$$

$$R_1 = 450 \cos 45^\circ$$

$$= 318.198 \text{ N}$$

$$\sum F_x = 0$$

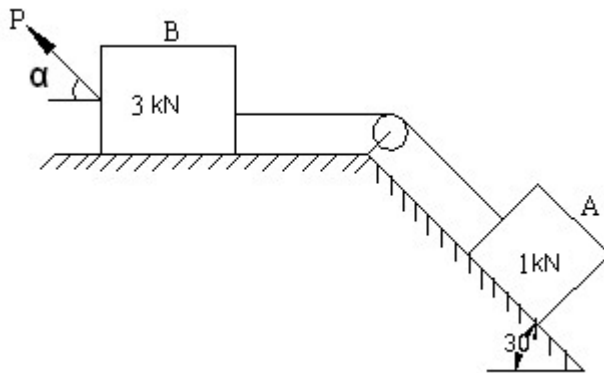
$$-\mu R_2 + P - T + 450 \sin 45^\circ$$

$$P = 0.25 \times 318.198 + 337.5 - 450 \sin 45^\circ$$

$$= 417.05 \text{ N} - 450 \sin 45^\circ$$

$$P = 98.852 \text{ N}$$

5. Find the least value of P required to cause system of block shown in figure to have impending motion to the left. $\mu = 0.2$

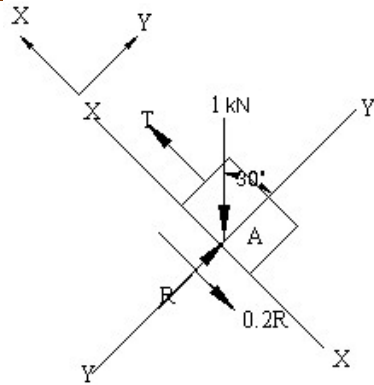


Consider F.B.D. of block A

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$$\sum F_y = 0$$

$$R_1 - 1 \times \cos 30^\circ = 0$$

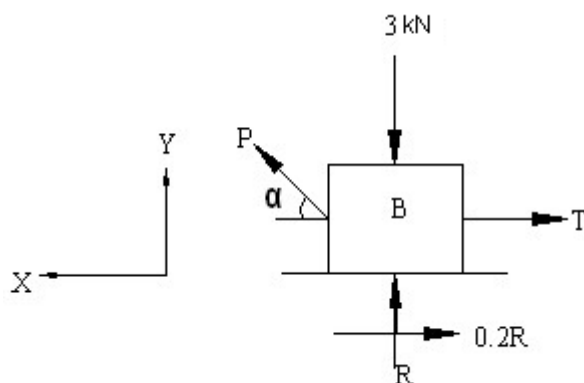
$$R_1 = 0.866 \text{ kN}$$

$$\sum F_x = 0$$

$$T - 0.2(0.866) - 1 \times \sin 30^\circ = 0$$

$$T = 0.673 \text{ kN}$$

Consider F.B.D. of block B



$$\sum F_y = 0$$



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$$R_2 = 3 - P \sin \alpha$$

$$\sum F_x = 0$$

$$P \cos \alpha - 0.673 - 0.2(3 - P \sin \alpha) = 0$$

$$P \cos \alpha + 0.2 P \sin \alpha - 1.273 = 0$$

$$P \cos \alpha + 0.2 P \sin \alpha = 1.273$$

$$P (\cos \alpha + 0.2 \sin \alpha) = 1.273$$

$$P = 1.273 / (\cos \alpha + 0.2 \sin \alpha)$$

P is least, when denominator is maximum

Denominator is maximum when,

$$\frac{d}{d\alpha} (\cos \alpha + 0.2 \sin \alpha) = 0$$

$$-\sin \alpha + 0.2 \cos \alpha = 0$$

$$\sin \alpha = 0.2 \cos \alpha$$

$$\tan \alpha = 0.2$$

$$\alpha = \tan^{-1}(0.2)$$

$$= 11.31^\circ$$

$$\therefore P = 1.273 / (\cos 11.31^\circ + 0.2 \sin 11.31^\circ)$$

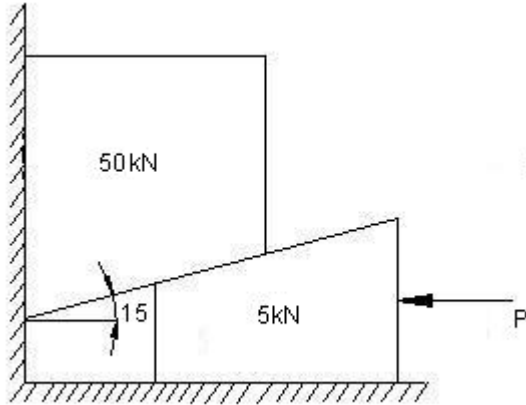
$$P = 1.248 \text{ kN}$$

5. A block of weight 50 kN is kept in equilibrium, by a wedge as shown in figure. If co-efficient of friction is 0.2 for all the surfaces and wedge has a weight of 5 kN, determine the force P necessary to cause a tendency in the block to move up.

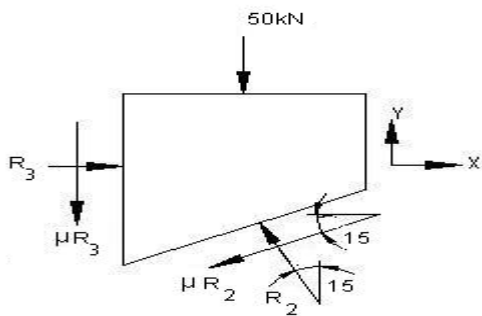
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Consider the F.B.D. of block



$$\sum F_y = 0$$

$$R_2 \cos 15 - 0.2R_2 \sin 15 - 0.2R_3 - 50 = 0 \quad \text{---(1)}$$

$$\sum F_x = 0$$

$$R_3 - R_2 \sin 15 - 0.2R_2 \cos 15 = 0 \quad \text{---(2)}$$

Solving eqns.(1) and(2),we get

$$R_2 = 60.69 \text{ kN}$$

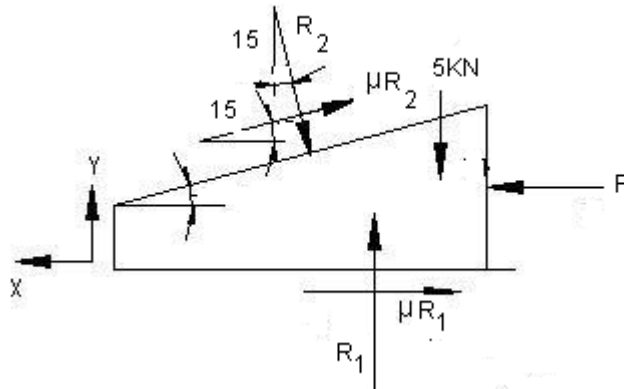
$$R_3 = 27.37 \text{ kN}$$

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Consider F.B.D. of wedge



$$\sum F_x = 0$$

$$-P + 0.2R_2 \cos 15 + R_2 \sin 15 + 0.2R_1 = 0$$

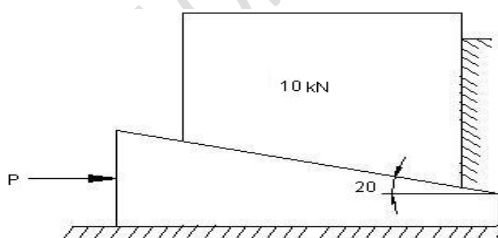
$$\sum F_y = 0$$

$$R_1 + 0.2R_2 \sin 15 - R_2 \cos 15 - 5 = 0$$

$$R_1 = 60.48 \text{ kN}$$

$$P = 39.516 \text{ kN}$$

6. What is the magnitude of force P read to raise the block having of weight 10kN. The coefficient of Friction for the entire surface is 0.25. Angle of wedge is 20°.

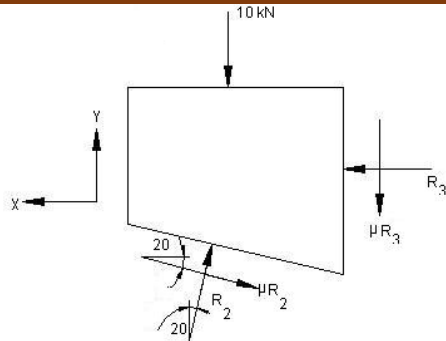


Consider the F.B.D. of Block

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$$\sum F_x = 0$$

$$R_2 \sin 20 + 0.25R_2 \cos 20 - R_3 = 0$$

$$0.576R_2 = R_3$$

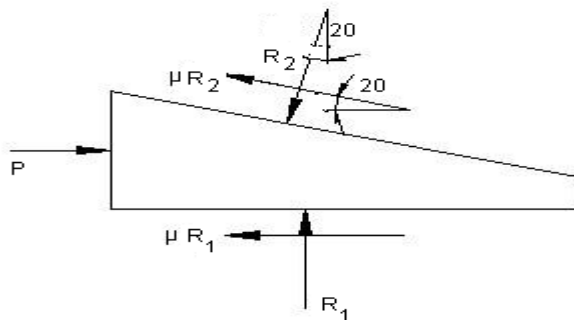
$$\sum F_y = 0$$

$$R_2 \cos 20 - 0.25R_2 \sin 20 - 0.25R_3 - 10 = 0$$

$$0.854R_2 - 0.25 \times 0.576R_2 - 10 = 0$$

$$R_2 = 14.08N$$

Consider F.B.D. of Wedge



$$\sum F_x = 0$$

$$P - 0.25R_1 - R_2 \sin 20 - 0.25R_2 \cos 20 = 0$$

$$P - 0.25R_1 - 8.1275 = 0$$

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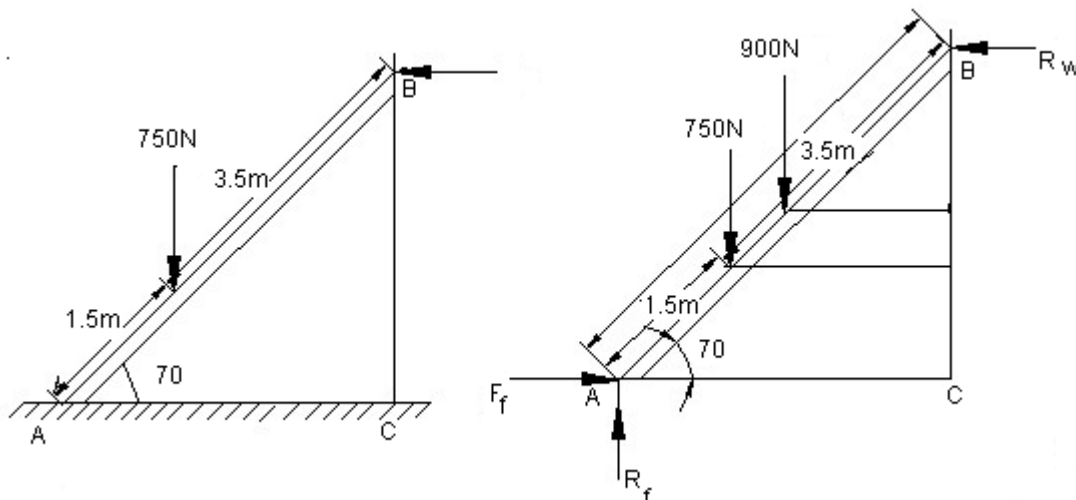
$$\sum F_y = 0$$

$$R_1 - R_2 \cos 20 + 0.25R_2 \sin 20 = 0$$

$$R_1 = 12.03 \text{ kN}$$

$$P = 0.25 \times 12.03 + 8.1275 = 11.135 \text{ kN}$$

7. A ladder 5m long rests on a horizontal ground and leans against a smooth wall at an angle of 70° with horizontal. The weight of ladder is 900N and a man weighing 750N stands on a ladder 1.5m from the bottom of ladder. Calculate coefficient of friction between ladder and floor.



Let μ_f = coefficient friction between ladder and floor

$$\sum F_y = 0$$

$$R_f = 750 + 900 = 1650 \text{ N}$$

$$F_f = \mu_f R_f = 1650 \mu_f$$

Now taking moment at B

$$1650 \times 1.710 - \mu_f \times 1650 \times 4.698 - 750 \times 1.197 - 900 \times 0.855$$

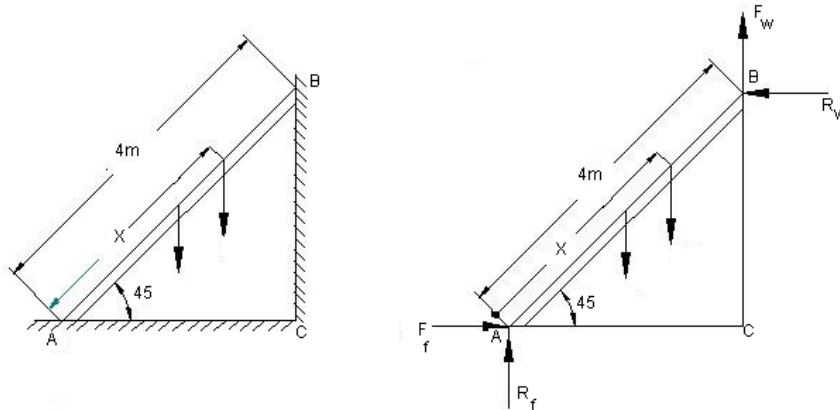
$$\mu_f = 0.149$$

8. A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45° . The coefficient of friction between ladder and wall is 0.4 and between ladder and floor is 0.5. If the man whose weight is one half of that of ladder ascends it how high will it be when the ladder slips?

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Given: $l=4M$, $\alpha =45$, $\mu_w=0.4$, $\mu_f=0.5$

Let x =distance between A& the man when the ladder is at the point of slipping.

$W = \omega \text{ of ladder}$

$\frac{W}{2} = \omega \text{ of man}$

From limiting friction, we have

$$F_f = \mu_f R_f = 0.5 R_f \text{ ----- (i)}$$

$$F_w = \mu_w R_w = 0.4 R_w \text{ ---(ii)}$$

Resolving the forces vertically,

$$R_f - W - 0.5W + F_w = 0$$

$$R_f + F_w = W + 0.5W$$

$$R_f + F_w = 1.5W$$

$$R_f + \mu_w R_w = 1.5W$$

$$R_f + 0.4 R_w = 1.5W \text{ -----(1)}$$

Resolving the forces horizontally,

$$F_f - R_w = 0$$

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$$\mu_f R_f - R_w = 0$$

$$0.5 R_f - R_w = 0$$

$$R_w = 0.5 R_f$$

$$R_f = 2R_w \text{ -----(2)}$$

Substitute equation (2) in equation (1)

$$2R_w + 0.4R_w = 1.5W$$

$$R_w = 0.625W$$

$$F_w = \mu_w R_w$$

$$F_w = 0.4 \times 0.625W$$

$$= 0.25W$$

Taking moments about A and equating,

$$(W \times 2 \cos 45^\circ) + (0.5W \times x \cos 45^\circ) - (F_w \times 4 \cos 45^\circ) - (R_w \times 4 \sin 45^\circ) = 0$$

$$(W \times 2 \cos 45^\circ) + (0.5W \times x \cos 45^\circ) = (F_w \times 4 \cos 45^\circ) + (R_w \times 4 \sin 45^\circ)$$

$$1.414 W + 0.353 W x = (0.25W \times 4 \cos 45^\circ) + (0.625W \times 4 \sin 45^\circ)$$

$$1.414 W + 0.353 W x = 0.707 W + 1.767 W$$

$$1.414 W + 0.353 W x = 2.474 W$$

$$0.353 W x = 2.474 W - 1.414 W$$

$$0.353 W x = 1.06 W$$

$$x = \frac{1.06 W}{0.353 W}$$

$$x = 3.0 \text{ m}$$

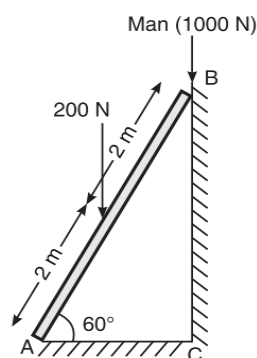
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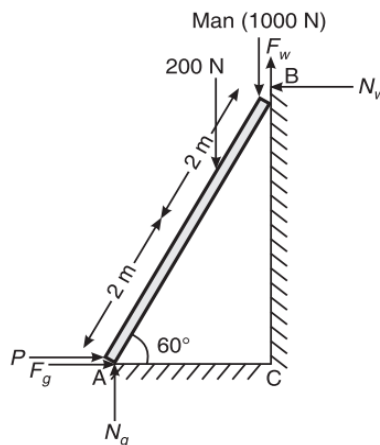
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9. A uniform ladder 4m long weighs 200N. It is placed against a wall making an angle of 60° with the floor. The coefficient of friction between the wall and the ladder is 0.25 and that between the ground and the ladder is 0.35. The ladder in addition to its own weight, has to support a man of 1000N at the top at B. Calculate: (i) The horizontal force P to be applied to the ladder at the ground level to prevent slipping. (ii) If the force P is not applied, what should be the minimum inclination of the ladder with the horizontal, so that it does not slip with the man at the top?

(Note: Normal Reaction can be taken as N or R)



i) Horizontal force P applied to ladder at the ground level to prevent slipping



Resolving forces horizontally,

$$P + F_g = N_w$$

Resolving forces vertically,

$$N_g + F_w = 200 + 1000$$

Taking moments about A,

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$$(1000 \times 4 \cos 60) + (200 \times 2 \cos 60) = (F_w \times 4 \cos 60) + (N_w \times 4 \sin 60)$$

$$2200 = 0.25 \times N_w \times 4 \cos 60 + N_w \times 4 \sin 60$$

$$N_w = 554.98 \text{ N}$$

$$F_w = \mu_w N_w$$

$$= 0.25 \times 554.98 = 138.75 \text{ N}$$

$$N_g + F_w = 200 + 1000$$

$$N_g + 138.75 = 200 + 1000$$

$$N_g = 1061.25 \text{ N}$$

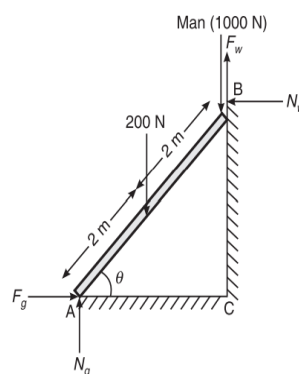
$$F_g = \mu_g N_g$$

$$F_g = 0.35 \times 1061.25 = 371.44 \text{ N}$$

$$P = N_w - F_g$$

$$P = 554.98 - 371.444 = 183.54 \text{ N}$$

ii) Inclination of ladder with the horizontal



Now consider the ladder inclined at an angle θ and without any horizontal force acting at the ground level.

Resolving horizontally,



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$$N_w = F_g = \mu_g N_g = 0.35 N_g$$

Resolving vertically,

$$N_g + F_w = 1000 + 200$$

$$N_g + 0.25 N_w = 1200$$

$$N_g + 0.35 \times 0.25 N_g = 1200$$

$$N_g = 1103.45 \text{ N}$$

$$N_w = 0.35 \times 1103.45 = 386.21 \text{ N}$$

$$\text{Similarly, } F_w = \mu_w N_w = 0.25 \times 386.21 = 96.55 \text{ N}$$

Taking moments about A and equating the same,

$$(1000 \times 4 \cos\theta) + (200 \times 2 \cos\theta) = (F_w \times 4 \cos\theta) + (N_w \times 4 \sin\theta)$$

Dividing throughout by $\cos\theta$

$$(386.21 \times 4) \tan\theta = (1000 \times 4) + (200 \times 2) - (96.55 \times 4)$$

$$\tan\theta = 2.598$$

$$\theta = 68.95^\circ$$

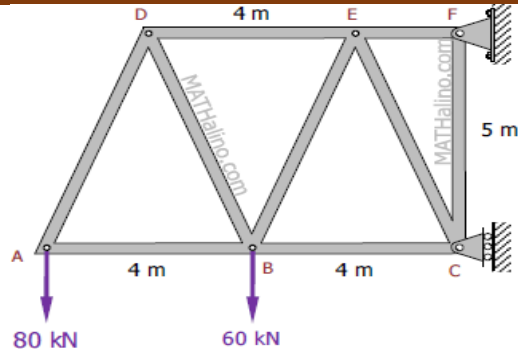
REVIEW QUESTIONS:

1. The truss is pinned to the wall at point F, and supported by a roller at point C. Calculate the force (tension or compression) in members BC, BE, and DE using method of sections.

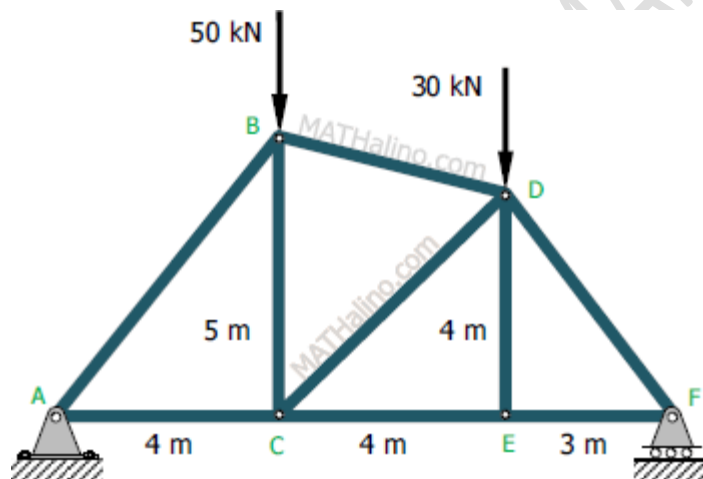
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2. Compute the force in all members of the truss using method of joints

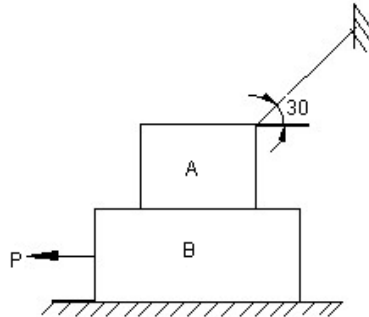


3. Two blocks A & B of weight 1 kN & 2 kN respectively are in equilibrium as shown in figure. If the coefficient of friction between two blocks A & B & floor is 0.3 find the force P required to move the block.

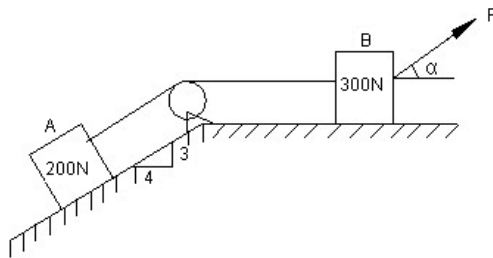
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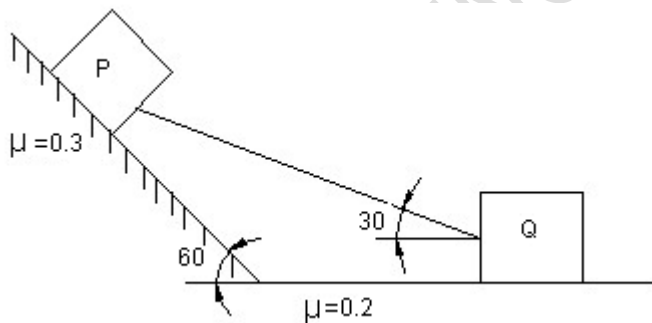
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4. Find the least value of P that will just start the system blocks shown in figure moving to the right. Take $\mu = 0.3$



5. Two blocks P & Q are connected by a rigid bar as shown in figure. If the weight of block P is 2800 N. Determine the minimum weight of block Q to prevent sliding.

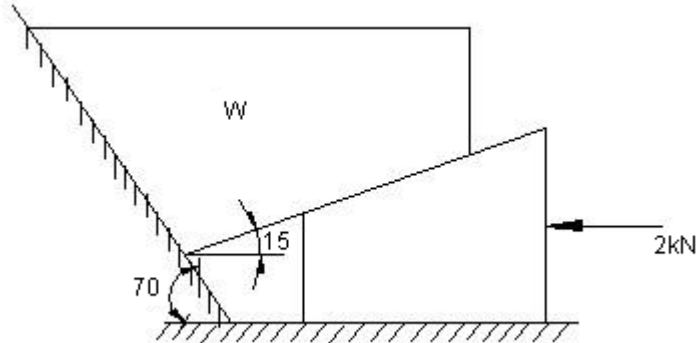


6. Find the Maximum weight of the block that can be just lifted if the wedge is pushed in by a force of 2KN for the wedge block system as shown in figure. Coefficient of friction between block and wall is 0.3, wedge and floor is 0.20 and that of contact between block and wedge is 0.25. Neglect the weight of the wedge.

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7. A ladder weighing 200N and 4m long is placed on the ground at an angle of 60° and at the top of it rests against the wall. A man weighing 1000N stands at the top of the ladder. The coefficient of friction between the wall and ladder is 0.30 and that between the ground and ladder is 0.35. If the ladder is not slip, find the horizontal force P required to be applied to ladder at its end on the floor.

