VISVESVARAYA TECHNOLOGICAL UNIVERSITY
BELGAUM

ENGINEERING MECHANICS

(Subject Code: BCIVC203)

LECTURE NOTES

(MODULE-5)

II-SEMESTER

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ENGINEERING MECHANICS

Subject Code: BCIVC203 **Department: Civil Engineering**

MODULE 5: KINEMATICS and KINETICS

SYLLABUS:

Kinematics:

Linear motion: Introduction, Displacement, speed, velocity, acceleration, acceleration due to gravity, Numerical examples on linear motion

Projectiles: Introduction, numerical examples on projectiles.

Kinetics: Introduction, D 'Alembert's principle of dynamic equilibrium and its application in-plane motion and connected bodies including pulleys, Numerical examples.

INTRODUCTION

Dynamics is the branch of science which deals with the study of behavior of body or particle in the state of motion under the action of force system.

Dynamic branches into two streams called kinematics and kinetics.

Kinematics is the study of relationship between displacement, velocity, acceleration and time of given motion without considering the forces that causes motion.

Kinetics is the study of relationships between the forces acting on the body, the mass of the body and motion of the body.

TECHNICAL TERMS RELATED TO MOTION

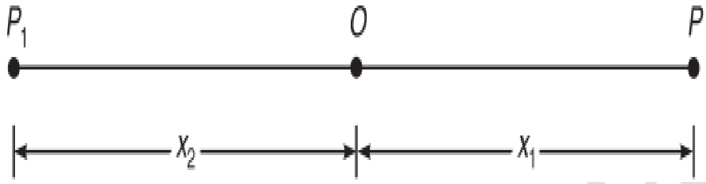
Motion: A body is said to be in motion if it is changing its position with respect to a reference point.

Path: It is the imaginary line connecting the position of a body or particle that has been occupied at different instances over a period of time. This path traced by a body or particle can be a straight line/liner or curvilinear.

Displacement and Distance Travelled: Displacement is a vector quantity, measure of the interval between two locations or two points, measured along the shortest path connecting them. Displacement can be positive or negative.

Distance is a scalar quantity, measure of the interval between two locations measured along the actual path connecting them. Distance is an absolute quantity and always positive.

A particle in a rectilinear motion occupies a certain position on the straight line. To define this position P of the particle we have to choose some convenient reference point O called origin. The distance x_1 of the particle from the origin is called displacement.



Let, P = Position of the particle at any time t_1

x_1 = Displacement of particle measured in +ve direction of O

x_2 = Displacement of particle measured in -ve direction of O

In this case the total distance travelled by a particle from point O to P to P_1 and back to O is not equal to displacement.

Total distance travelled = $x_1 + x_l + x_2 + x_2 = 2(x_1 + x_2)$

Whereas the net displacement is zero.

Velocity: Rate of change of displacement with respect to time is called velocity denoted by v.

$$\text{Mathematically } v = \frac{dx}{dt}$$

Average velocity: When an object undergoes change in velocities at different instances, the average velocity is given by the sum of the velocities at different instances divided by the number of instances. That is, if an object has different velocities $v_1, v_2, v_3, \ldots, v_n$, at times $t = t_1, t_2, t_3, \ldots, t_n$, then the average velocity is given by

$$V = \frac{[v_1 + v_2 + v_3 + \cdots + v_n]}{n}$$

Instantaneous velocity: It is the velocity of moving particle at a certain instant of time. To calculate the instantaneous velocity Δx is considered as very small.

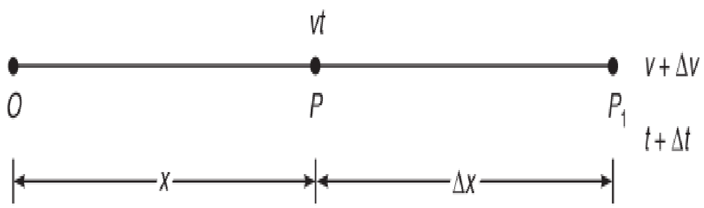
$$\text{Instantaneous velocity } v = \Delta t \rightarrow 0 \ \frac{\Delta x}{\Delta t}$$

Speed: Rate of change of distance travelled by the particle with respect to time is called speed.

Acceleration: Rate of change of velocity with respect to time is called acceleration. Mathematically a = dv/dt

Average Acceleration

Consider a particle P situated at a distance of x from O at any instant of time t having a velocity v. Let P_1 be the new position of particle at a distance of $(x + \Delta x)$ from origin with a velocity of $(v + \Delta v)$.



Acceleration due to gravity: Each and every body is attracted towards the centre of the earth by a gravitational force and the uniform acceleration with which the body is pulled towards the centre of the earth due to gravity is known as Acceleration due to gravity and denoted by 'g'. The value of g is normally taken as 9.81 m/s^2.

Newton's Laws of Motion

Newton's first law: This law states that 'everybody continues in its state of rest or of uniform motion, so long as it is under the influence of a balanced force system'.

Newton's second law: This law states that 'the rate of change momentum of a body is directly proportional to the impressed force and it takes place in the direction of force acting on it.

Newton's third law: This law states that 'action and reaction are equal in magnitude but opposite in direction'.

Types of Motion

1. Rectilinear motion
2. Curvilinear motion
3. Projectile motion

Graphical representation: The problems in dynamics can be analysed both analytically and graphically without compromising on the accuracy. Most of the times graphical representations can lead to simpler solutions to complicated problems. Using the simple terms defined in the initial portions of the section, we can draw different types of graphs.

Displacement-time graph: The representation with graph in Figure shows that the displacement is uniform with time. Hence it is understood that the body is under rest as the displacement is constant with respect to time. The representation with graph in Figure shows that the plot is having a constant slope and the variation of displacement is uniform with time. The slope indicates the ratio. of displacement to time which is equal to velocity of the body;

Figure shows variation of displacement with time as a curve. The tangent to this curve at any point indicates the velocity of the body at that instant. As can be seen the slope of the tangent is changing with respect to time and ever increasing, it indicates that the velocity is changing with respect to time and also indicates that the velocity is increasing with respect to time. This increasing velocity with respect to time is termed acceleration.

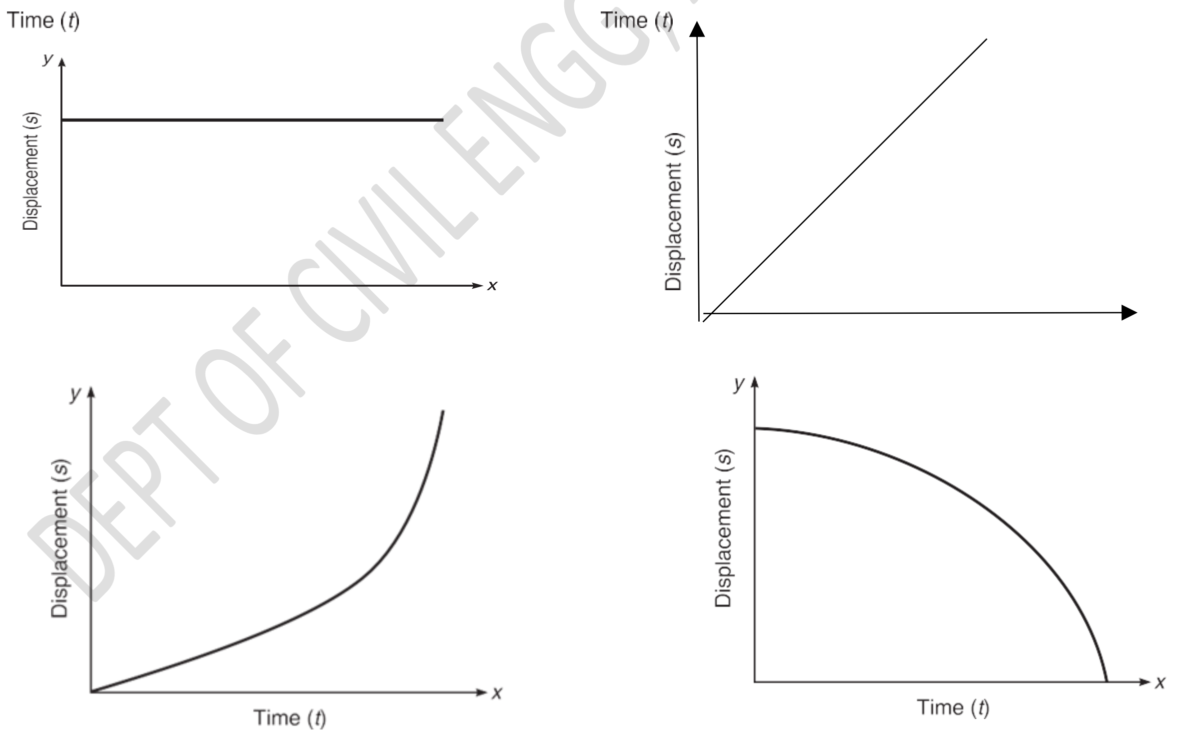
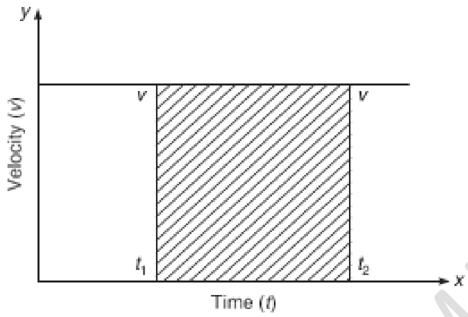


Fig: Variation of displacement with time.

In case of the curvature is decreasing, and the slope of the tangent is decreasing with respect to time and rate change of velocity is decreasing. This is termed as deceleration

Velocity time graph: A plot of velocity with respect to time is termed as velocity-time graph



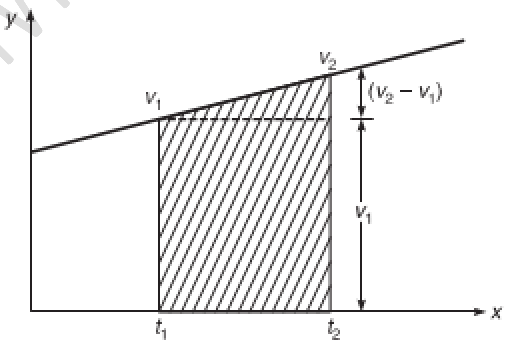
Variation of velocity with time.

Unit of velocity = v = LT^{-1}
Unit of time = T
Velocity x Time = LT^{-1} xT=L Distance
Hence, the area under V-T graph will produce the distance traveled by the body/particle from time t_1 to t $_2$,

s = v x (t_2 –t_1,) = vt . . ……………………..(1)

This is applicable only **when the velocity is uniform.**

In case if, the velocity is varying uniformly with respect to time as seen from sloped straight line.



Variation of velocity with time

The slope of the line gives acceleration

$$a = (v_2 - v_1) / (t_2 - t_1)$$

$(v_2-v_1) = a\,(t_2-t_1)$

$v_2 = v_1 + a\,(t_2-t_1)$

$v = u + at$ …… (1)

where,

v = final velocity,

u = initial velocity

and $t = (t_2-t_1)$

As seen from earlier graph, the total distance traveled is given by the area under the curve and hence the area is given as

$$S = v_1 * t + 0.5(v_2-v_1)t$$

But acceleration = $a = (v_2-v_1)/t$

Substituting, we get

$$S = v_1 \times t + \frac{1}{2}at^2 = ut + \frac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where u is the initial velocity or velocity at time t_1

Acceleration-time graph: It is a plot of acceleration versus time graph as shown in Figure

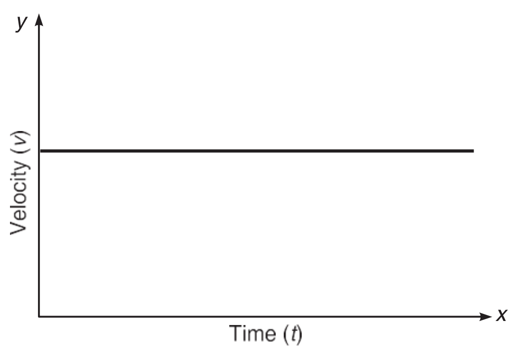
It is seen that die acceleration is constant with respect to time t. The same can be connected to velocity-time graph, wherein the velocity variation is constant.

The coordinates in acceleration-time graph show the area under the velocity-time curve.

In Figure, it is seen that the acceleration line in acceleration-time plot, it shows the variation of acceleration to be uniform.

Variation of acceleration with time.

Variation of velocity with time.



The curve in velocity-time graph will be simplified as a straight line in acceleration-time graph.

Using Eqs (1) and (2), to get an equation without tim, we substitute for t from Eq. 1 in Eq. 2, we get

$$s = u\frac{(v-u)}{a} + \frac{1}{2}\times a\left[\frac{(v-u)}{a}\right]^2 = \frac{u(v-u)}{a} + \frac{(v-u)^2}{2a}$$

$$2as = 2uv - 2u^2 + v^2 + u^2 - 2uv = v^2 - u^2$$

$$v^2 - u^2 = 2as \qquad \text{………………………….(3)}$$

Rectilinear Motion

When a particle or a body moves along a straight-line path, then it is called linear motion or rectilinear motion.

Equations of motion along a straight line

$v = u + at$

$v^2 - u^2 = 2as$

$S = ut + \frac{1}{2}at^2$

<u>Derivation of the Equations of Motion:</u>

1) First equation of Motion: V = u + at

Consider a body of mass **m** having initial velocity **u** and after time **t** its final velocity is **v** due to uniform acceleration **a**.

Acceleration = change in velocity/Time taken

Acceleration = Final Velocity-Initial velocity / time taken

a = v-u /t

at = v-u

or v = u + at

(2) Second equation of Motion: $s = ut + ½ at^2$

Let the distance travelled by the body be **(s)** distance = Average velocity x Time

Distance (s) = (u+v)/2 x t ……. eq. (1)

wkt: v = u + at

substituting this value of "v" in eq. (1),

we get = (u+u+at)/2 x t
s = (2u+at)/2 x t
s = $(2ut+at^2)/2$
s = ut +1/2 at^2
Proved

(3) Third equation of Motion: $v^2 = u^2 + 2as$
We know that v = u + at
v-u = at
t = (v-u)/a
Distance = average velocity X Time.
s = [(v+u)/2] x [(v-u)/a]
s = $(v^2 - u^2)/2a$
2as = $v^2 - u^2$
$v^2 - u^2 = 2as$
Hence proved.

Example 1: The motion of a particle is given by the equation $x = t^3 - 3t^2 - 9t + 12$. Determine the time, distance travelled and acceleration of particle when velocity becomes zero.

Solution:

$X = t^3 - 3t^2 - 9t + 12$ (1)

Differentiating Eq. (1) with respect to 'x', we get

$v = dx/dt = 3t^2 - 6t - 9$ (2)

when v = 0

The above equation is in the form of

$ax^2+bx+c=0$

and the solution is

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} \quad .... \quad (3)$$

Substituting the respective values in Eq. (3), we get

t = -1 or t = 3 s (negative value of t can be discarded)

Substitute t = 3 s in (1), we get

x = -15 m Differentiating

Eq. (2), we get a =12 m/s^2

Example 2: The motion of a particle is defined by the relation $x = t^3 - 9t^2 + 24t - 6$. Determine the position, velocity and acceleration when t = 5 s.

Solution: $x=t^3-9t^2+24t-6$ (1)

Differentiating Eq. (1), we get

$$\frac{dx}{dt} = v = 3t^2 - 18t + 2 \qquad (2)$$

Differentiating Eq. (2). we get

$$\frac{d^2x}{dt^2} = a = 6t - 18$$

Substitute t = 5 s in Eqs. (1), (2) and (3), we get x = 14 m

v = 9 m/s

a =12 m/s^2

Example 3: A car is moving with a velocity of 15 m/s. The car is brought to rest by applying brakes in 5s. Determine (i) Retardation (ii) Distance travelled by the car after applying the brakes.

(i) Retardation

We know that v = u + at

0= 15 + a (5) a = -3 m/s^2

(ii) Distance travelled by the car after applying the brakes.

We know that $s = ut + 0.5at^2$

$s= 15\times5+ 0.5\times(-3) \times (5)^2$

s = 37.5m

MOTION UNDER GRAVITY

We know that everybody on the earth experiences a force of attraction towards the centre of the earth is known as gravity. When a body is allowed to fall freely, it is acted upon by acceleration due to gravity and its velocity goes on increasing until it reaches the ground. The force of attraction of the earth that pulls all bodies towards the centre of earth with uniform acceleration is known as acceleration due to gravity. The value of acceleration due to gravity is constant in general and its value is considered to be 9.81 m/s^2 and is always directed towards the centre of earth. Acceleration due to gravity is generally denoted by 'g'.

When the body is moving vertically downwards, the value of g is considered as positive and if the body is projected vertically upwards, then acceleration due to gravity is considered as negative. Evidently, all equations of motion are applicable except by replacing uniform acceleration V with acceleration due to gravity 'g1 and are written as

(i) When a body is projected vertically downward, under the action of gravity, the equations of motion are

$$v = u + gt$$

$$v^2 = u^2 + 2gh \ h$$

$$= ut + 0.5gt^2$$

(ii) When a body is projected vertically upward, under the action of gravity, the equations of motion are

$$v = u - gt$$

$$v^2 = u^2 - 2gh$$

$$s = ut - 0.5gt^2$$

1. A ball is thrown vertically upward into air with an initial velocity of 35 m/s. After 3 s another ball is thrown vertically. What initial velocity must the second ball have to pass the first ball at 30 m from the ground.

Solution: Consider the first ball, we know that

$h = u_1t - 0.5gt^2$

$30 = 35t - 0.5*9.81*t^2$

$t^2 - 7.135t + 6.116 = 0$

$t = 6.138$ sec

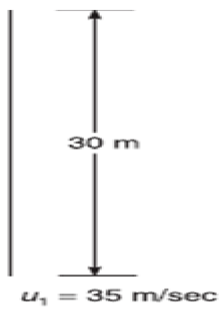
Consider the second ball

$t_2 = (6.138 - 3) = 3.138$sec

$h = u_2t_2 - 0.5gt_2^2$

$h = 30$ m

$u_2 = 24.91$m/sec



2) A stone is thrown upward with a velocity of 35m/s. Determine the time when the stone is at a height of 8 m and is moving downwards.

Solution: $u = 35$m/s $t = ?$

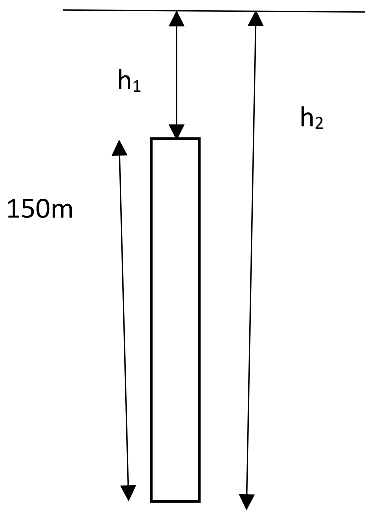
$h = ut - ½ gt^2$

$8 = 35t - ½ (9.81) t^2$ solving $t = 0.23$ sec or $t = 6.89$ sec

The smaller value of 't' is for upward motion and larger value for downward motion.

Therefore, $t = 6.89$ sec.

3) A stone is thrown vertically into the air from the top of the tower 150m high. At the same instant a second stone is thrown upward from the ground. The initial velocity of the first stone is 60 m/s and of the second stone is 80 m/s. When and where will the stones be at the same height from the ground.



Solution

Consider the first stone,

$h_1 = ut - ½ gt^2$

$h_1 = 60t - ½ (9.81)t^2$

$h_1 = 60t - 4.905t^2$ ……………….(1)

Consider the second stone,

$h_2 = 80t - 4.905t^2$ ……………….(2)

$h_2 - h_1 = 150$

$(80t - 4.905t^2) - (60t - 4.905t^2) = 150$

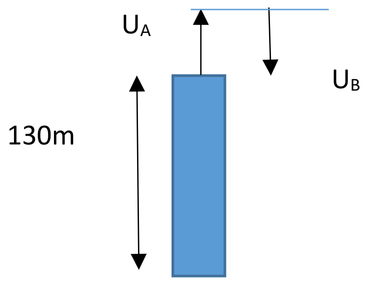
t = 7.5 sec

Substitute the value of t in the equation (1) and (2)

$h_1 = 174.09$ m

and $h_2 = 324.09$ m

4) Two objects A and B are projected vertically at 130m above the ground level. 'A' is projected up with a velocity 30 m/s and B is projected downwards with the same velocity. Find the time taken by each object to reach the ground. Also find the height from which the object 'A' must be released from rest in order that the two objects hit the ground simultaneously.



Solution: $U_A = U_B = 30$ m/s

Case (i) Consider the object 'A'
Let t_1 be the time taken by 'A' to reach the ground.
Initially velocity u = 30 m/s
h = -130m
$h = ut_1 - ½ gt_1^2$

$-130 = 30t_1 - ½ (9.81) t_1^2$

$4.90t_1^2 - 30t_1 - 130 = 0$

t = 9.053 sec

Consider the object 'B'
Let t_2 be the time taken by 'B' to reach the ground.
$h = ut_2 - ½ gt_2^2$

$-130 = 30t_2 - ½ (9.81) t_2^2$

$4.90t_2^2 + 30t_2 - 130 = 0$

t = 2.929 sec

Case (ii) Let 'h' be the height at which the body 'A' is released from rest so that the two objects reach the ground simultaneously

$h = \frac{1}{2} g t_2^2$

$= \frac{1}{2} (9.81) (2.929)^2$

h = 42.04 m

5. A balloon is moving upwards with velocity 10m/s. It releases a stone which comes down to the ground in 11s. The height of the balloon at the moment when the stone was dropped is?

Solution:

Let the height from which the stone is released be h.
The upward velocity u of the stone when dropped from height h = 10 m/s.
The further height d to which the stone will rise further can be found from the relation;
$v^2 - u^2 = 2 g d$
$0^2 - 10^2 = - 2\times 10\times d$; ==> -20 d = -100, or d = 5 m.
Time taken to go up 5m can be found using
v = u + a t. At the highest point v=0 m/s, u = 10 m/s, t =1 s.
The stone starts to fall from a height (h +5) m, with an initial velocity zero, and an acceleration +10 m/s², and in time t =11 s -1s = 10s reaches the ground. Using the relation
$s = u t + \frac{1}{2} a t^2$, we can find substituting for u, a and we get s, we get
$s = 0\times 10 + \frac{1}{2} \times 10\times 10^2 = 500$m
s = h + 5 = 500 m; ==> h = 495 m.
The height of the balloon when the stone was dropped from it = 495 m.

Method 2:

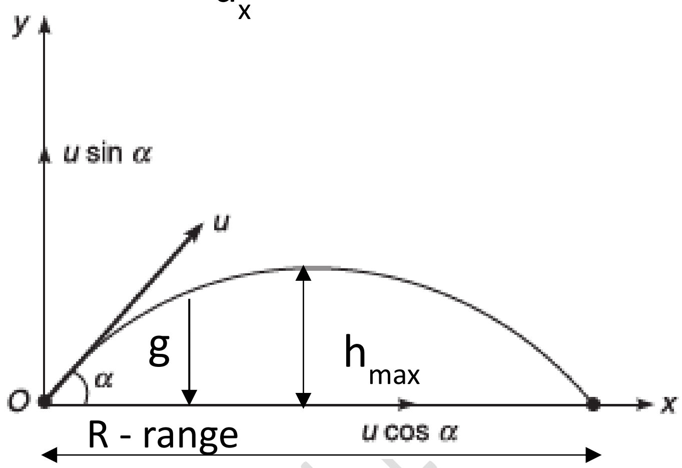
Let the point from which the stone is dropped be taken as origin of coordinate system, upwards direction is taken as positive and downward direction is taken as negative.
Let the stone be released from the balloon at a height h above the ground, when the stone reaches the ground, its displacement is -h. an upward velocity u = +10 m/s², and an acceleration due to gravity = -10 m/s², t = 11 second
$-h = 10\times 11 - \frac{1}{2} \times 10\times 11^2 = 110 - 5\times 121 = 110 - 605 = - 495$ m.
ie h= 495 m.
ie the height from where the stone is dropped = 495 m.

PROJECTILES

Whenever a particle is projected upwards with some inclination to the horizontal (but not vertical), it travels in the air and traces a parabolic path and falls on the ground point (target) other than the point of projection. The particle itself is called projectile and the path traced by the projectile is called trajectory.

$u_y = u \sin \alpha$

$u_x = u \cos \alpha$



Terms used in projectile

1. Velocity of projection (u): It is the velocity with which projectile is projected in the upward direction with some inclination to the horizontal.

2. Angle of projection (α): It is the angle with which the projectile is projected with respect to horizontal.

3. Time of flight (T): It is the total time required for the projectile to travel from the point of projection to the point of target.

4. Horizontal range (R): It is the horizontal distance between the point of projection and target point.

5. Vertical height (h): It is the vertical distance/height reached by the projectile from the point of projection.

Some relations

Time of flight:

Let T be the time of flight. We know that the vertical ordinate at any point on the path of projectile after a time T is given by

$$y = (u \sin \alpha)t - 0.5gt^2$$

When the projectile hits the ground, say at B: y = 0 at t = T

Above equation becomes

$$0 = (u \sin \alpha)t - 0.5gt^2$$

$$(u \sin \alpha) = 0.5gt$$

$$T = (2u \sin \alpha)/g$$

Horizontal range of the projectile:

During the time of flight, the horizontal component of velocity of projectile = u cos α {Horizontal distance of the projectile} = R= {Horizontal component of velocity of projection}

{Time of flight} = u cos α x T

$$R = (u \cos \alpha * 2u \sin \alpha)/g = (u^2 \sin (2\alpha))/g$$

sin (2α) will be maximum only when sin 2α = 1

sin 2 α = sin 90 or α = 45°

Hence maximum horizontal range is given by

$$R_{max} = (u^2 \sin 90)/g = u^2/g$$

Maximum height attained by the projectile: When the projectile reaches its maximum height, vertical component of velocity of projection becomes zero.

$$V^2 - u^2 = 2gs$$

$$0 - u^2 \sin^2 \alpha = -2gh_{max} \quad h_{max}$$

$$= u^2 \sin^2 \alpha / 2g$$

Time required to reach the maximum height is given by

$$v = u + at$$

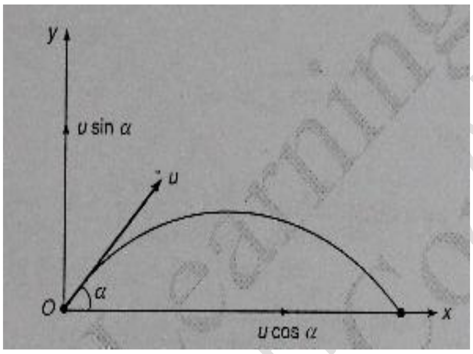
$$0 = u \sin \alpha - gt$$

$$t = u \sin \alpha / g$$

Motion of projectile: Let a particle be projected upward from a point O at an angle a with horizontal with an initial velocity of u m/s as shown in Figure. Now resolving this velocity into two components, we ge

$$u_x = u \sin\alpha$$

$$u_y = u \cos\alpha$$



The vertical component of velocity is always affected by acceleration due to gravity. The particle will reach the maximum height when vertical component becomes zero. The horizontal component of velocity will remain constant since there is no effect of acceleration due to gravity. The combined effect of horizontal and vertical components of velocity will move the particle along some path in air and then fall on the ground other than the point of projection.

Equation for the path of projectile (Trajectory equation): Let a particle is projected at a certain angle from point O. The particle will move along a certain path OPA in the air and will fall down at A.

Let u = velocity of projection α

= angle of projection

After t seconds, let a particle reach any point 'P' with x and y as coordinates as shown in

Figure

We know that, horizontal component of velocity of projection = u cosα

Vertical component of velocity of projection = u sin α

Therefore, $x = u \cos\alpha t$

$y = u \sin \alpha t - 0.5gt^2$

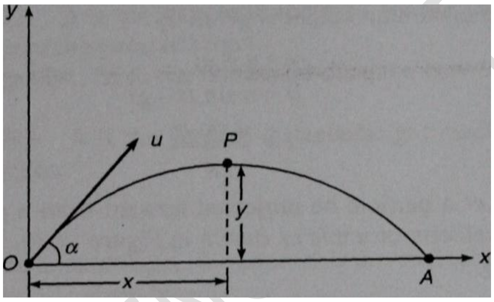
From Eq. (1)

$t = x/(u \cos \alpha)$

substitute in Eq. (2), we get

$y = u \sin\alpha[x/(u \cos\alpha)] - 0.5g[x/u\cos\alpha]^2$

$y = x \tan\alpha - [gx^2/(2u^2\cos^2\alpha)]$



1. A particle is projected at an angle of 60° with horizontal. The horizontal range of particle is 5 km. Find

   (i) Velocity of projection (ii) Maximum height attained by the particle

Solution Data given; R = 5 km = 5000 m, g = 9.81 m/s2 and α = 60°

To find: u and h_{max}

We know that

$$R = (u^2 \sin 2\alpha)/g \qquad \ldots\ldots\ldots\ldots(1)$$

Substituting the known values in Eq. (1), we get

$$u = 237.98 \text{ m/s}$$

Again, maximum height attained by the particle

$$h_{max} = (u^2 \sin \alpha)/2g = 2164.9\text{m}$$

Motion of a body thrown horizontally from a certain height into air

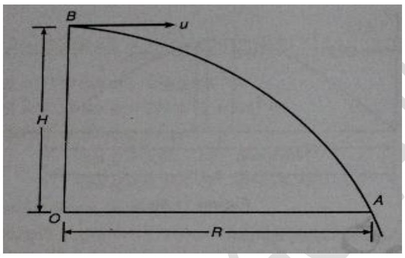
The figure shows a body thrown horizontally from certain height 'H' into air. At 'B' there is only horizontal component of velocity. As the body moves in the air towards the ground, the body has both horizontal and vertical components of velocity.

The horizontal component of velocity from B to A remains constant and will be equal to u. But the vertical component of velocity in the downward direction will be subjected to gravitational force and hence will not be a constant.

Resultant velocity = R = $\sqrt{(u^2 + v^2)}$ & $\Theta = \tan^{-1}(v/u)$

(i) Vertical downward distance travelled by the body is given

H = (vertical component of velocity at B) t + $0.5gt^2$



(ii) The horizontal distance travelled by the body

R = (horizontal component of velocity at B)t

R = ut

(iii) The vertical component of velocity at point A is obtained from the equation

v = u + gt

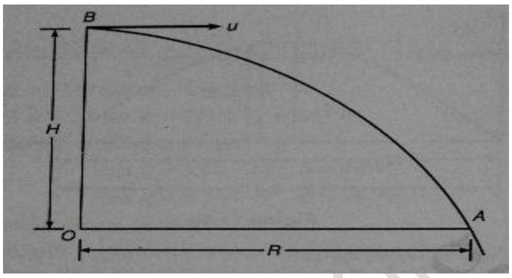
or v = gt

Resultant Velocity at A = R = $\sqrt{(u^2 + v^2)}$

1) An aircraft is moving horizontally at a speed of 108 km/h at an altitude of 1000 m towards a target on the ground releases a bomb which hits the target. Estimate the horizontal distance of aircraft from the target when it releases a bomb. Calculate also the direction and velocity with which bomb hits the target.

Solution:

Speed of aircraft = (108x100)/(60x30) = 30m/s



Horizontal velocity of bomb = u = 200 m/s

Height H= 1000m

Let t be the time required for the bomb to hit the target

We know that

$$H = 0.5gt^2$$

$$1000 = 0.5 \times 9.81 \times t^2 \quad \text{or} \quad t = 14.278 \text{ s}$$

(i) Horizontal distance of aircraft from the target when it releases a bomb.

$$R = u \times t = 30 \times 14.278 = 428.57 \text{ m}$$

(ii) Velocity with which bomb hits the target.

Vertical component of velocity = v = gt = 9.8 1 x 14.278 = 139.9 m/s

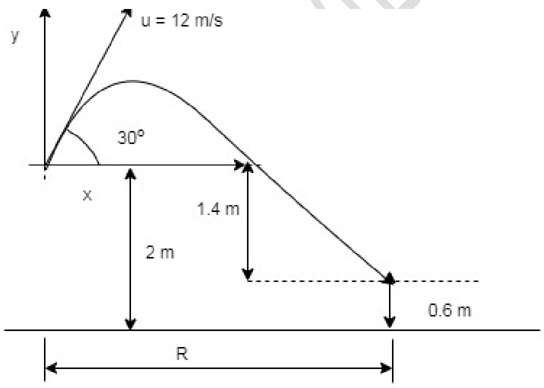
Horizontal component of velocity = u = 30 m/s

Resultant velocity = R = $\sqrt{(u^2 + v^2)}$= 143.08 m/s

Direction = $\Theta = \tan^{-1}(v/u) = 77^o$

2) A cricket ball thrown by a player from a height of 2m above the horizontal ground at an angle of 30^o to the horizontal and with the velocity of 12m/s. The ball hits the wicket at a height of 0.6m above the ground. How far is the player from the wicket?

Solution:



$\alpha = 30^o$

y = -1.4 m

$u = 12$ m/s

$y = ut_2 - ½ gt_2^2$

- $1.4 = u\sin\alpha t – 1/2gt^2$
- $1.4 = 12\sin30^o\, t – ½ (9.81)\, t^2$

$t = 1.42$sec

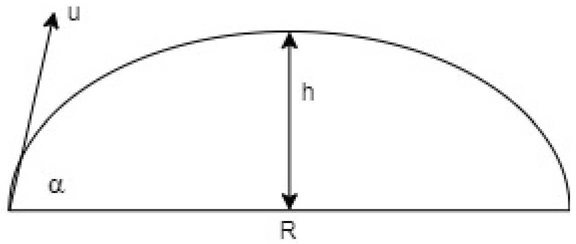
The distance of the player from the wicket,

$R = u \cos \alpha t$

$R = 12 \cos 30^o (1.42)$

$R = 14.75$ m

3) Determine from the first principal the angle at which the bullet has to be fired over a horizontal plane such that the greatest height attained by it is equal to the greatest distance travelled on the plane.



We have R = H……………..(1)

$R = (u^2 \sin 2\alpha)/g$

$H = (u^2 \sin^2\alpha)/2g$

Substituting values in eqn.(1)

$(u^2 \sin 2\alpha)/g = (u^2 \sin^2\alpha)/2g$

$\sin 2\alpha = \sin^2\alpha$

$4 \sin\alpha \cos\alpha = \sin^2\alpha$

$4 = \tan\alpha$

$\alpha = 75.96^o$

4) A projectile is fired from the top of cliff 150m height with an initial velocity of 180m/s at an angle of elevation of 30^o to the horizontal. Neglect the air resistance,

i) determine the greatest elevation above the cliff

ii) The greatest elevation above the ground reached by the particle

iii) The horizontal distance from the gun to the point where the projectile strikes the ground

Solution:

i) $H = (u^2 \sin^2\alpha)/2g$

$H = (180^2 \sin^2 30^o)/ 2 \times 9.81$

$H = 412.84$ m

ii) $H_{max} = 150 + H$

$= 150 + 412.84$

$= 562.84$ m

iii) Equation of path is

$$Y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$-150 = x\tan 30^0 - \frac{9.81x^2}{2(180)^2\cos^2 30}$$ solving the equation we get

X = 3100 m

5) A projectile is projected from a point at an angle of elevation of 30° with a velocity of 600m/s. Find the velocity and direction of motion of the particle at the end of i) 25 second ii) 40 seconds

Solution:

$\mathbf{v_x} = u \cos \boldsymbol{\alpha}$

$\mathbf{v_y} = u \sin\alpha - gt$

$u = 600$ m/s $\alpha = 30^o$

i) t =25 sec

$v_x = u \cos\alpha$

$= 600 \cos 30^o$

$v_x = 519.6$ m/s

$\mathbf{v_y} = 600 \sin 30^o - 9.81(25)$

$\mathbf{v_y} = 54.75$ m/s

$$v = \sqrt{vx^2 + \mathbf{vy}^2}$$

V = 522.48 m/s

$\Theta = \tan^{-1}\left(\left|\frac{vy}{vx}\right|\right)$ $\Theta = 6^{o}$

ii) t = 40 sec

$v_x = u \cos \alpha$

= 600 cos 30

= 519.6 m/s

We also have

$\mathbf{v_y} = 600 \sin 30^{o} - 9.81(40)$

$\mathbf{v_y} = -92.4$ m/s

$$v = \sqrt{vx^{2} + \mathbf{vy}^{2}}$$

V = 527.75 m/s

$\Theta = \tan^{-1}\left(\left|\frac{vy}{vx}\right|\right)$ $\Theta = 10^{o}$

6. A projectile is fired from the surface of the earth with a velocity of 5 m/s and angle theta with the horizontal another projectile fired from another planet with a velocity of 3 m/s at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth the value of the acceleration due to gravity on the planet is

Sol: Since both the projectiles follow the same path and same trajectories, their maximum height and range will be same.
Let the acceleration due to gravity on the planet be g'
Let u = 5 m/s and u' = 3 m/s
Equating the ranges of both the projectiles we get

$$\frac{u^2 Sin\, 2\theta}{g} = \frac{u^2 Sin\, 2\theta}{g'}$$

or,

or, $g' = 9.8 \times \frac{u'^2}{u}$

or, $g' = 9.8 \times \frac{3^2}{5^2} \Rightarrow$ g' = 3.528 m/s²

7. A bullet is fired horizontally with a velocity of 80m/s. Then what happens during the first second?

Solution:
Since it has no initial vertical velocity, we have.
s = −1/2gt2
s=−1/2gt2
So, after 1 second,
s≈−5s≈−4.9 metres.

8. A bullet is fired on a wall with a velocity of 100m/s. If the bullet stops at a depth of 10cm inside the wall, then find the retardation produced by the wall.
Solution:
Initial velocity of bullet = 100m/s
Bullet strike with wall and after 10cm moving in wall bullet will be rest.
final velocity = 0
use kinematic equation,
$v^2 = u^2 + 2aS$
$0 = 100^2 + 2a\times 0.01$
[10 cm = 10/100 m]
-10000 = (20/100) × a
a = -50000 m/s² Retardation produced by wall = -50000 m/s²

9. A bullet shot is fired with an initial velocity of 40m/s at an angle of 60° with horizontal. Find time of flight, maximum height and horizontal range of the bullet shot.
Solution:
u = 40 m / s
θ = 60°
Horizontal range $R = (u^2 \sin 2\alpha)/g = (40)^2 \sin 2(60)/ 9.81 = 141.24$ m
Maximum height $H = (u^2 \sin^2\alpha)/2g = 61.16$ m
Time of flight $T = (2u \sin \alpha)/g = (2(40) \sin 60) / 9.81 = 7.06$ s

D'ALEMBERT'S PRINCIPLE alternative form of Newton's second law of motion introduced by Jean le Rond **d'Alembert.**

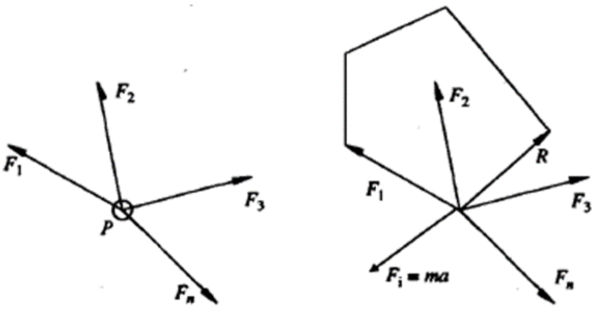
It states that a moving body can be brought to equilibrium by adding inertia force to the system. In magnitude this inertia force is equal to the product of mass and acceleration and takes place in a direction opposite to that of acceleration.

By Newton's second law of motion,

$\Sigma F = ma$

$\Sigma F - ma = 0$

$\Sigma F - \Sigma F_i = 0$ where, ΣF_i is inertia force



A force is acting on a particle P and inertia force F_i = - ma which is imagined to be acting on the particle. Then F and F_i are in equilibrium. This is known as D'Alembert's principle.

Then F and F_i are acting on a particle, the same will remain in equilibrium. To distinguish this equilibrium from static equilibrium it is termed as dynamic equilibrium.

When number of forces are acting on the particle, the principle can be applied as below. When F_1, F_2,……Fn be the forces acting on a particle as shown in Fig above. We can calculate the resultant of all the forces i.e R. Therefore, we can write as

R = ma

Where 'a' is the resultant acceleration of the particle.

REVIEW QUESTIONS:

1. Derive all the basics equations of motion in kinematics.
2. What is super elevation and what are its necessity?
3. Derive the equation to the path of projectile.
4. Explain with the sketch for projectile motion: Range, Time of flight, Maximum height, Angle of projection.
5. Define uniform velocity, Rectilinear motion, Curvilinear motion, Projectile
6. Define displacement, speed, velocity, acceleration.
7. A car is moving with a velocity of 15 m/s. The car is brought to rest by applying brakes in 5 s. Determine (i) Retardation (ii) Distance travelled by the car after applying the brakes.
8. A ball is thrown vertically upward into air with an initial velocity of 35 m/s. After 3 s another ball is thrown vertically. What initial velocity must be the second ball has to pass the first ball at 30 m from the ground.
9. State and prove D'Alembert's principle