VISVESVARAYA TECHNOLOGICAL UNIVERSITY
BELGAUM

ENGINEERING MECHANICS

(Subject Code: BCIVC203)

LECTURE NOTES

(MODULE-4)

II-SEMESTER

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DEPARTMENT OF CIVIL ENGINEERING

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ENGINEERING MECHANICS

Subject Code: BCIVC203 **Department: Civil Engineering**

MODULE 4

Centroid of Plane areas: Introduction, Locating the centroid of rectangle, triangle, circle, semicircle, quadrant and sector of a circle using method of integration, centroid of composite areas and simple built-up sections, Numerical examples.

Moment of inertia of plane areas: Introduction, Rectangular moment of inertia, polar moment of inertia, product of inertia, radius of gyration, parallel axes theorem, perpendicular axis theorem, moment of inertia of rectangular, triangular and circular areas from the method of integration, moment of inertia of composite areas and simple built-up sections, Numerical examples.

Centre of gravity:

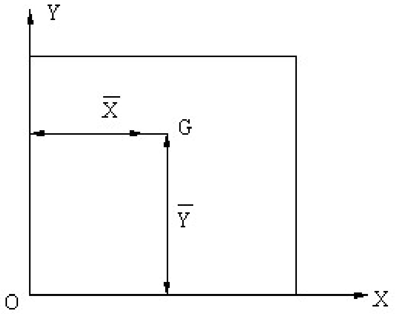
It is the point where the whole weight of the body is supposed to be concentrated.

It is denoted by 'C.G.' or 'G'

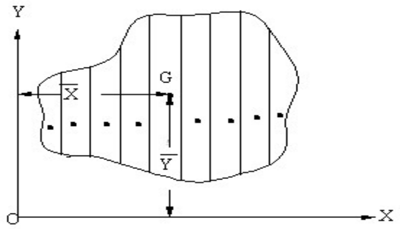
Centroid: (C.G) or (G)

It is the point where the whole area of the plane figure is supposed to be concentrated.

Calculation of centroid means determination of $\bar{x}$ and $\bar{y}$



Determination of centroid by the method of Moments



Let us consider a plane figure of total area A. The centroid of whole figure is located at a distance of $\bar{x}$ from y axis and $\bar{y}$ from x axis.

Let us divide the whole figure in to a number of elemental strips of areas a_1, a_2, a_3, a_4………

Whose centroid is located at a distance of x_1, x_2, x_3 ……..from oy axis and y_1, y_2, y_3, y_4 ……from ox axis

Apply the theorem of moments about oy axis

$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 +$ ………

$\bar{x} = (a_1x_1+a_2x_2+a_3x_3+\ldots..)/A$

$\bar{x} = \frac{\sum ax}{\sum a}$ $\bar{x} = \int \frac{xdA}{A}$

Apply theorem of moments about x axis

$A\bar{y} = a_1y_1+a_2y_2+a_3y_3+\ldots\ldots$

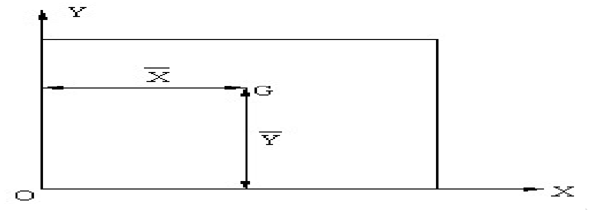
$\bar{y} = (a_1y_1+a_2y_2+a_3y_3+\ldots.)/A$

$\bar{y} = \frac{\sum ay}{\sum a}$

$\bar{y} = \int \frac{ydA}{A}$

Axis of reference:

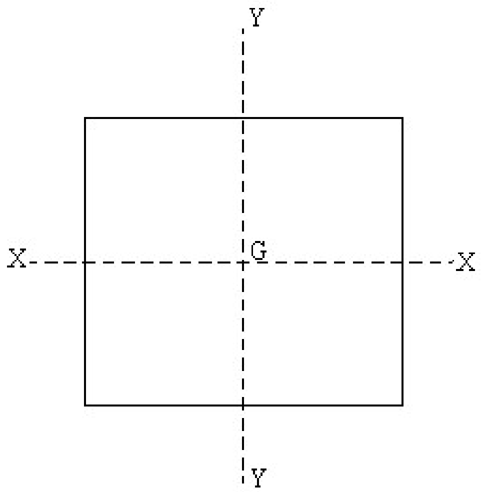
It is the axis with respect to which the centroid of given figure is determined.



Generally the left line and bottom line is considered as reference axis w.r.t which the centroid of given figure is measured

Centroidal axis:

The axis which passes through the centroid of given figures is known as Centroidal axis.

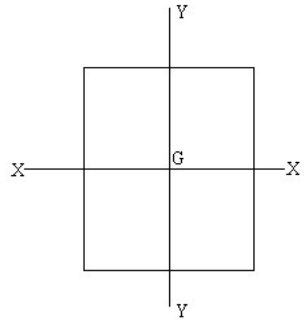


Symmetrical axis:

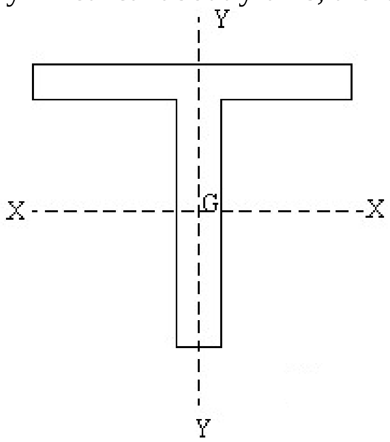
It is the axis, which divides the whole figure in to equal parts

a) Symmetrical about both the axis:

$\bar{x}=0$, $\bar{y}=0$

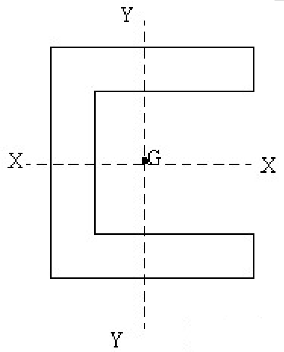


b) Symmetrical about y-axis, therefore $\bar{x}=0$

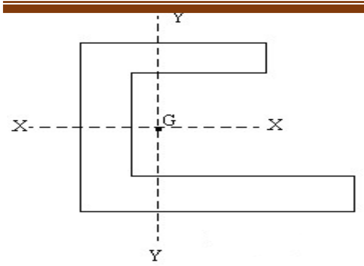


The area in the left portion and the right portion of the y axis are equal.

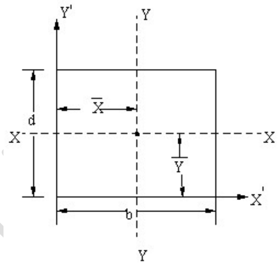
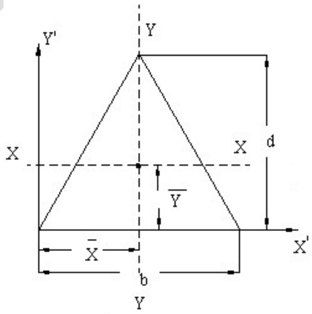
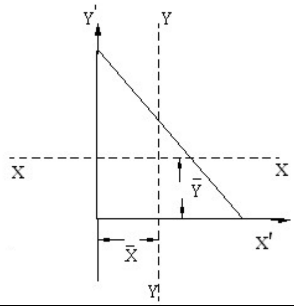
c) Symmetrical about x-axis, therefore $\bar{y}=0$

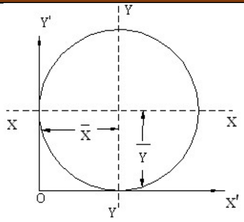
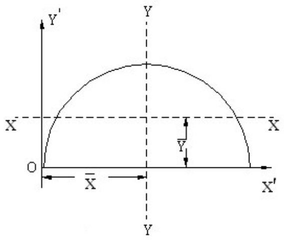
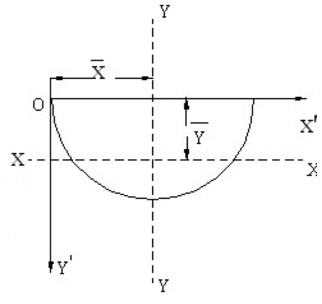
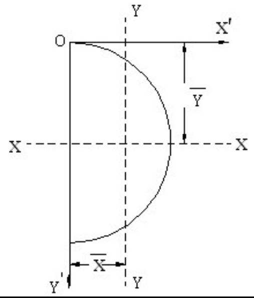
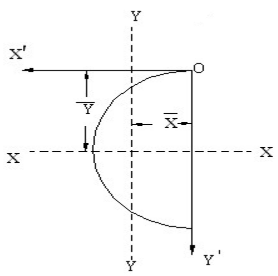


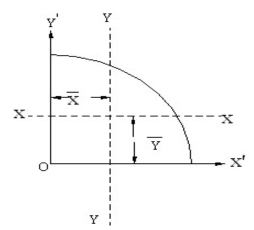
d) No axis of symmetry. Calculate both $\bar{x}$ and $\bar{y}$



Centroid of some important geometrical figures:

Shape	Area	$\bar{x}$	$\bar{y}$	Figure
Rectangle: (Same for square)	bd	b/2	d/2	
Triangle:	1/2bd	b/2	$1/3 \times d$	
Right angled triangle:	1/2bd	$1/3 \times b$	$1/3 \times d$	
Circle:	πr^2	$\bar{x}$ =r	$\bar{y}$ =r	

				
Semicircle:	$\frac{\pi r^2}{2}$	d/2	$\frac{4r}{3\pi}$	
		d/2	$\frac{-4r}{3\pi}$	
		$\frac{4r}{3\pi}$	-d/2	
		$\frac{-4r}{3\pi}$	d/2	

Quarter circle:	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	

Reference Table:

Component	Centroidal distance from the reference y-axis(x)	Centroidal distance from reference x-axis (y)	Area (a)	ax	Ay
			Σa	Σax	Σay

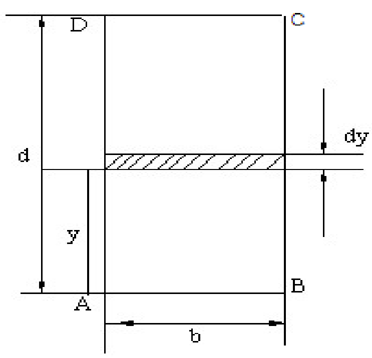
$$\bar{x} = \frac{\sum ax}{\sum a} \qquad \Sigma ax = a_1x_1 + a_2x_2 + a_3x_3 + \ldots\ldots$$

$$\bar{y} = \frac{\sum ay}{\sum a} \qquad \Sigma ay = a_1y_1 + a_2y_2 + a_3y_3 + \ldots\ldots$$

$$\Sigma a = a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots$$

Derivation of Centroid of Some important Geometrical Figures:

Rectangle:



Let us consider a rectangular lamina of area b × d as shown in figure. Now consider a horizontal elemental strip of area b× dy. which is at a distance of y from the reference axis AB.

Moment of area of elemental strip about AB

= b × dy × y

Sum of moments of such elemental strips @ AB is given by

$$\int_0^d b \times dy \times y \qquad = b \int_0^d dy \; x \; y$$

$$= b\int_0^d y.dy$$

$$= b \times \left[\frac{y^2}{2}\right]_0^d \; b \; x \; \frac{d^2}{2} = bd \; x \; y$$

$$= \frac{bd^2}{2}$$

Moment of total area about AB= bd × $\bar{y}$

Apply the principle of moments about AB,

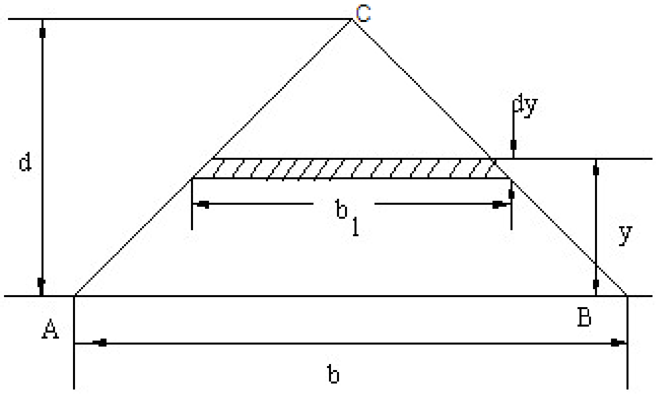
$$\frac{bd^2}{2} = bd \times \bar{y}$$

$$\bar{y} = \frac{d}{2}$$

By considering the vertical strip, similarly we can prove that

$$\bar{x} = \frac{b}{2}$$

Triangle:



Consider a triangular lamina of area ½ × b × d as shown in figure

Now consider an elemental strip of area $b_1 \times dy$ which is at a distance of y from reference axis AB.

Using similar triangle property

$$\frac{b_1}{b} = \frac{d-y}{d}$$

$$b_1 = \frac{(d-y)b}{d}$$

Area of the elemental strips $= b_1 \times dy$

$$= \frac{(d-y)b.dy}{d}$$

Moment of area of strip about AB

$= \text{area} \times y$

$$= \frac{(d-y)b.dy.y}{d}$$

$$= \frac{bdy.dy}{d} - \frac{by^2.dy}{d}$$

$$= by.dy - \frac{by^2.dy}{d}$$

Sum of moments of such elemental strips is given by

$$\int_0^d by.dy - \int_0^d \frac{by^2}{d}.dy$$

$$= b \times \left[\frac{y^2}{2}\right]_0^d - \frac{b}{d}\left[\frac{y^3}{3}\right]_0^d$$

$$= \frac{bd^2}{2} - \frac{bd^2}{3d}$$

$$= \frac{bd^2}{2} - \frac{bd^2}{3}$$

$$= \frac{bd^2}{6}$$

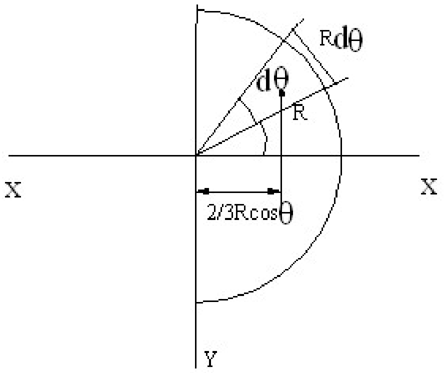
Moment of total area @ AB = $\frac{1}{2}bd \times \bar{y}$

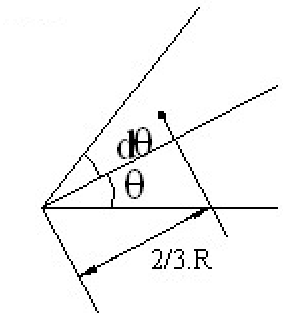
Applying the principle of moments,

$$\frac{bd^2}{6} = \frac{1}{2} \times bd \times \bar{y}$$

$\bar{y} = \frac{d}{3}$

Semicircle:





Consider a semicircular lamina of $\frac{\pi r^2}{2}$ as shown in figure. Now consider a triangular elemental strip of area $\frac{1}{2} \times R \times R \times d\theta$, This strip makes an angle of θ with the x-axis, whose center of gravity is at a distance of $\frac{2}{3}R$ from 0 and its projection along x-axis = $2/3R\cos\theta$

Moment of area of elemental strip about Y-axis

$= \; = \frac{1}{2} \times R^2.d\theta \times 2/3R\cos\theta$

$= \frac{R^3.\cos\theta.d\theta}{3}$

Sum of moments of such elemental strips about y axis

$= \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{R^3}{3}\cos\theta.d\theta$

$= \frac{R^3}{3}[\sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= \frac{R^3}{3}\left[\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right]$

$= \frac{2R^3}{3}$

Moment of total Area about y axis

$= \frac{\pi R^2}{2} \times \bar{x}$

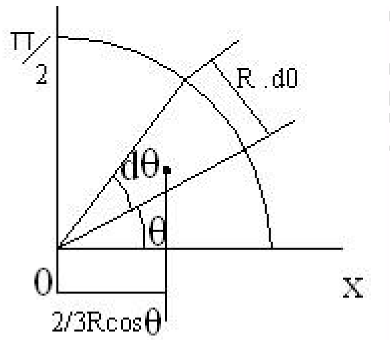
Using principle of moments

$$\frac{2R^3}{3} = \frac{\pi R^2}{2} \times \bar{x}$$

$$\therefore \bar{x} = \frac{2R^3 \times 2}{3R^2 \pi}$$

$$\bar{x} = \frac{4R}{3\pi}$$

Quarter circle:



Consider a quarter circular lamina of area $\frac{\pi R^2}{4}$ as shown in the figure. Consider a triangular elemental strip of area $\frac{1}{2} \times R \times R \times d\theta$ at an angle of θ from the x-axis, whose center of gravity is at a distance of $\frac{2}{3}R$ from 0 and its projection of x-axis = $\frac{1}{2}R\cos\theta$

Moment of area of elemental strip about Y-axis

$$= \frac{\frac{1}{2} \times R^2.d\theta}{\frac{2}{3}R\cos\theta}$$

$$= \frac{R^3.\cos\theta.d\theta}{3}$$

$$= \int_0^{\frac{\pi}{2}} \frac{R^3}{3} \cos\theta.d\theta$$

$$= \frac{R^3}{3}\left[\sin\frac{\pi}{2}\right]$$

$$= \frac{R^3}{3}$$

Sum of moments of such elemental strips about y axis

Moment of total Area about y axis

$$= \frac{\pi R^2}{4} \times \bar{x}$$

Using principle of moments

$$\frac{R^3}{3} = \frac{\pi R^2}{4} \times \bar{x}$$

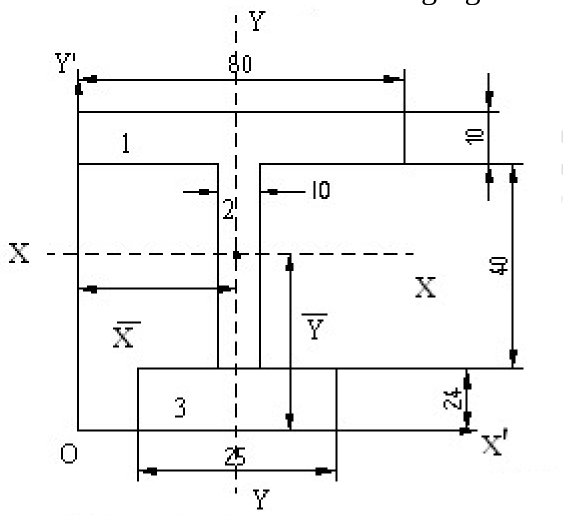
$$\therefore \bar{x} = \frac{4R^3 \times 2}{3R^2 \pi}$$

$$\bar{x} = \frac{4R}{3\pi}$$

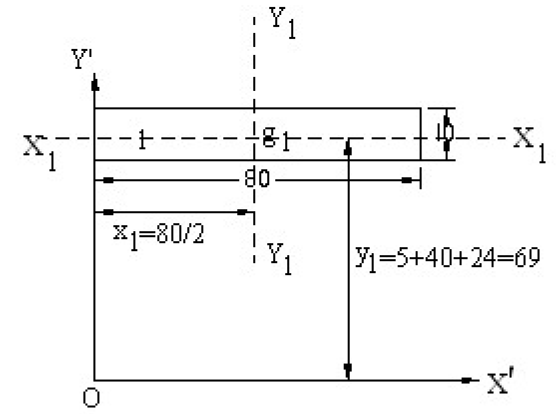
Similarly, we can prove that $\bar{y} = \frac{4R}{3\pi}$

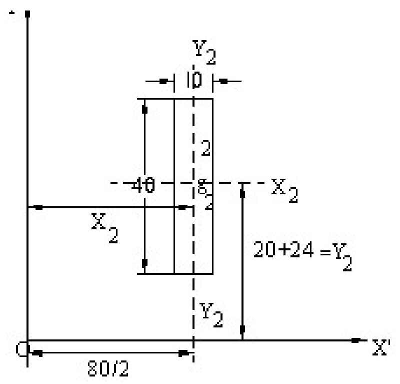
PROBLEMS:

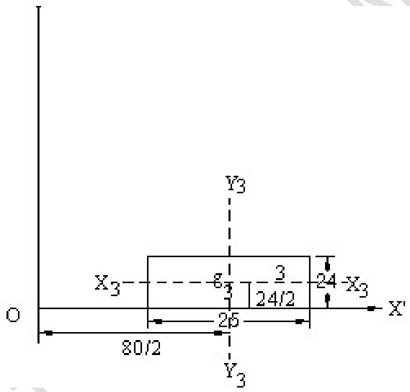
1. Find centroid of the following figure.



All dimensions in mm





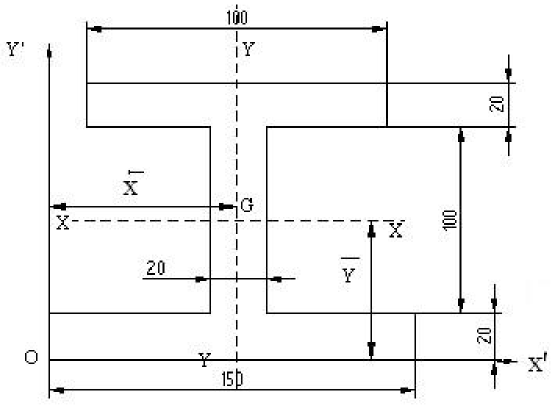


Component	Area a	Centroidal distance from x axis (y)	Centroidal distance from y axis (x)	ax	ay
Rectangle-1	10×80=800	5+40+24=69	40	32000	55200
Rectangle -2	10×40=400	20+24=44	40	16000	17600
Recangle-3	25×24=600	12mm	40	24000	7200
	Σa=1800			Σax=72000	Σay=80000

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{72000}{1800} = 40 \text{ mm}$$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{80000}{1800} = 44.44 \text{ mm}$$

2. Determine the centroid of the figure shown below.



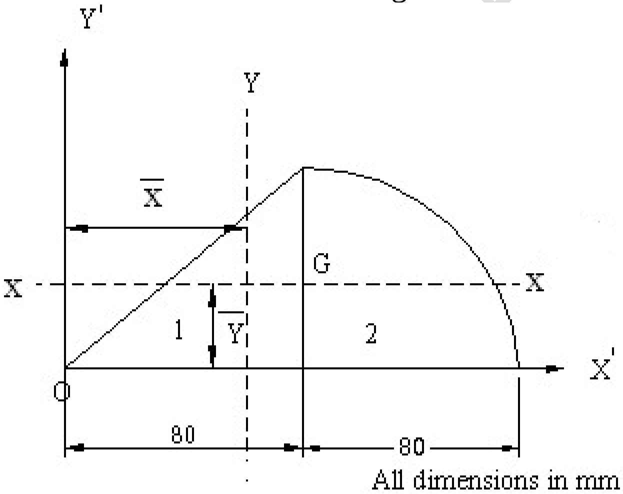
All dimensions are in mm

Component	Area	X	y	ax	Ay
Rectangle-1	100×20=2000	100/2+25=75	20+100+10=130	150000	260000
Rectangle-2	100×20=2000	75	70	150000	140000
Rectangle-3	150×20=3000	75	10	225000	30000
	Σa=7000			Σax=525000	Σay=430000

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{525000}{7000} = 75\,\text{mm}$$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{430000}{7000} = 61.429\,\text{mm}$$

3. Determine centroid of the figure shown below



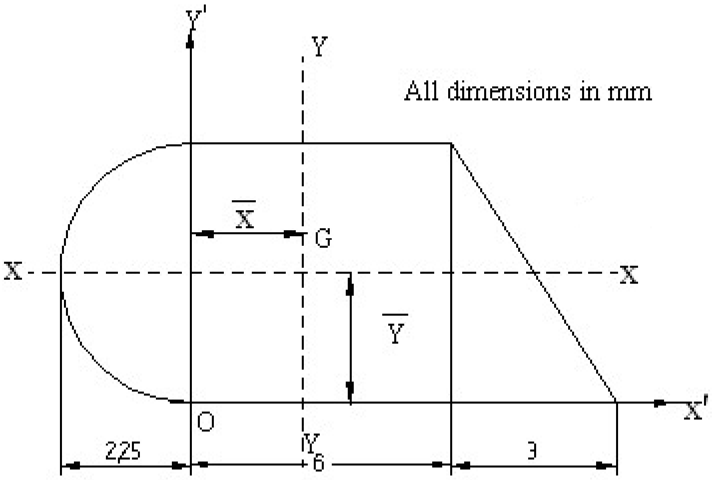
All dimensions in mm

Component	Area	X	y	ax	Ay
Triangle	1/2×80×80 =3200	2/3×80 =53.333	1/3×80 =26.667	170665.6	85334.4
Quarter circle	$\frac{\pi \times 80^2}{4}$ =5026.548	33.953+80 =113.953	$\frac{4\times 80}{3\times\pi}$ =33.953	572790.224	170666.384
	Σa=8226.548			Σax=743455.824	Σay=256000.784

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{743455.824}{8226.548} = 90.373 \text{mm}$$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{256000.784}{8226.548} = 31.119 \text{mm}$$

4. Calculate the centroid of the figure shown

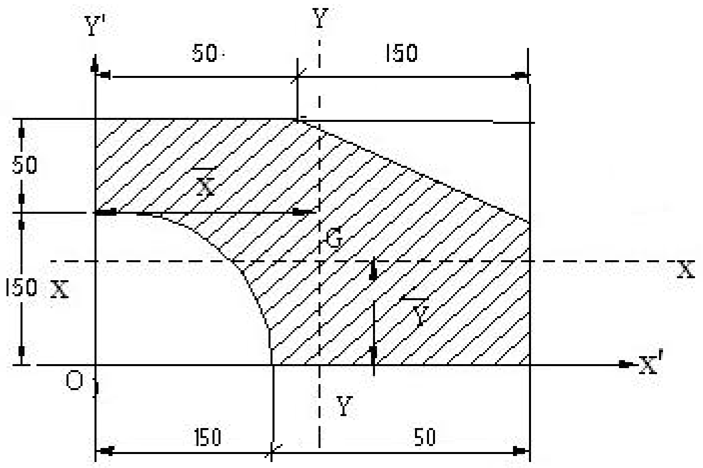


Component	Area	x	y	Ax	Ay
Rectangle	6×4.5=27	3	2.25	81	60.75
Triangle	1/2×3×4.5=6.75	1/3×3+6=7	1/3×4.5=1.5	47.25	10.125
Semicircle	$\frac{\pi\times(2.25)^2}{2}$ =7.95	$\frac{-4\times 2.25}{3\times\pi}$ = -0.955	2.25	-7.594	17.892
	Σa=41.702			Σax=120.65	Σay=88.767

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{120.656}{41.702} = 2.893\text{mm}$$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{88.767}{41.702} = 2.129\text{mm}$$

5. Determine the centroid for the figure shown.



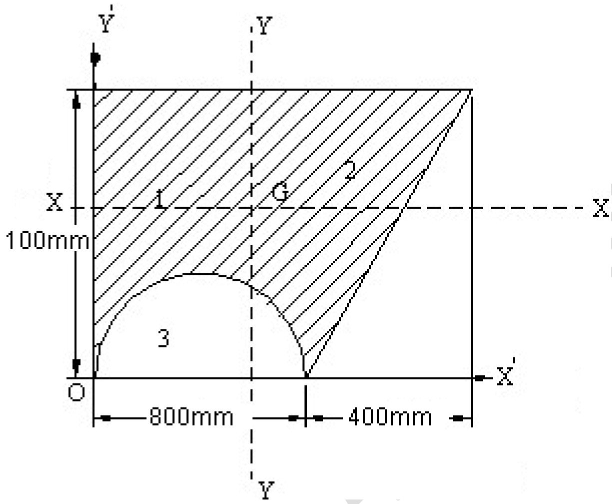
All dimensions in mm

Component	Area	x	y	ax	ay
Rectangle	200×200 =40000	100	100	4000000	4000000
Quarter circle	$\frac{\pi \times (150)^2}{4}$ =17671.459	$\frac{4\times150}{3\times\pi}$ =63.662	63.662	1125000.423	1125000.423
Triangle	$\frac{1}{2}\times150\times150$ =11250	$\frac{2}{3}\times150+50$ =150	$50+\frac{2}{3}\times150$ =150	1687500	1687500

$$\bar{x} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3} = \frac{1187499.577}{11078.541} = 107.189 \text{ mm}$$

$$\bar{y} = \frac{1187499.577}{11078.541} = 107.189mm$$

6. Locate the center of shaded area shown:

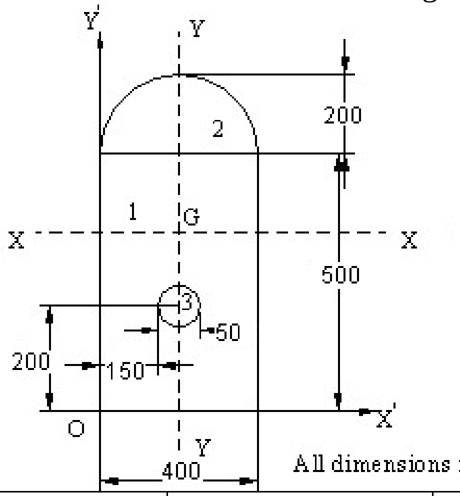


Component	Area(mm²)	x(mm)	y(mm)	ax	Ay
Rectangle	800000	400	500	320000000	400000000
Triangle	$\frac{1}{2}\times 400\times 1000$ =200000	$800+\frac{1}{3}\times 400$ =933.333	$\frac{2}{3}\times 1000$ = 666.667	186666666.667	133333333.33
Semicircle	$-\frac{\pi r^2}{4} = -251327.41$	400	$\frac{4r}{3\pi} = 169.765$	-100530964.8	-42666518.1
	Σa=748672.58			Σax=406135701.8	Σay=490666735.2

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{406135701.8}{748672.588} = 542.474 \text{ mm}$$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{490666735.2}{748672.588} = 655.382 \text{ mm}$$

7. Determine the centroid of the figure shown below



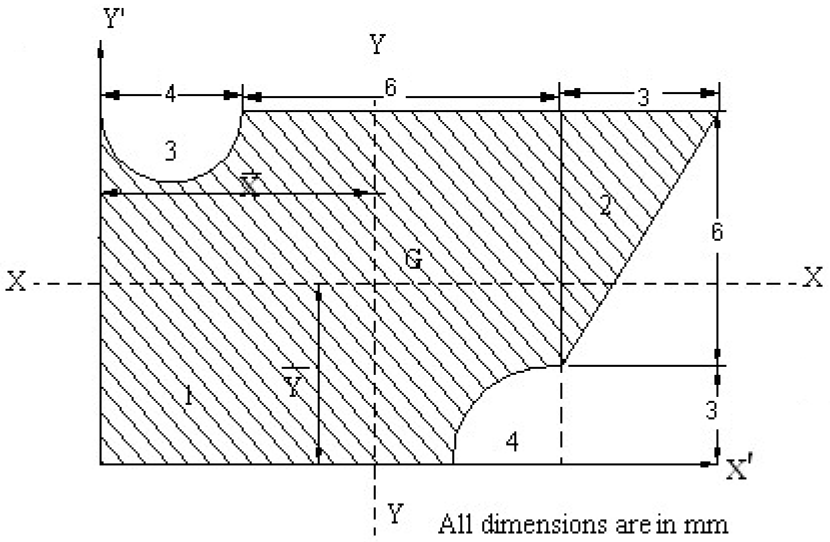
All dimensions in mm

Component	Area	x	y	ax	Ay
Rectangle-1	200000	200	250	40000000	50000000
Semicircle-2	$\frac{\pi\times(200)^2}{2}$ =62831.853	200	$\frac{500+4\times200}{3\pi}$ =84.883	12566370.6	36749282.68
Circle-3	$\pi\times(25)^2$ =--1963.495	175	200	-343611.696	-392699
	Σa=260868.358			Σax=52222758.9	Σay=86356583.68

$$\bar{x}=\frac{\sum ax}{\sum a}=\frac{52222758.9}{260868.358}=200.188\text{ Units}$$

$$\bar{y}=\frac{\sum ay}{\sum a}=\frac{86356583.68}{260868.358}=331.035\text{ Units}$$

8. Locate the centroid of the area shown in figure. All dimensions are in mm.

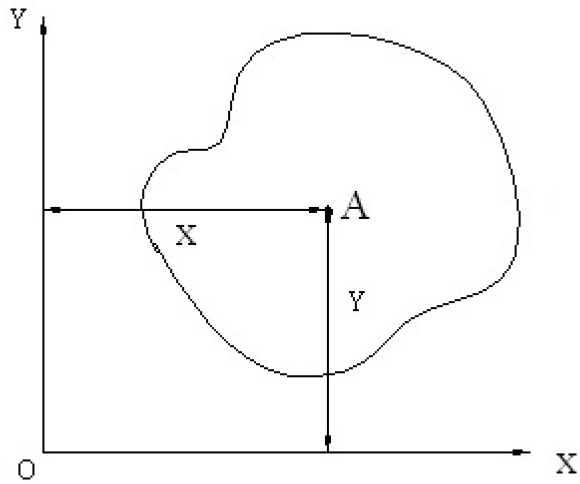


Component	Area	x	y	ax	ay
Rectangle-1	10×9=90	5	4.5	450	405
Triangle-2	3 × 6×1/2 = 9	10+1/3×3= 11	3 + 2/3 ×6=7	99	63
semicircle-3	$\frac{\pi\times(2)^2}{2}=6.283$	4/2 =2	$\frac{9-4\times2}{3\pi}=8.151$	-12.566	-51.213
Quarter circle-4	$\frac{\pi(3)^2}{4}=7.068$	$\frac{10-4\times3}{3\pi}=8.727$	$\frac{4\times3}{3\times\pi}=1.273$	-61.682	-8.998
	Σa = 85.649			Σax = 474.752	Σay = 407.789

$$\bar{x}=\frac{\sum ax}{\sum a}=5.543\,\text{mm}$$

$$\bar{y}=\frac{\sum ay}{\sum a}=4.761\text{mm}$$

MOMENT OF INERTIA



Let us consider an irregular plane lamina of area A, whose center of gravity is at distance of X from the reference Y axis and Y from reference X axis.

Moment of area @ Y axis = (first moment of area). If the first moment of area is multiplied by perpendicular distance X, it gives aX^2 known as second moment of area or moment of inertia.

Moment of inertia about Y-axis , $I_{yy} = AX^2$

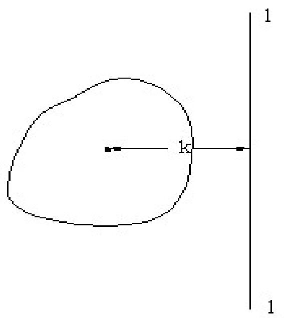
Moment of inertia about Y-axis , $I_{xx} = AY^2$

Least and greatest M.I

I_{xx} and I_{yy} are the moment of inertia of a plane figure about X axis and Y axis. If I_{xx} is greater than I_{yy} then I_{xx} is known as greatest moment of inertia and I_{yy} is called least moment of inertia.

Unit of moment of inertia – mm^4 or cm^4

Radius of gyration (K):



It is the distance from the given axis where the whole area of plane figures are assumed to be concentrated so as not to alter the moment of inertia about the given axis.

Moment of inertia about axis-11

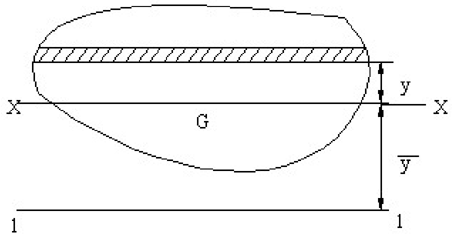
$$I = AK^2$$

$$K = \sqrt{\frac{I}{A}}$$

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}}$$

Parallel Axis Theorem:



This theorem states that the moment of Inertia of plane figure about an axis parallel to the centroidal axis I_{1-1} is equal to sum of moment of inertia about Centroidal axis ie. I_{xx} and the product of area of the plane figure and square of the distance between the two axes.

Proof: Let us consider a plane figure of total area A as shown in figure. Let I_{xx} is the moment of inertia about x axis and I_{1-1} is the moment of inertia about 1-1 axis.

Let us choose an elemental strip of area da at a distance of y from the centroid axis.

Moment of inertia of the strip about x axis

Moment of inertia of strip @ strip x-x axis = da. y^2

Moment of inertia of total area @ xx-axis = I_{xx} = $\sum da.y^2$

Moment of inertia of the strip about 1-1 axis = $da\left(y+\bar{y}\right)^2$

Moment of inertia of total area about 1-1-axis

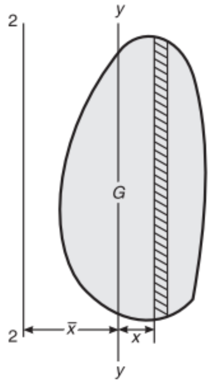
$$I_{1-1} = \sum da\left(y^2 + \bar{y}^2 + 2y\bar{y}\right)$$

$$I_{1-1} = \sum day^2 + \sum da\bar{y}^2 + 2\bar{y}\left(\sum day\right)$$

y = 0.

Because the distance of C.G of whole area from the centroidal axis = 0.

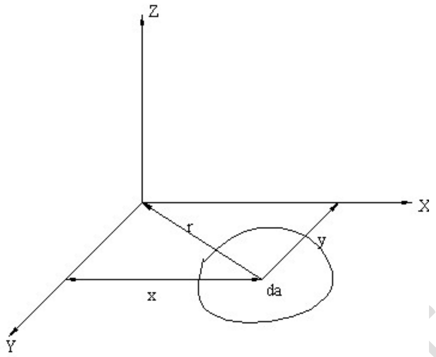
$$I_{1-1} = I_{xx} + A\bar{y}^2$$



Similarly

$I_{2-2} = I_{yy} + A\bar{x}^2$

Perpendicular axis Theorem:



This theorem states that moment of inertia of a plane figure about an axis which is perpendicular to the plane of the figure is equal to sum of moment of inertia about two mutually perpendicular axes

Proof: Let us consider an irregular figure of total area A as shown in figure. Let us choose an elemental strip of area da at a distance of x from y axis, y from x- axis and r from z axis respectively.

$r^2 = x^2 + y^2$

Moment of inertia of strip about x axis = $da \times y^2$

Moment of inertia of whole area @ x axis = $I_{xx} = \sum da.y^2$

Similarly, moment of inertia of strip about y axis = $da \times x^2$

Moment of inertia of whole area @ y axis = $I_{yy} = \sum da.x^2$

Moment of inertia of strip about z-axis = $da \times r^2$

Moment of inertia of whole area @ z axis = $\sum da.r^2$

$= \sum da\left(x^2 + y^2\right)$

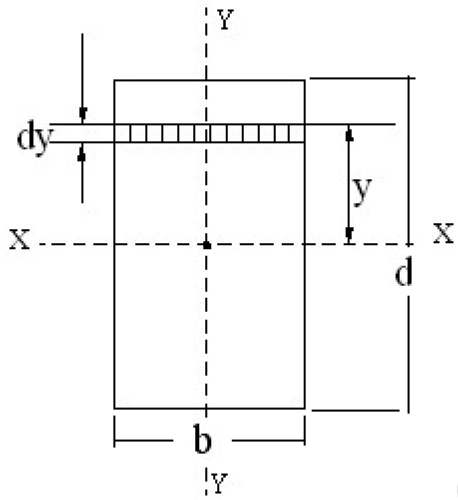
$= \sum da.x^2 + \sum da.y^2$

$= I_{yy} + I_{xx}$

$I_{zz} = I_{xx} + I_{yy}$

Moment of inertia of important figures:

Rectangle:



Let us consider a rectangular lamina of breadth b and depth d whose moment of inertia is to be determined Now consider an elemental strip of area(b.dy) at a distance of y from centroidal x- axis. The moment of inertia of strip @ xx-axis = $b.dy \times y^2$

Moment of inertia of whole figure @ x-x axis = $\int_{-\frac{d}{2}}^{\frac{d}{2}} b \times dy.y^2$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{\frac{d}{2}}$$

$$= b\left[\frac{d^3}{24} + \frac{d^3}{24}\right]$$

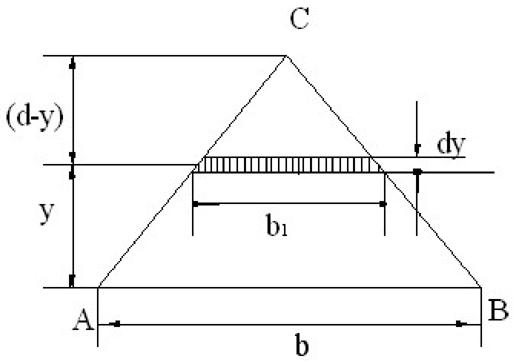
$$= \frac{b \times d^3}{12}$$

$$i,e \ I_{xx} = \frac{bd^3}{12}$$

$$Similarly \quad I_{yy} = \frac{db^3}{12}$$

Triangle:

Let us consider a triangular lamina of base b and depth d as shown in figure. Let us consider an elemental strip of area $b_1 \times d_y$ which is at a distance of y from base AB.



Using similar triangle property

$$\frac{b_1}{b} = \frac{d-y}{d}$$

$$b_1 = \frac{(d-y)b}{d}$$

Area of the strip = $\frac{(d-y)b}{d}.dy$

Moment of inertia of the strip about AB = $\frac{(d-y)b}{d} \times d.dy \times y^2$

$$= \frac{bdy^2.dy}{d} - \frac{by^3.dy}{d}$$

$$= by^2.dy - \frac{by^3.dy}{d}$$

Moment of inertia of the whole area about AB,

$$I_{AB} = \int_0^d by^2 dy - \int_0^d \frac{b}{d} y^3 dy$$

$$= b\left[\frac{y^3}{3}\right]_0^d - \frac{b}{d}\left[\frac{y^4}{4}\right]_0^d$$

$$= \frac{bd^3}{3} - \frac{b}{d}\frac{d^4}{4}$$

$$= \frac{bd^3}{3} - \frac{d^3}{4}$$

$$I_{AB} = \frac{bd^3}{12}$$

M.I.bout centroidal X axis,

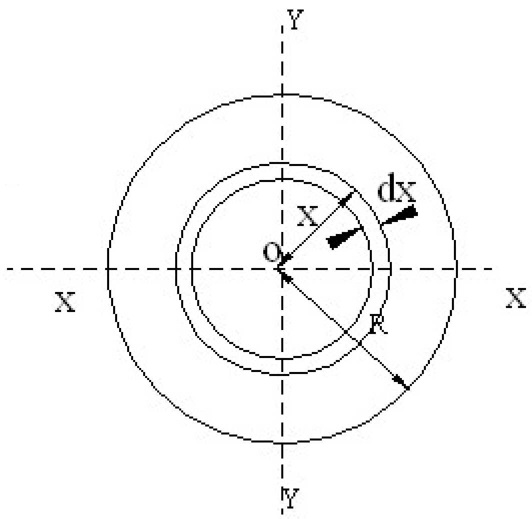
$$I_{AB} = I_{XX} + Ay^2$$

$$I_{XX} = I_{AB} - Ay^2$$

$$= \frac{bd^3}{12} - \frac{1}{2}bd\left(\frac{1}{3}d\right)^2 = \frac{bd^3}{36}$$

$$M.I\ of\ the\ triangle\ about\ centroidal\ y\ axis = \frac{db^3}{36}$$

Circle: Derive the expression of moment of inertia of a circle about diametrical axis



Let us consider a circular lamina of radius R as shown in figure.
Let us choose a circular elemental strip of thickness dx at a distance of x from the center.
Area of the strip =2 π .dx
Moment of inertia @ zz axis=2 π .dx .x^2

Moment of inertia @ zz axis for whole circle=Izz= $\int_0^R 2\pi x^3 .dx$

$$= 2\pi\left[\frac{x^4}{4}\right]_0^R$$

$$= \frac{2\pi R^4}{4}$$

$$= \frac{\pi R^4}{2}.$$

For the circular lamina, $I_{xx}=I_{yy}$,

∴ Using perpendicular axis theorem, we have

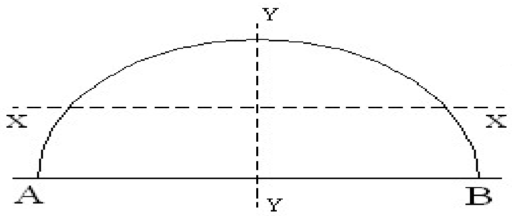
$I_{zz}=I_{xx}+I_{yy}$

$I_{zz}=2I_{xx}$

$I_{xx}=I_{zz}/2$

$$I_{xx}= \frac{\pi R^4}{2\times 2} = \frac{\pi R^4}{4} = Iyy$$

Semicircle:



Let us consider a semicircular lamina of radius R as shown in the figure.

Moment of inertia of semicircle about the diametrical axis about AB = $\frac{1}{2}\times\frac{\pi R^4}{4}$

$$= \frac{\pi R^4}{8}$$

$I_{AB} = I_{xx} + A\bar{y}^2$

$I_{xx} = I_{AB} - \frac{\pi R^2}{2}\left(\frac{4R}{3\pi}\right)^2$

$= \frac{\pi R^4}{8} - \frac{\pi R^2 \times 16R^2}{2\times 9\pi^2}$

$= \frac{\pi R^4}{8} - \frac{8\pi R^4}{9\pi^2}$

$= \frac{\pi R^4}{8} - \frac{8R^4}{9\pi}$

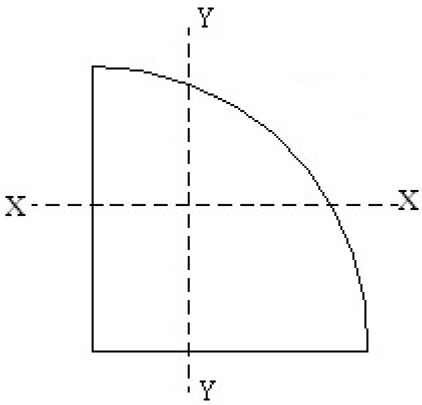
$I_{xx} = 0.11R^4$

Moment of inertia about y axis,

$I_{yy} = \frac{\pi R^4}{8} = \frac{1}{2}\times\frac{\pi R^4}{4}$

Quarter Circle:

$I_{xx} = I_{yy} = 0.055R^4$



Reference Table:

Comp.	Area	X	Y	ax	ay	ax^2	ay^2	Igx	Igy
	$\sum a$			$\sum ax$	$\sum ay$	$\sum ax^2$	$\sum ay^2$	$\sum Igx$	$\sum Igy$

Igx, Igy- Moment of inertia of the individual figures about centroidal x and y axis respectively.

$\bar{x} = \frac{\sum ax}{\sum a}$ $\bar{y} = \frac{\sum ay}{\sum a}$

$I_{1-1} = I_{xx} + A\bar{y}^2$

$I_{1-1} = \sum Igx + \sum ay^2$

$I_{1-1} - A\bar{y}^2 = I_{xx}$

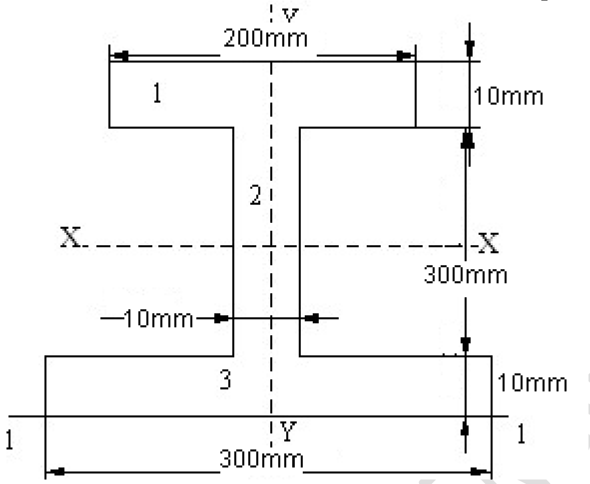
$I_{2-2} = I_{yy} + A\bar{x}^2$

$I_{2-2} = \sum Igy + \sum ax^2$

$I_{yy} = I_{2-2} - A\bar{y}^2$

PROBLEMS:

1. Determine the moment of inertia of unequal I-section shown in figure about its centroidal axis



Comp.	Area	Y	ay	ay^2	Igx
Rectangle-1	200×10	Y_1=315mm	6.3×10^5	$1.98\text{x}10^8$	$\frac{200\times10^3}{12}$
Rectangle-2	300×10 =3000	Y_2=160mm	4.8×10^5	$0.798\text{x}10^8$	$\frac{10\times(300)^3}{12}$ $=22.5\times10^6$
Rectanle-3	300×10 =3000	Y_3=5mm	0.15×10^5	$75\text{x}10^3$	$\frac{300\times(10)^3}{12}$ 2.5×10^4
	$\sum a = 8000$		$\sum ay = 11.25\times10^5$	$\sum ay^2 = 2.75\times10^8$	ΣIgx=$2.254\text{x}10^7$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{11.25\times10^5}{8000} = 140.625mm$$

$I_{1-1} = I_{XX} + A\bar{y}^2$

$I_{1-1} = \sum Igx + \sum ay^2$

$= 2.975\times10^8$

$I_{XX} = I_{1-1} - A\bar{y}^2$

$= 2.975\times10^8 - 8000\times(140.625)^2$

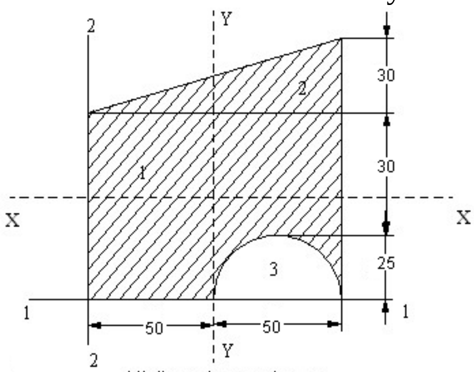
$= 1.393\times10^8$mm.

$$Iyy = \frac{d_1b_1^3}{12} + \frac{d_2b_2^3}{12} + \frac{d_3b_3^3}{12}$$

$$= \frac{10\times(200)^3}{12} + \frac{300\times(10)^3}{12} + \frac{10\times(300)^3}{12}$$

$$= 2.919\times10^7\,mm^4$$

2. Find the least radius of the Gyration about X- axis and Y- axis of shaded area shown in figure.



All dimensions are in mm

Comp.	Area	X	Y	ax	ay	ax^2
Rectangle-1	55×100=5500	50	27.5	2.75×10^5	1.513×10^5	13.75×10^6
Triangle-2	$\frac{1}{2}\times30\times100$ $=1500$	$\frac{2}{3}\times100$ $=66.667$	$55+10=65$	1×10^5	0.975×10^5	6.667×10^6
Semicircle-3	$-\frac{\pi\times(25)^2}{2}$ $=-981.748$	75	$\frac{4\times25}{3\pi}$ $=10.61$	-0.736×10^5	-0.104×10^5	-5.522×10^6

	$\Sigma a=6018.252$			$\Sigma ax=3.014\times10^5$	$\Sigma ay=2.384\times10^5$	$\Sigma ax^2=14.895\times10^6$

ay^2	Igx	Igy
4.159×10^6	$\frac{b_1 d_1^3}{12}=\frac{100\times55^3}{12}=1.386\times10^6$	$\frac{d_1 b_1^3}{12}=\frac{55\times100^3}{12}=4.583\times10^6$
6.338×10^6	$\frac{b_2 d_2^3}{36}=\frac{100\times30^3}{36}=0.075\times10^6$	$\frac{d_2 b_2^3}{12}=\frac{30\times100^3}{36}=0.833\times10^6$
-0.111×10^6	$-0.11(25)^4=0.043\times10^6$	$\frac{-\pi(25)^4}{8}=-0.153\times10^6$
$\Sigma ay^2=10.386\times10^6$	$\Sigma Igx=1.418\times10^6$	$\Sigma Igy=5.263\times10^6$

$$\bar{x}=\frac{\sum ax}{\sum a}=\frac{3.014\times10^5}{6018.252}$$

$$=50.08mm$$

$$\bar{y}=\frac{\sum ay}{\sum a}=\frac{2.384\times10^5}{6018.252}$$

$$=39.613mm$$

$I_{XX}=I_{1-1}-A\bar{y}^2$

$$I_{1-1}=\sum Igx+\sum ay^2$$

$$=1.418\times10^6+10.386\times10^6$$

$$=11.804\times10^6$$

$$I_{22}=\sum Igy+\sum ax^2$$

$$=5.263\times10^6+14.895\times10^6$$

$$=20.158\times10^6$$

$I_{XX}=I_{1-1}-A\bar{y}^2$

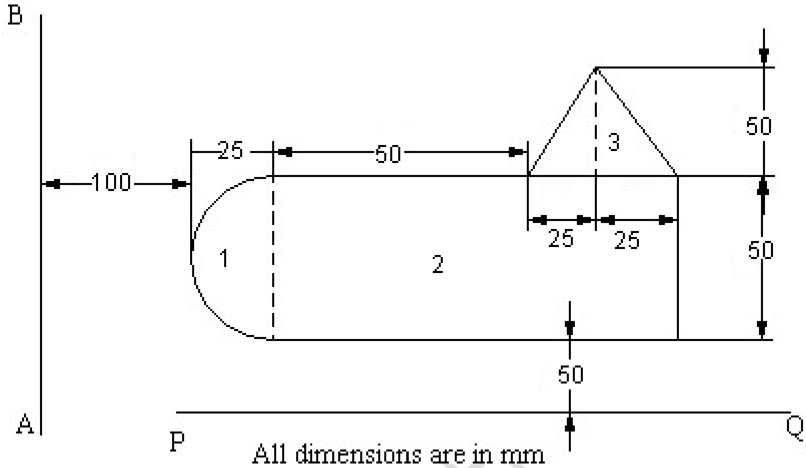
$=11.804\times10^6-6018.252\times(39.613)^2$

$=2.36\times10^6mm^4$

$I_{YY} = I_{2-2} - A\bar{x}^2$
$= 20.158\times10^6 - 6018.252\times(50.081)^2$
$= 5.064\times10^6 mm^4$

$$K_{xx} = \sqrt{\frac{Ixx}{A}} = \sqrt{\frac{2.36\times10^6}{6018.252}} = 19.803mm$$
$$K_{yy} = \sqrt{\frac{Iyy}{A}} = \sqrt{\frac{5.064\times10^6}{6018.252}} = 29.008mm$$

3. Determine moment of inertia about PQ and AB



Comp.	Area	x	Y	ax	Ay	ax^2
Semicircle -1	$\frac{\pi\times(25)^2}{2} = 981.748$	100+5-4r/3Π	25+50 =75	112.301×10^3	73.631×10^3	12.846×10^6
Rectangle-2	100×50 =5000	50+25 +100	25+50 =75	875×10^3	375×10^3	153.125×10^6
Triangle -3	$\frac{1}{2}\times50\times50 = 1250$	25+50 +25+100 =200	$\frac{1}{3}\times50+100$ =116.66	250×10^3	145.834×10^3	50×10^6
	Σa =7231.748			Σax $=1237.3\text{x}10^3$	Σay $=594.465\text{x}10^3$	$\Sigma ax^2 = 215.971\times10^6$

ay^2	Igx	Igy
5.522×10^6	$\frac{\pi(25)^4}{8} = 0.153\times10^6$	$0.11(25)^4=0.043\times10^6$
28.125×10^6	$\frac{b_1 d_1^{\ 3}}{12} = \frac{100\times50^3}{12} = 1.042\times10^6$	$\frac{100^3\times50}{12} = 4.167\times10^6$
17.014×10^6	$\frac{b_2 d_2^{\ 3}}{36} = \frac{50\times50^3}{36} = 0.174\times10^6$	$\frac{50\times50^3}{36} = 0.174\times10^6$
$\Sigma ay^2=50.661\times10^6$	$\Sigma Igx=1.369\times10^6$	$\Sigma Igy=4.384\times10^6$

$I_{PQ}=\Sigma Igx + \Sigma Igy^2$

$=1.369\times10^6 + 50.661\times10^6$

$=52.03\times10^6 mm^4$

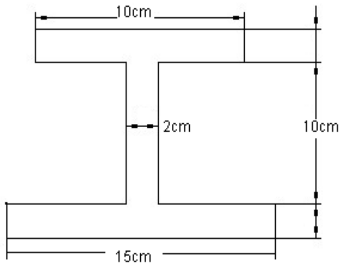
$I_{AB}=\Sigma Igy + \Sigma ax^2$

$= (4.384+215.971)\times10^6$

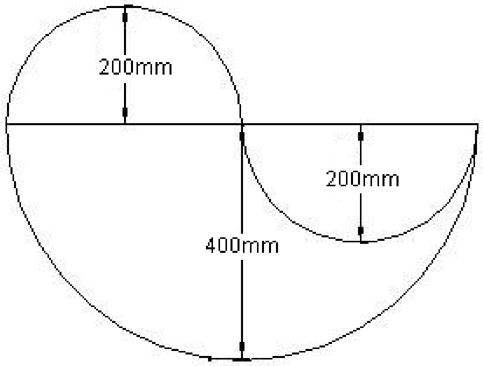
$=220.355\times10^6 mm^4$

REVIEW QUESTIONS.

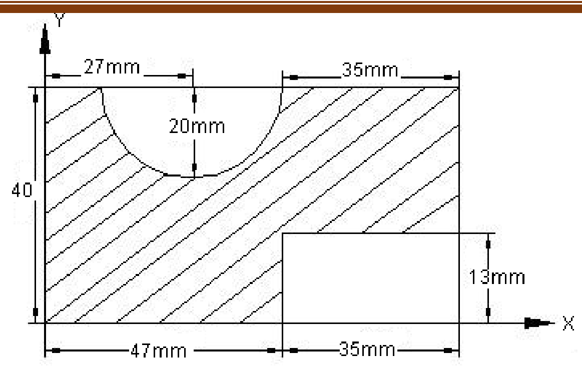
1. State and prove parallel axis theorem.
2. Derive an expression for moment of inertia of a triangle with respect to horizontal centroidal axis.
3. Determine the centroid of semi-circular lamina of radius R by method of integration.
4. Derive an expression for the moment of inertia of a rectangle from first principles about its vertical centroidal axis.
5. Derive an expression for the centroid of a quarter circle on its diametrical axis.
6. Find the moment of inertia of the section shown in figure at the centroidal axis xx perpendicular to the web



7. Find the centroid of figure shown



8. Find the centroid of shaded area shown in the figure.



9. In the figure shown in below determine the coordinates of the center of 12mm diameter circular hole cut in the plate, so that this point will be the centroid of the remaining shaded area