VISVESVARAYA TECHNOLOGICAL UNIVERSITY
BELGAUM

ANALYSIS OF STRUCTURES

(Subject Code: BCV401)

LECTURE NOTES

(MODULE-1)

IV-SEMESTER

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Module -1

Introduction and Analysis of Plane Trusses

Introduction and Analysis of Plane Trusses: Structural forms, Conditions of equilibrium, Compatibility conditions, Degree of freedom, Linear and nonlinear analysis, Static and kinematic indeterminacies of structural systems, Types of trusses, Assumptions in analysis, Analysis of determinate trusses by method of joints and method of sections.

Structures are built to facilitate the performance of various activities connected with residence, office, education, healthcare, sports and recreation, transportation, storage, power generation, irrigation, etc. We see a variety of structures in our midst. Some are monumental, some residential, some commercial, some recreational, some mobile, etc. All of them have certain common features; they form systems consisting of a load-resisting component, which is called super structure and a load-dis- tributing component to the ground which is known as substructure.

All the structures should sustain the loads coming on them during their service life by possessing adequate strength and also limit the deformation by possessing enough stiffness. Strength of a structure depends on the characteristics of the material with which it is constructed. Stiffness depends on the cross section and the geometrical configuration of the structure. A structure is not a single entity; it consists of many parts that are assembled together as a system. The parts are called elements or members. The loads coming on a structure degenerate into forces in these elements because of the deformation they undergo. The members should be designed to resist these forces induced in them as per the relevant codes of practices prevalent in a country. Besides, the structure should be stable against overturning moments caused by some kind of horizontal loads like that caused by earth- quake or wind. Moreover, all the loads applied on the structure should be safely transmitted to the ground through its foundation. Therefore, safety is of prime importance in the existence of structures. Because human beings occupy the structure eventually one should not compromise on the safety aspect of the structure. Otherwise distress in the structure will endanger lives of occupants. Transmission of loads coming on the global system through its local members to the subsystem consisting of the foundation for eventual distribution on the ground is called load path. Any interruption in the load path will lead to collapse of the structure. So, the safety of a structure can be assured with the right choice of appropriate load path.

Structural analysis, therefore, deals with the mechanism of degeneration of loads applied on the system into local element forces, using various theories and theorems enunciated by eminent engineers and investigators. It also deals with the computation of deformations these members suffer under the action of the induced forces.

FORMS OF STRUCTURES

We have constructed structures of many forms and shapes. All structural forms used for load transfer from one point to another are three-dimensional (3D) in nature. Generally, they can be categorized as linear forms (Fig. 1.1) and curvilinear

Fig. 1.1 Chicago downtown buildings (linear form).

forms (Fig. 1.2). The type of functions and aesthetics dictate the forms of structures. For instance, linear forms are preferred for residential, official, and educational purposes. The linear form is called skeletal structures. They are artic- ulated structures assembled with parts consisting of linear elements, such as bars and beams, the connection between them being bolted or riveted or welded.

Fig. 1.2 Balloon structure (curvilinear form).

Assemblage of members forming a frame to support the forces acting is called the *framed structure*. A framework is the skeleton of the complete structure and it supports all intended loads safely and economically. Some structural examples are frames [Fig. 1.3(a)], high-rise structures [Fig. 1.3(b)], trusses [Fig. 1.3(c)], industrial shed [Fig. 1.3(d)], bridge deck [Fig. 1.3(e)], plates [Fig. 1.3(f)], etc. Generally, these structures are two-dimensional (2D) lying in one plane along two coordinate axes. However, the parts by which they are assembled are one-dimensional (1D) lying in a single plane along one coordinate axis.

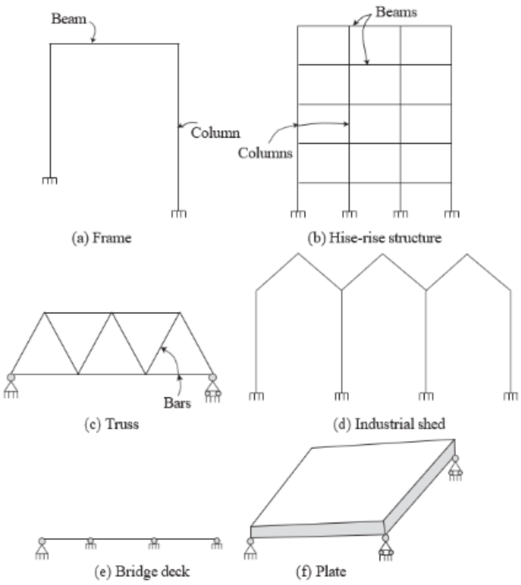


Fig. 1.3 Skeletal structures.

Curvilinear forms as single entities mostly occupy a space. For structural analysis purposes these structures are idealized as continuous system. Continuous system structures transfer loads through the in-plane or membrane action to the boundaries. Assemblages of continuous members like shells, domes, etc., are called continuous system. They are 3D structures. The examples for continuous system are domes, shells, arches, cables, cylindrical members, cooling towers, space crafts, aircrafts, etc. These are shown in Fig. 1.4. Structures in curvilinear form are called surface structures.

The most suitable structural form is the one which provides satisfactory solutions to functional, economic, sociological, aesthetic, and other requirements to the highest degree and that can be economically and reliably built, using the most appropriate structural materials and construction methods that are available.

On the basis of the dominant stress conditions developed under their most significant design loads and conditions, structural forms may be classified as uniform stress forms and varying stress forms. When the stress across a section is uniform over the depth of a member or over the thickness of a panel, e.g., cables, arches, truss members, membranes, and shells, such a form is called a uniform stress form. When the stress varies over the depth or thickness, from a maximum compressive stress on one surface to a maximum tensile stress on the other, e.g., in the case of beams, rigid frames, slabs, plates, etc., such a form is called a varying stress form.

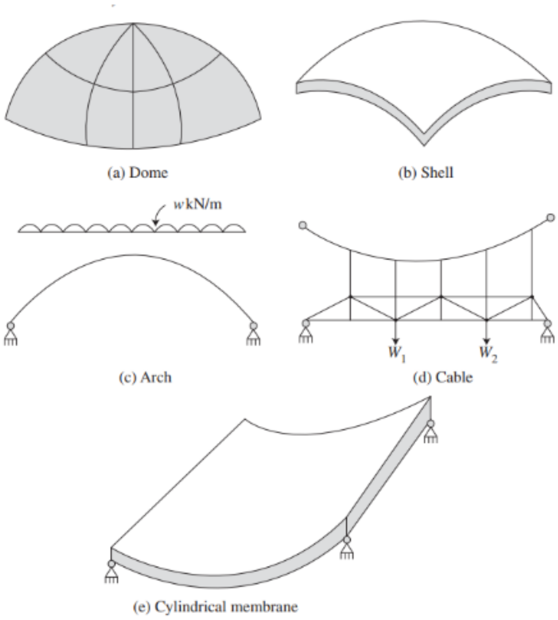


Fig. 1.4 Surface structures.

CONDITIONS OF EQUILIBRIUM

The basic tool in structural analysis is the use of equilibrium equations which states that the structure or part of it remains in its stationary position. Hence, if the entire structure is considered, the reactions from the support and the loads on the structure should be in static equilibrium. The equations of static equilibrium are as follows:

i) The summation of all the forces along any ais is zero.

ii) The summation of all the moments about any axis is also zero.

The equations of static equilibrium are based on Newton's law. For a three-dimensional system, the equations of equilibrium are as follows:

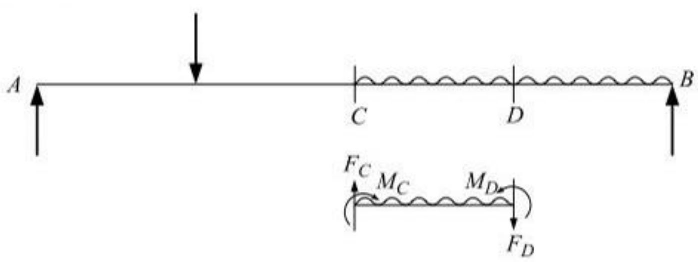
$\sum Fx = 0 \quad \sum Fy = 0$ and $\sum Fz = 0$

$\sum Mx = 0 \quad \sum My = 0$ and $\sum Mz = 0$

For a two-dimensional system with x and y as the orthogonal axis, the equations of equilibrium are:

$\sum Fx = 0 \quad \sum Fy = 0$ and $\sum M = 0$

The above equilibrium conditions may be applied to a part of the structure also provided that, in such case, apart from the external loads, the reactive forced from the removed part are also considered.



Compatibility Conditions

Compatibility conditions means requirement of continuity, such as in joints where two or more members meet. The following two compatibility conditions are to be satisfied at any joint:

1. The members meeting at a joint will continue to meet at the same joint even after deformation takes place.
2. At rigid-joints, the angle between any two members remains the same even after deformation takes place. The compatibility conditions will help in formulating additional equations.

Statically Determinate and Indeterminate Structures

The structures are grouped into statically determinate and statically indeterminate structures. A structural system which can be analysed by using equations of statical equilibrium only is called as statically determinate structure, e.g., beams or trusses with both ends simply supported, one end hinged and another on rollers and the cantilever type. A structure which cannot be analysed by using equations of equilibrium only is called a statically indeterminate structure, e.g., fixed beams, continuous beams, propped cantilevers. To analyse indeterminate structures, apart from using equations of equilibrium one has to determine the various deformations and make use of compatibility conditions. Indeterminate structures are also called redundant structures.

Difference between Determinate Structure and Indeterminate Structure

Determinate Structure	Indeterminate Structure
1) The structure can be analysed by using equations of equilibrium ($\sum V=0$, $\sum H=0$, $\sum M=0$) then it is called statically determinate structure.	1) The structure can not be analysed by using conditions of equilibrium, then it is called statically indeterminate structure.
2) The determination of internal force in structure do not require cross section area and moment of inertia.	2) The determination of internal force in structure requires cross-sectional area and moment of inertia.
3) There is no stress caused due to change in temperature.	3) Stresses are caused due to change in temperature.
4) There is no stress caused due to settlement of supports. E.g., Simply supported beam, cantilever beam	4) There is stress due to settlement of supports. E.g., Propped cantilever beam, fixed beam.

Linear and Non-Linear Systems

A system is called a linear system if its material has linear stress-strain relationship and a small deflection. In such cases, the law of superposition holds good. A system will be treated as a non-linear system if its material does not have linear stress-strain relationship or its deformation is so large that a change of geometry cannot be neglected in the analysis. If the non-linearity is due to stress-strain relationship, it is called material non-linearity and if the non-linearity is due to considerable changes in the geometry, it is called geometric non-linearity. In some cases, both material non-linearity and geometric non-linearity need to be considered. Non-linear analysis is lengthy and repetitive type with minor changes in each cycle. Hence, it is ideally suited for computer-aided analysis.

Difference between Linear and Non-Linear Structures

Linear Structures	Non-Linear Structures
1) A system is said to be Linear system if its material has linear stress-strain relationship.	1) A system is called non-linear system if its material does not have linear stress-strain relationship.
2) Linear system undergoes small deflection.	2) Non-Linear system undergoes very large deflection.
3) Principle of superposition can be applied.	3) Principle of superposition cannot be applied.
4) Linear system allows investigations to make certain mathematical assumptions and approximations allowing for easier computation of result.	4) In Non-Linear system these assumptions cannot be made.
5) Dimension and configuration do not change after the application of load.	5)Dimension and configuration changes on the application of load.

Sign Convention

As the forces and displacements are direction-dependent we need to adopt a sign convention to sum up the results of various actions. We adopt here the sign convention as shown in Fig. 1.16 for a 3D structure. Here X, Y, and Z are the coordinate axes and are shown in positive directions. When we move from X to Y in the horizontal plane, the Z-axis must advance in its positive direction. This is called left-hand system. We assume that forces directed along the positive direction of axes are positive. For a left-hand system the couple should be a left-hand screw progressing in the direction of the coordinate axes. So, an anticlockwise moment is taken as positive here.

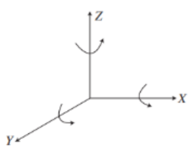


Fig. 1.16 Sign convention.

DEGREES OF FREEDOM, DETERMINATE, AND INDETERMINATE STRUCTURES

In this section, we now explain these terms that are frequently referred in structural analysis.

Degrees of Freedom

The degrees of freedom (DOF) can be defined as a set of independent displacements that specify completely the deformed position and orientation of the body or system under loading. Here, displacements include deflections and rotations as well. A rigid body that moves in 3D space in linear directions has three translational displacement components as DOFs. The rigid body can also undergo angular motion, which is called rotation. So, the body has three rotational DOFs. Altogether a rigid body can have at most six DOFS, three translations, and three rotations. Translation refers to the ability of a body to move without rotating whereas rotation refers to its angular motion about some axis.

When a structure is loaded, the joints, also called nodes, will undergo unknown displacements. These displacements are referred to as the DOF for the structures.

Determinate Structures

The conditions of equilibrium discussed in Section 1.5 are necessary and sufficient conditions to establish the equilibrium of structures. When structures are loaded, they pass on these loads to the support as reactions. The applied forces and the resulting reactions keep the structure in equilibrium. However, these reactions are mostly unknowns. We normally evaluate these reactions by using the equations of equilibrium. If all the reactions in a structure can be determined strictly only by the application of equilibrium equations, the structure is referred to as statically determinate. In other words, we can define a determinate structure as the one which can be fully analysed and all internal forces and stresses determined through the use of one or more of the six equations of equilibrium without recourse to stiffness, deflection, or other criteria for analysis.

Given a set of forces and reactions in equilibrium, the structural geometry of determinate structures takes care of itself. In other words, force–deformation compatibility for such structures is automatically satisfied for any set of forces and reactions in equilibrium. For example, the support reactions and hence, the moments and shears in a simple beam (Chapter 3) or a three-hinged arch (Chapter 20) can be found statically without paying any attention to their deformed shapes. As may be verified easily, a determinate structure has only as many support reactions as absolutely necessary for its stability. The removal of even a single reaction makes the structure unstable.

Figure 1.17 shows the determinate structures. In Fig. 1.17(a), the frame has three support reactions which can be calculated easily by Eq. (1.1). The arch in Fig. 1.17(b) has four support reactions against the three equations of equilibrium available for solution. So, it seems that reactions cannot be computed statically. However, the condition that the moment at the hinge C be zero provides the additional fourth equation for finding the four unknown reactions. Such additional equations are called condition equations. A statically determinate structure may also be defined alternatively as the one in which the number of unknown reactions R equals the sum of the number of applicable equations of equilibrium n and that of the condition equations c, i.e.,

$$R = n + c$$

Equation (1.3) is called the equations of statics.

The qualification 'applicable' is important because equilibrium equations which are applicable to a problem need only be counted in assessing its determinacy. For example, in the continuous beam as shown in Fig. 1.17(c), as the loading is only vertical, only two conditions, namely, $\sum F_V = 0$ and $\sum M = 0$ are applicable. Therefore, $\sum F_H = 0$ is meaningless in the absence of horizontal loads on the beam.

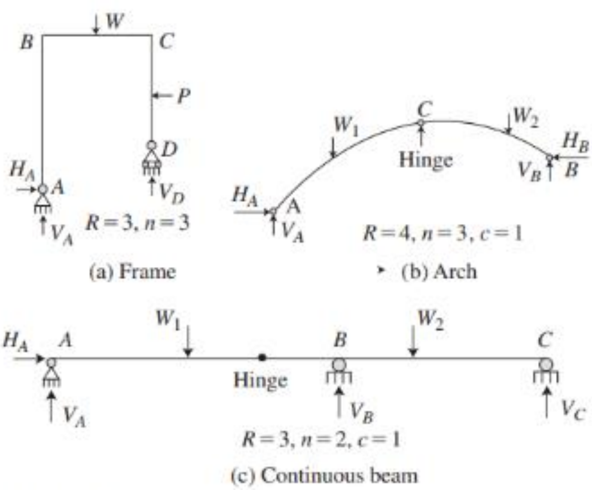


Fig. 1.17 Determinate structures.

Indeterminate Structures

Structures in which the reactions cannot be evaluated by the application of static equilibrium equations alone are defined a statically indeterminate or hyperstatic structures. They are also known as redundant structures. In these structures, the number of unknown reactions is greater than the number of available equations of static equilibrium. However, sometimes it is quite possible that the support reactions are statically determinate, but internal forces remain indeterminate. For example, we consider a truss shown in Fig. 1.18(a). We will discuss in Chapter 2 as how to evaluate the forces and reactions in a truss. Accordingly, the truss in Fig. 1.18(a) is statically determinate both for support reactions and forces in the members. In contrast, the truss shown in Fig. 1.18(b) is statically determinate only with reference to the calculations of support reactions.

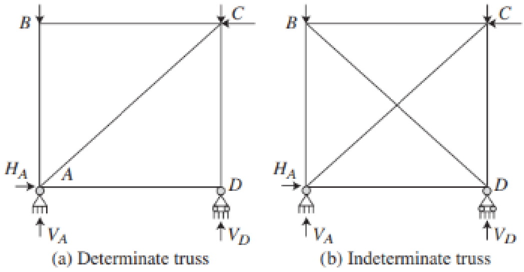


Fig. 1.18 Determinate and indeterminate structures.

We now consider, for example, a continuous beam (discussed in Chapter 9) shown in Fig. 1.19. It has six unknown support reactions as shown in Fig. 1.18 as against three equilibrium equations, namely, $\sum F_V = 0$, $\sum F_H = 0$, and ©M = 0 available for the determination of these six reactions. Unless we deter- mine these six reactions, it is not possible to evaluate the internal forces in the beam. Three extra equations should be set up to circumvent this difficulty. We can develop these equations from the geometrical conditions. For example, we can specify that the vertical deflections at B, C, and D are zero. These additional equations are called equations of compatibility and their number determines the degree of indeterminacy D of the structure. The reactions, for the solution of which the compatibility equations are developed, are termed as redundant R.

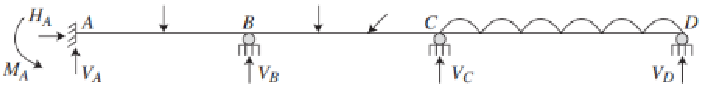


Fig. 1.19 Continuous beam.

It may be observed in Fig. 1.19 that the supports B, C, and D may be removed without affecting the stability of the beam. This action reduces the beam into a determinate one which is called a primary or released structure. So, we can conclude here that an indeterminate structure has more support reactions than are absolutely necessary for its stability. This characteristic may be used to deter- mine the degree of indeterminacy of structures. We can divide indeterminate structures into three categories as follows:

1. Externally indeterminate structures
2. Internally indeterminate structures
3. Structures with combined indeterminacies.

LINEAR AND NON-LINEAR STRUCTURES

We use wood, concrete, steel, etc., in the construction of structures. The load resistance and deformation characteristics of structures significantly depend on the properties of these materials. Each of these materials has different properties that should be taken into consideration in the analysis and design of the structure. Typical stress–strain curve for these materials is shown in Fig. 1.9. As is clear from Fig. 1.9, the ultimate tensile strength (UTS) of different materials is different. Therefore, their resistance to loading, which depends on UTS, is also varied. The initial slope of the curve for each material is different. This slope characterizes the modulus of elasticity or Young’s modulus E of the material. The modulus of elasticity of each material must be known for the calculation of dis- placement of structures.

As can be observed in Fig. 1.9 that each material in the initial stage behaves linearly, i.e., the stress and strain are proportional up to a certain limit which is called an elastic limit. This is also called a linear range. Subsequently, at higher stress the behaviour becomes non-linear, i.e., there is disproportionate increase in strain for a corresponding increase in stress. This zone is called a non-linear range. We call a system a linear structure when the stresses developed in it are within the elastic limit, i.e., the stresses in the system lie within the linear range. A system is called a non-linear structure if the stresses developed in it fall in the plastic or a non-linear range. Such a classification is based on the behaviour of material. The behaviour of a material in the plastic regime is characterized as material non-linearities for representation in structural analysis.

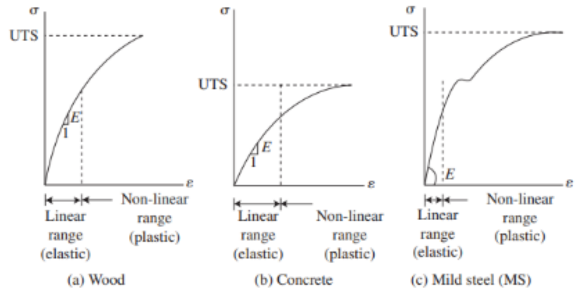


Fig. 1.9 Typical stress–strain relation of various materials.

In addition to material non-linearity, some structures may exhibit non-linear characteristics in its overall behaviour due to changes in its shape under loading. This necessitates that the structure should displace by a significant amount to maintain its overall equilibrium. This kind of behaviour of the structure is called *geometrical non-linearity*. A classic example of this type of non-linearity can be observed in cable structures discussed in Chapter 21. Also, a cantilever structure shown in Fig. 1.10 is another example of geometric non-linearity.

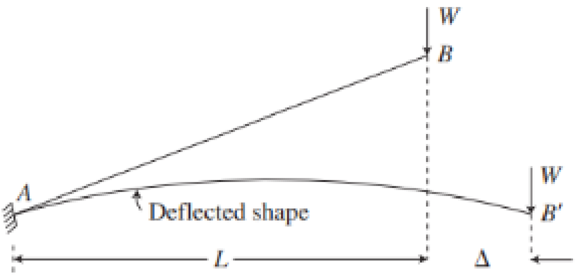


Fig. 1.10 Geometric non-linearity.

An important property of a linear structure is that when it is loaded, the stress in the material increases along a linear path till the elastic limit in the material is reached. Suppose we unload the structure or remove the load on the structure within this stage, the stress diminishes and it retraces the same linear path and the structure returns to its original position without leaving any residual deformation in the structure as shown in Fig. 1.11(a).

In contrast, the stress level in a non-linear structure goes beyond elastic limit and mostly it remains in plastic regime. If the load is removed from the structure once the stresses have crossed elastic limit, then the structure returns to the original position by a different path as shown in Fig. 1.11(b) leaving some residual deformation in the structure. This is called a permanent set.

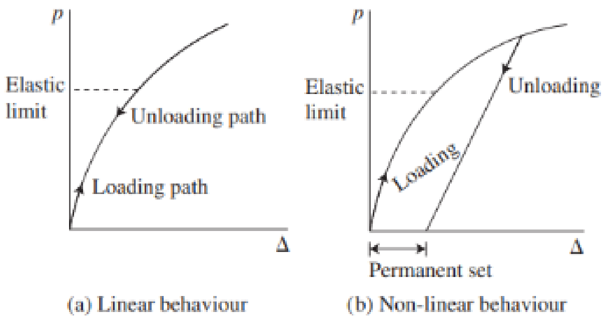


Fig. 1.11 Linear and non-linear structures.

STATIC AND KINEMATIC INDETERMINACIES OF STRUCTURAL SYSTEMS

Before analyzing a structure, the analyst must ascertain whether the reactions can be computed using equations of equilibrium alone. If all unknown reactions can be uniquely determined from the simultaneous solution of the equations of static equilibrium, the reactions of the structure are referred to as **statically determinate**. If they cannot be determined using equations of equilibrium alone then such structures are called **statically indeterminate structures.** If the number of unknown reactions are less than the number of equations of equilibrium then the structure is statically unstable.

The degree of indeterminacy is always defined as the difference between the number of unknown forces and the number of equilibrium equations available to solve for the unknowns. These extra forces are called redundants. Indeterminacy with respect external forces and reactions are called **externally indeterminate** and that with respect to internal forces are called **internally indeterminate.**

A general procedure for determining the degree of indeterminacy of two-dimensional structures are given below:

NUK= Number of unknown forces

NEQ= Number of equations available

IND= Degree of indeterminacy

NJ = Number of joints

IND= NUK – NEQ

Indeterminacy of Planar Trusses

Members carry only axial forces NEQ = 2NJ

NUK= NM+NR

IND= NUK – NEQ

IND= NM+NR-2NJ

Indeterminacy of 3D FRAMES

A member or a joint has to satisfy 6 equations of equilibrium NEQ = 6NM + 6NJ-NC (Number of additional condition)

NUK= 12NM+NR

IND= NUK – NEQ

IND= 6NM+NR-6NJ-NC

Indeterminacy of 3D Trusses

A joint has to satisfy 3 equations of equilibrium

NEQ = 3NJ

NUK= NM+NR

IND= NUK – NEQ

IND= NM+NR - 3NJ

Stable Structure:

Another condition that leads to a singular set of equations arises when the body or structure is improperly restrained against motion. In some instances, there may be an adequate number of support constraints, but their arrangement may be such that they cannot resist motion due to applied load. Such situation leads to instability of structure. A structure may be considered as externally stable and internally stable.

Externally Stable:

Supports prevents large displacements

No. of reactions ≥ No. of equations

Internally Stable:

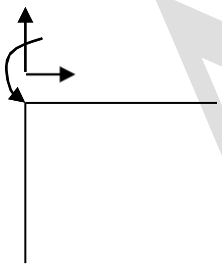
Geometry of the structure does not change appreciably

For a 2D truss NM ≥ 2Nj -3 (NR ≥ 3)

For a 3D truss NM ≥ 3Nj -6 (NR ≥ 3)

Degree of freedom or Kinematic Indeterminacy

Members of structure deform due to external loads. The minimum number of parameters required to uniquely describe the deformed shape of structure is called **"Degree of Freedom".** Displacements and rotations at various points in structure are the parameters considered in describing the deformed shape of a structure. In framed structure the deformation at joints is first computed and then shape of deformed structure. Deformation at intermediate points on the structure is expressed in terms of end deformations. At supports the deformations corresponding to a reaction is zero. For example hinged support of a two dimensional system permits only rotation and translation along x and y directions are zero. Degree of freedom of a structure is expressed as a number equal to number of free displacements at all joints. For a two-dimensional structure each rigid joint has three displacements as shown in Fig. 1.5



In case of three-dimensional structure each rigid joint has six displacements.

- Expression for degrees of freedom

1. 2D Frames: NDOF = 3NJ – NR NR ≥3
2. 3D Frames: NDOF = 6NJ – NR NR ≥6
3. 2D Trusses: NDOF= 2NJ – NR NR ≥3
4. 3D Trusses: NDOF = 3NJ – NR NR ≥6

Where, NDOF is the number of degrees of freedom

In 2D analysis of frames sometimes axial deformation is ignored. Then NAC=No. of axial condition is deducted from NDOF

Plane Trusses

Trusses are articulated frames used extensively in the construction of a variety of structures. The spectrum consists of industrial sheds to airport hangers at one end and lean-to-roof to high-rise buildings at the other. Different types of trusses and frames existing across the globe and mostly they are constructed in wood and steel.

It is quite obvious that a truss or a frame is an assemblage of bars or rods. Sometimes tubes, angles, and channels are also used. We know that a bar is always subjected to a load along its longitudinal axis. It can be a tensile force or a compressive force. We can join two bars with a pin. They may be subjected to either a tensile force or a compressive force. In this configuration as the bars are connected only at one end, the other ends are free and hence they can rotate freely. Therefore, it forms an unstable system. Moreover, this arrangement cannot resist any load. In order to position, fix the free ends and make the system to resist load, we connect the free ends of both the bars by another bar with pins.

We know that a triangular configuration is a stable and rigid system. Therefore, the triangular arrangement of bars provides a stable structure which can resist load without any relative displacement between the bars. Loads are invariably applied only at joints. A truss can either be in one plane or in space which constitutes multiple planes. A truss lying in one plane is called a plane truss. In contrast, a truss lying in multiple planes or space is called space truss.

Plane trusses are used to support roofs and bridges. Roof trusses are often used as part of an industrial building frame. In bridges, the truss is the main structural element.

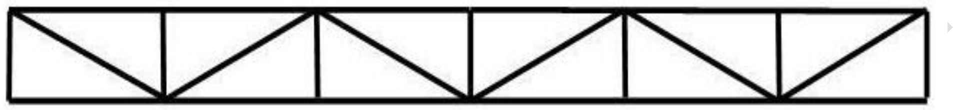
The distance between the supports is called the span of the truss. The top member of a truss is called top or upper chord. It can either be horizontal or inclined. Similarly, the bottom member is called lower or bottom chord. Both the top and bottom chords are connected by vertical and diagonal members. The space enclosed between the top and the bottom chord members, and diagonals is called a panel. The pattern of bars arranged in a triangular configuration is chosen to yield a light-weight, efficient, and load-bearing structure. The ends of the bars are connected to gusset plates with the help of bolts or welds. Such connections are called joints of the truss. Loads are applied only at these joints. Truss members act in direct stress, i.e., tension or compression; they carry load efficiently and hence their cross sections are relatively small and also slender.

Types of trusses

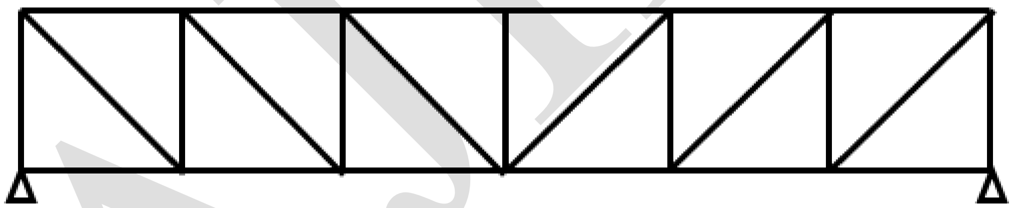
Triangular configuration, which is the basic unit of a truss, can be extended to enclose more space. Such an exercise will produce a variety of trusses that can be used in roofs as well as in bridges. The common types of roof trusses are shown in figure.

1. Warren truss
2. Pratt truss
3. Bowstring truss
4. King post truss
5. Lenticular truss

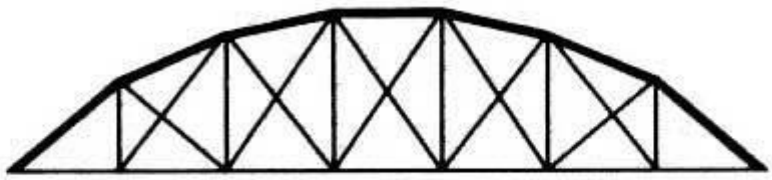
1. **Warren truss -** Truss members form a series of equilateral triangles, alternating up and down.



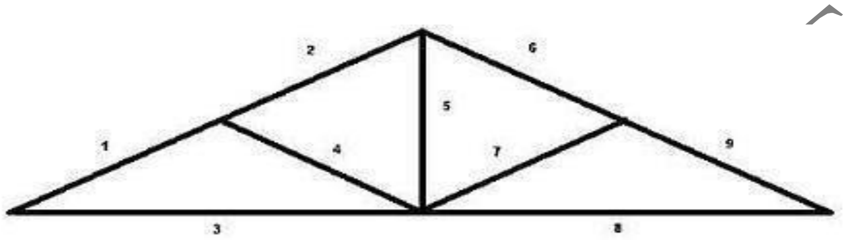
2. **Pratt truss** - Pratt trusses are commonly used in long span buildings ranging from 20 to 100 m in span. In a conventional Pratt truss, diagonal members are in tension for gravity loads. This type of truss is used where gravity loads and uplift loads are predominant



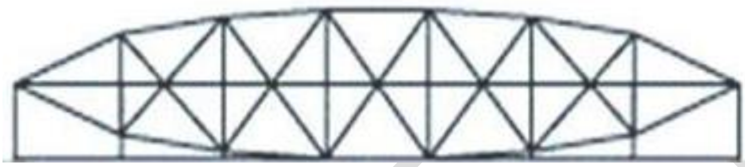
3. **Bowstring truss** - Named for their shape, bowstring trusses were first used for arched truss bridges, often confused with tied-arch bridges. Thousands of bowstring trusses were used during World War II for holding up the curved roofs of aircraft hangars and other military buildings.



4. **King post truss** - One of the simplest truss styles to implement, the king post consists of two angled supports leaning into a common vertical support.



5. **Lenticular truss** - have the top and bottom chords of the truss arched, forming a lens shape. A lenticular pony truss bridge is a bridge design that involves a lenticular truss extending above and below the road bed.



Assumptions in analysis

1. The members cannot develop moments at the ends.
2. The members are subjected to purely axial forces.
3. The connections to other members are perfectly pinned/hinged through frictionless pins.
4. Each member is of uniform cross-sectional area.
5. The entire structure is in one plane if it is a plane truss
6. Loads act at the joints only. Any loads that act between joints are split into equivalent support end reactions and added to the joint loads.
7. Self-weight of the truss can be ignored or at least assumed to be equally distributed as loads at the joints.
8. Even if the members are connected at the ends with gusset plates and welded, no fixity is assumed. A nominal moment that actually develops due to imperfect hinge connection is simply ignored.

Analysis of determinate trusses by method of joints and method of sections.

1. **Method of Joints -** The free-body diagram of any joint is a concurrent force system in which the summation of moment will be of no help. two equilibrium equations can be written as $\Sigma Fx=0$ and $\Sigma Fy=0$.

   This means that to solve completely for the forces acting on a joint, we must select a joint with no more than two unknown forces involved. This can be started by selecting a joint acted on by only two members. We can assume any unknown member to be either tension or compression. If negative value is obtained, this means that the force is opposite in action to that of the assumed direction. Once the forces in one joint are determined, their effects on adjacent joints are known. We then continue solving on successive joints until all members have been found.

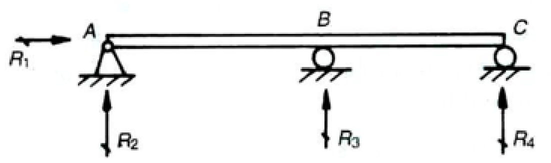
2. **Method of Sections** - In this method, we will cut the truss into two sections by passing a cutting plane through the members whose internal forces we wish to determine. This method permits us to solve directly any member by analyzing the left or the right section of the cutting plane.

   To remain each section in equilibrium, the cut members will be replaced by forces equivalent to the internal load transmitted to the members. Each section may constitute of non-concurrent force system from which three equilibrium equations can be written as $\Sigma Fx=0$, $\Sigma Fv=0$ and $\Sigma M=0$. Because we can only solve up to three unknowns, it is important not to cut more than three members of the truss. Depending on the type of truss and which members to solve, one may have to repeat Method of Sections more than once to determine all the desired forces.

Numerical Problems

1. Determine Degrees of Statical indeterminacy and classify the structures.

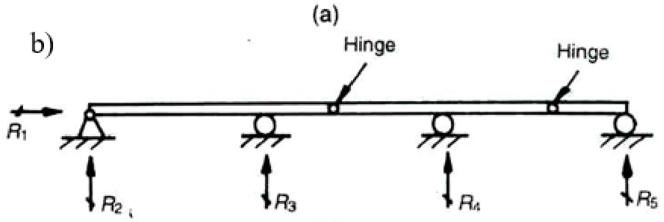
a)



(a)

NM=2; NJ=3; NR =4; NC=0
IND=3NM+NR-3NJ-NC
IND=3 x 2 + 4 – 3 x 3 -0 = 1
INDETERMINATE

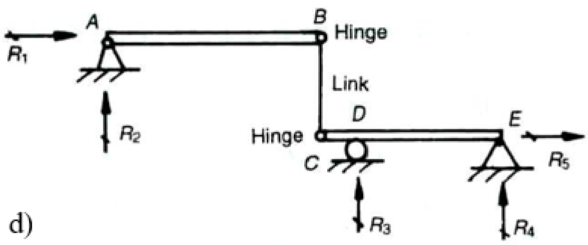
b)



(b)

NM=3; NJ=4; NR =5; NC=2
IND=3NM+NR-3NJ-NC
IND=3 x 3 + 5 – 3 x 4 -2 = 0
DETERMINATE

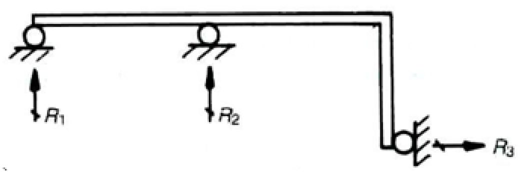
c)



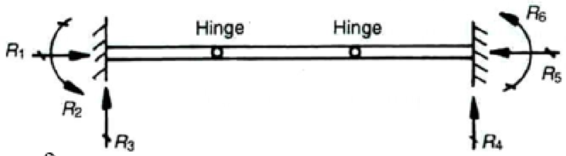
(c)

NM=3; NJ=4; NR =5; NC=2
IND=3NM+NR-3NJ-NC
IND=3 x 3 + 5 – 3 x 4 -2 = 0
DETERMINATE

d)

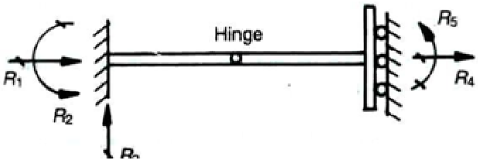


NM=3; NJ=4; NR =3; NC=0
IND=3NM+NR-3NJ-NC
IND=3 x 3 + 3 – 3 x 4 -0 = 0
DETERMINATE



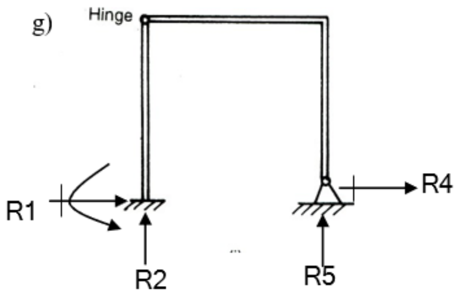
f) (e)

NM=1; NJ=2; NR =6; NC=2
IND=3NM+NR-3NJ-NC
IND=3 x 1 + 6 – 3 x 2 -2 = 1
INDETERMINATE

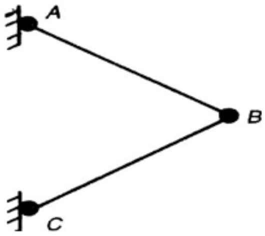


NM=1; NJ=2; NR =5; NC=1
IND=3NM+NR-3NJ-NC
IND=3 x 1 + 5 – 3 x 2 -1 = 1
INDETERMINATE

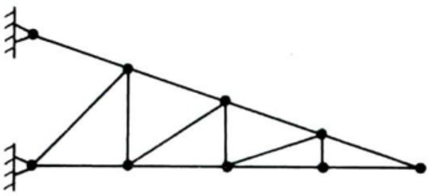
g)



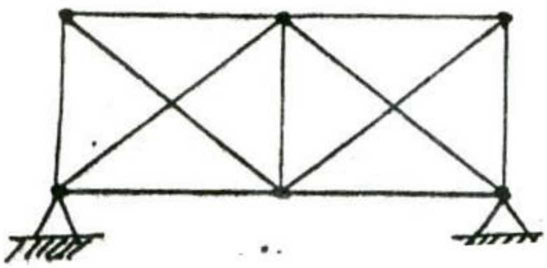
NM=1; NJ=2; NR =5; NC=1
IND=3NM+NR-3NJ-NC
IND=3 x 1 + 5 – 3 x 2 -1 = 1
INDETERMINATE



Truss
NM=2; NJ=3; NR =4;
IND=NM+NR-2NJ
IND= 2 + 4 – 2 x 3 = 0
DETERMINATE



Truss
NM=14; NJ=9; NR =4;
IND=NM+NR-2NJ
IND= 14+ 4 – 2 x 9 = 0
DETERMINATE



Truss
NM=11; NJ=6; NR =4;
IND=NM+NR-2NJ
IND= 11 + 4- 2 x 6 = 3
INDETERMINATE

Problems on method of joints

1. Find the force acting in all members of the truss shown in Figure T-01. using methods of joints.

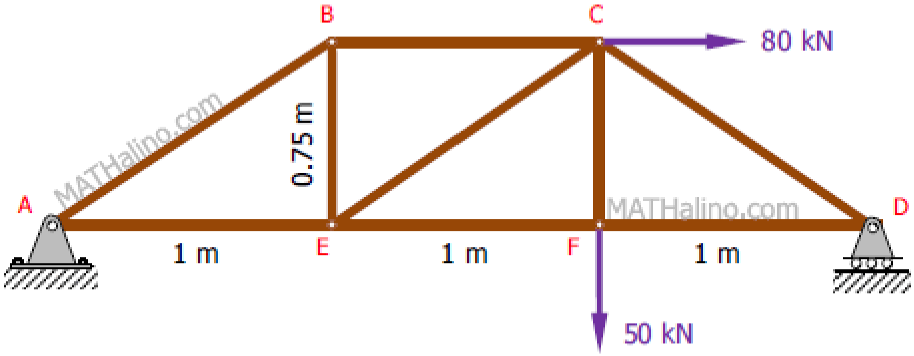
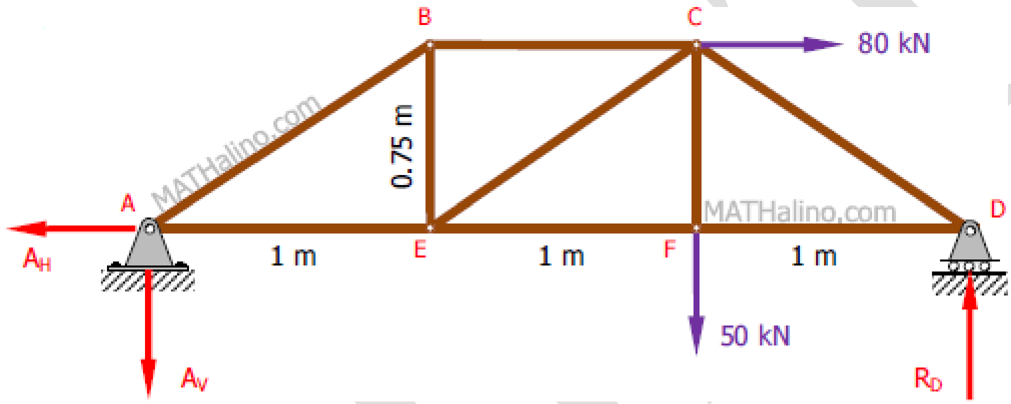


Figure T-01



$\Sigma M_D=0$

$3A_V+50(1)=80(0.75)$

$A_V=3.33$ kN

$\Sigma F_H=0$

$A_H=80$ kN

$\Sigma M_A=0$

$3R_D=50(2)+80(0.75)$

$R_D=53.33$ kN

At joint A

$\Sigma F_V=0$

$3/5F_{AB}=3.33$

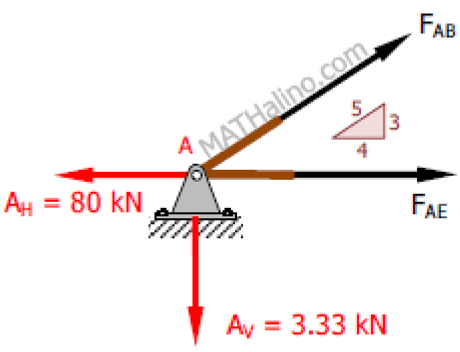
$F_{AB}=5.56$ kN tension

$\Sigma F_H=0$

$F_{AE}+45F_{AB}=80$

$F_{AE}+45(5.56)=80$

$F_{AE}=75.56$ kN tension



FBD of Joint A

At joint B

$\Sigma F_H=0$

$F_{BC}=4/5\ F_{AB}$

$F_{BC}=4/5\ (5.56)$

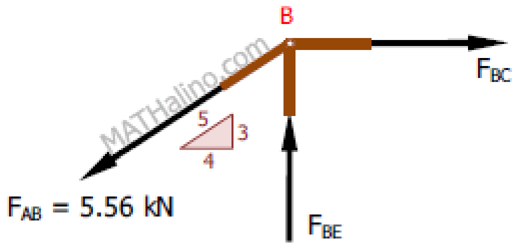
$F_{BC}=4.45$ kN

$\Sigma F_V=0$

$F_{BE}=3/5\ F_{AB}$

$F_{BE}=3/5\ (5.56)$

$F_{BE}=3.34$ kN compression



FBD of Joint B

Joint E

$\Sigma F_V=0$

$35F_{CE}=F_{AE}$

$35F_{CE}=3.3435$

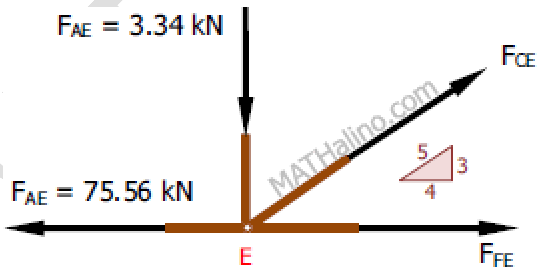
$F_{CE}=5.57$ kN tension

$\Sigma F_H=0$

$F_{FE}+4/5\ F_{CE}=F_{AE}$

$F_{FE}+4/5\ (5.57)=75.56$

$F_{FE}=71.11$ kN tension



FBD of Joint E

At joint F

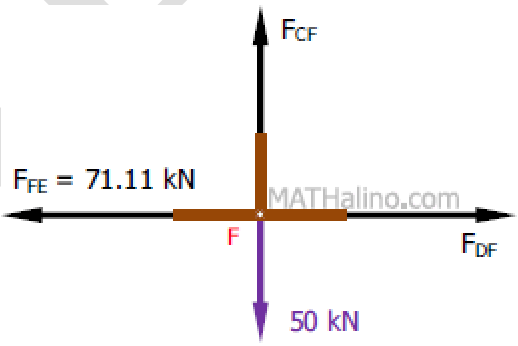
$\Sigma F_V=0$

$F_{CF}=50$ kN tension

$\Sigma F_{FH}=0$

$F_{DF}=F_{FE}$

$F_{DF}=71.11$ kN tension



FBD of Joint F

At joint C

$\Sigma F_H=0$

$4/5\ F_{CD}+4/5\ F_{CE}+F_{BC}=80$

$4/5\ F_{CD}+4/5\ (5.57)+4.45=80$

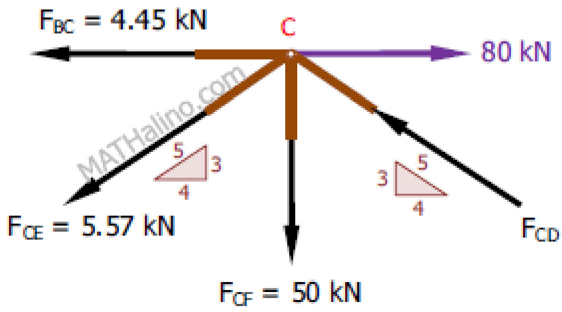
$F_{CD}=88.87$ kN compression

$\Sigma F_V=0$

$3/5\ F_{CD}=3/5\ F_{CE}+F_{CF}$

$3/5\ (88.87)=3/5\ (5.57)+50$

$53.3 = 53.3$



FBD of Joint C

At joint D

$\Sigma F_H=0$

$4/5\ F_{CD}=F_{DF}$

$4/5\ (88.87)=71.11$

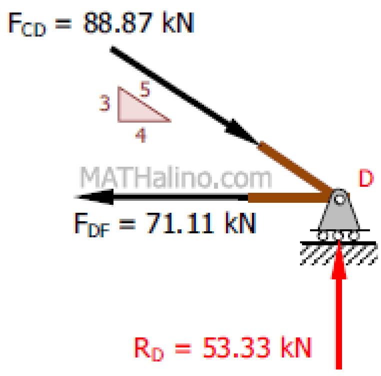
$71.1=71.1$ *check*

$\Sigma F_V=0$

$R_D=3/5\ F_{CD}$

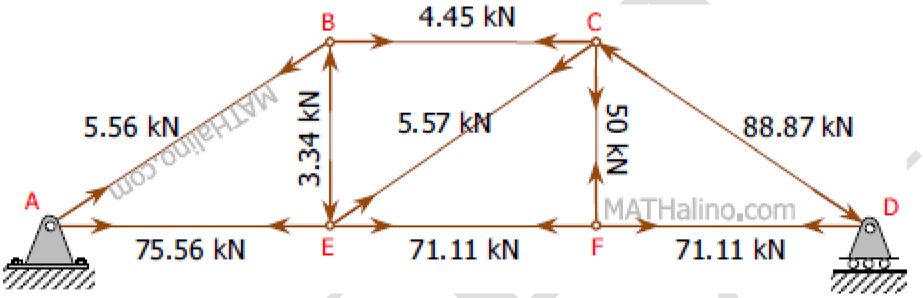
$53.33=3/5\ (88.87)$

$53.3=53.3$ *check*



FBD of Joint D

Summary



F_{AB}=5.56 kN tension

F_{AE}=75.56 kN tension

F_{BC}=4.45 kN tension

F_{BE}=3.34 kN compression

F_{CD}=88.87 kN compression

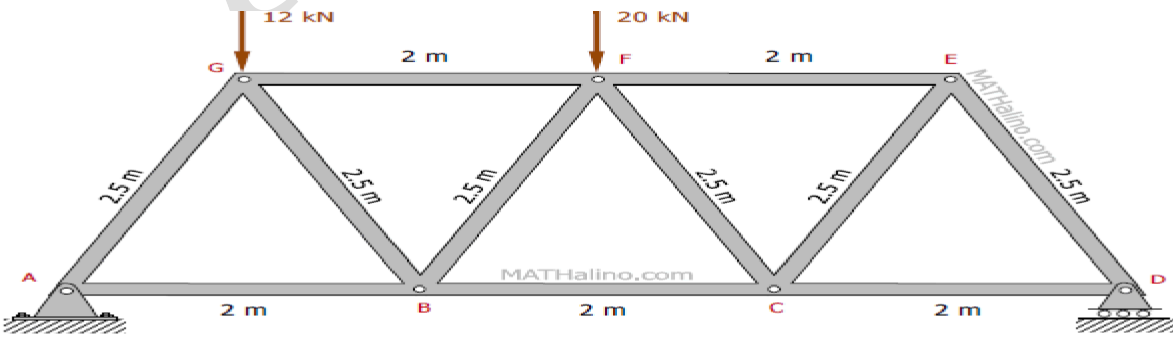
F_{CE}=5.57 kN tension

F_{CF}=50 kN tension

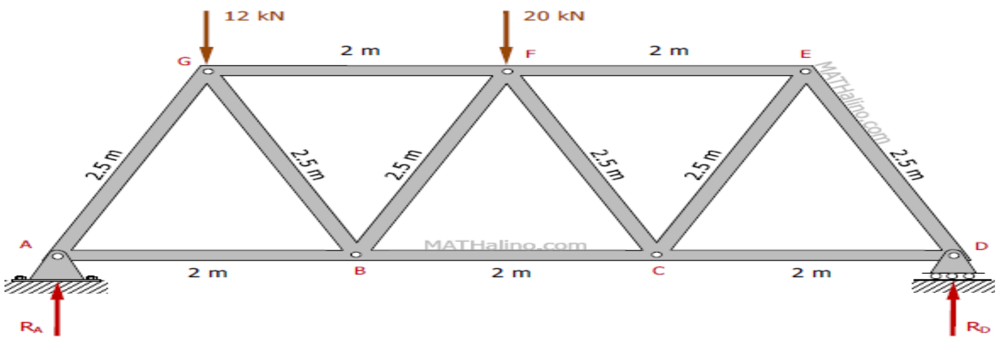
F_{DF}=71.11 kN tension

F_{FE}=71.11 kN tension

2. **The structure in Fig. T-02 is a truss which is pinned to the floor at point A, and supported by a roller at point D. Determine the force to all members of the truss. using method of joints.**



Solution



$\Sigma M_D=0$

$6R_A=5(12)+3(20)$

$R_A=20$ kN

$\Sigma MA=0$

$6R_D=1(12)+3(20)$

$R_D=12$ kN

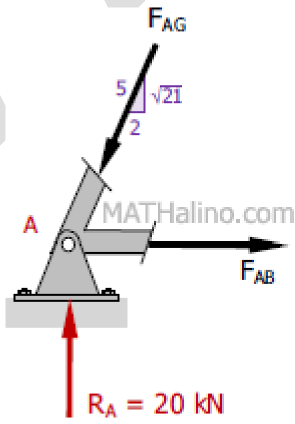
At joint A

$\Sigma F_V=0$

$21/\sqrt{5}\ F_{AG}=R_A$

$21/\sqrt{5}\ F_{AG}=20$

$F_{AG}=21.82$ kN compression



FBD of Joint A

$\Sigma F_H=0$

$F_{AB}=2/5\ F_{AG}$

$F_{AB}=2/5\ (21.82)$

$F_{AB}=8.73$ kN tension

At joint G

$\Sigma FV=0$

$21/\sqrt{5}\ F_{BG}+12=21/\sqrt{5}\ F_{AG}$

$21/\sqrt{5}\ F_{BG}+12=21/\sqrt{5}\ (21.82)$

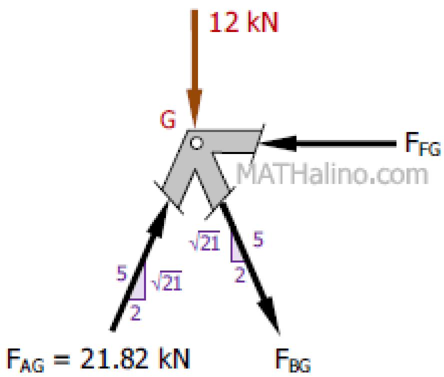
$F_{BG}=8.73$ kN tension

$\Sigma F_H=0$

$F_{FG}=2/5\ F_{AG}+2/5\ F_{BG}$

$F_{FG}=2/5\ (21.82)+2/5\ (8.73)$

$F_{FG}=12.22$ kN compression



FBD of Joint G

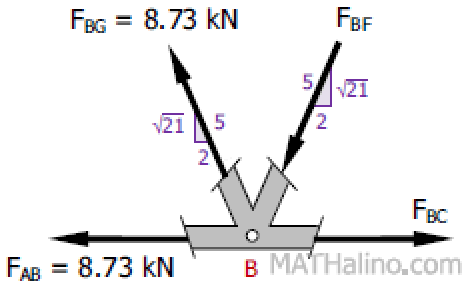
At joint B

$\Sigma F_V=0$

$21/\sqrt{5}\ F_{BF}=21/\sqrt{5}\ F_{BG}$

$F_{BF}=F_{BG}$

F_{BF}=8.73 kN compression



FBD of Joint B

$\Sigma F_H=0$

$F_{BC}=F_{AB}+2 / 5\ F_{BG}+2 / 5\ F_{BF}$

$F_{BC}=8.73+2 / 5\ (8.73) +2/ 5\ (8.73)$

F_{BC}=15.71 kN tension

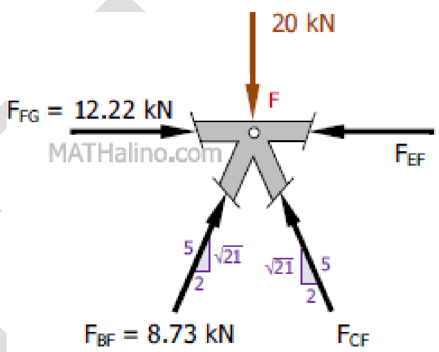
At joint F

$\Sigma F_V=0$

$21 / \sqrt{5}\ F_{CF}+ 21/ \sqrt{5}\ F_{BF}= 20$

$21 / \sqrt{5}\ F_{CF}+21 / \sqrt{5}\ (8.73) =20$

F_{CF}=13.09 kN compression



FBD of Joint F

$\Sigma F_H=0$

$F_{EF}+2 / 5\ F_{CF}= 2 / 5\ F_{BF}+ F_{FG}$

$F_{EF}+2 / 5\ (13.09) =2 / 5\ (8.73) +12.22$

F_{EF}=10.48 kN compression

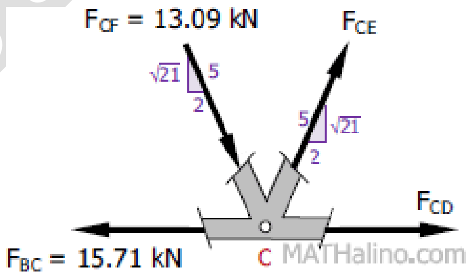
At joint C

$\Sigma F_V=0$

$21/ \sqrt{5}\ F_{CE}=21/ \sqrt{5}\ F_{CF}$

$F_{CE}= F_{CF}$

F_{CE}=13.09 kN tension



FBD of Joint C

$\Sigma F_H=0$

$F_{CD}+25F_{CE}+25F_{CF}=F_{BC}$

$F_{CD}+25(13.09) +25(13.09) =15.71$

F_{CD}=5.24 kN tension

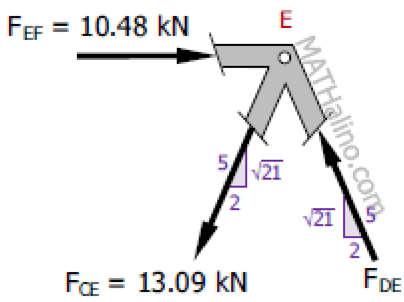
At joint E

$\Sigma F_V=0$

$21 / \sqrt{5}F_{DE}=21 / \sqrt{5}F_{CE}$

$F_{DE}=F_{CE}$

F_{DE}=13.09 kN compression



FBD of Joint E

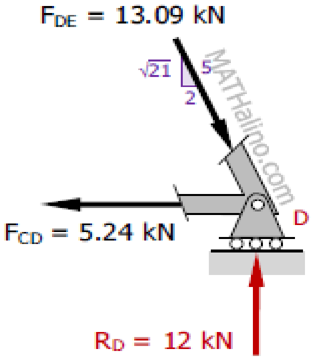
$\Sigma F_H=0$

$F_{EF}=25F_{CE}+25F_{DE}$

10.48=25(13.09) +25(13.09)

10.5=10.5 *check*

At joint D



FBD of Joint D

$\Sigma F_V=0$

$R_D=21 / \sqrt{5}\ F_{DE}$

$12=21/ \sqrt{5}\ (13.09)$

$12=12$ *check*

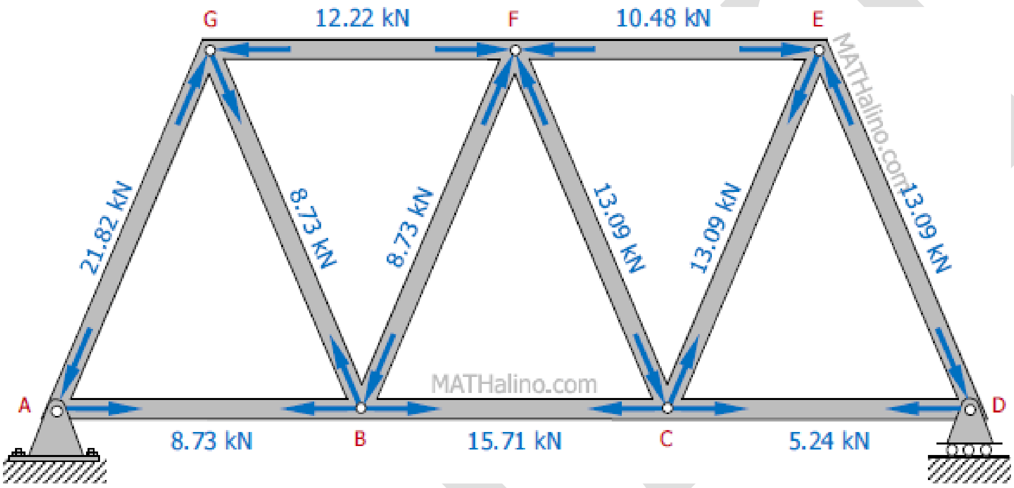
$\Sigma F_H=0$

$F_{CD}=2 / 5\ F_{DE}$

$5.24=2 / 5\ (13.09)$

$5.24=5.24$ *check*

Summary



F_{AB} = 8.73 kN tension

F_{AG} = 21.82 kN compression

F_{BC} = 15.71 kN tension

F_{BF} = 8.73 kN compression

F_{BG} = 8.73 kN tension

F_{CD} = 5.24 kN tension

F_{CE} = 13.09 kN tension

F_{CF} =13.09kN compression

F_{DE} = 13.09 kN compression

F_{EF} = 10.48 kN compression

F_{FG} = 12.22 kN compression

Problems on method of sections

1. From the truss in Fig. T-01, determine the force in members BC, CE, and EF.

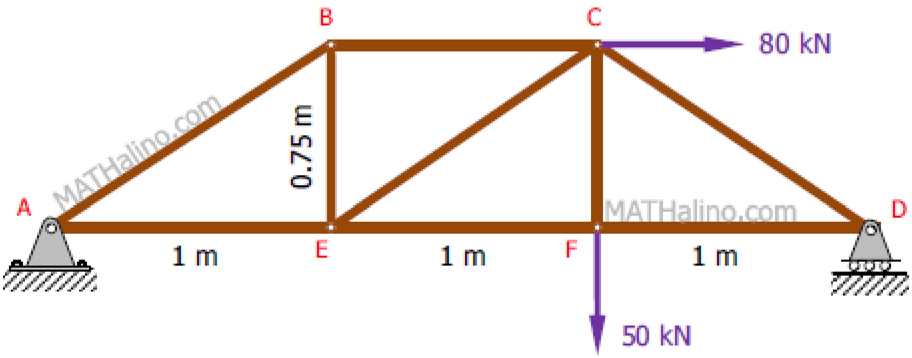
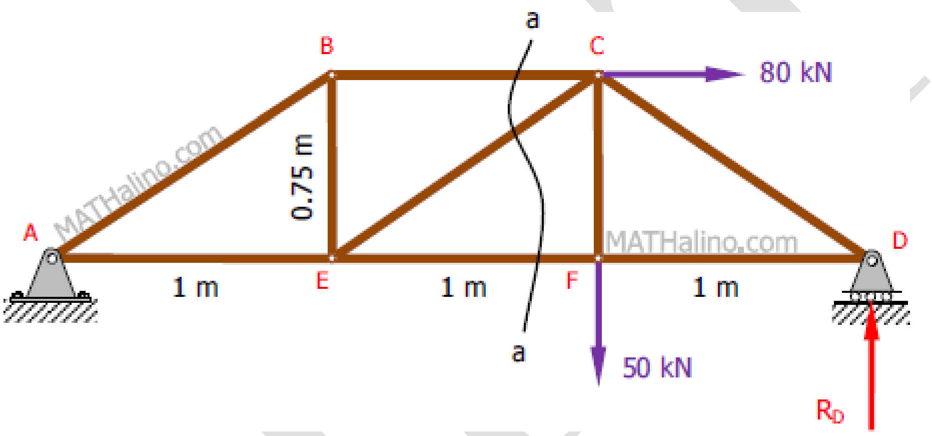


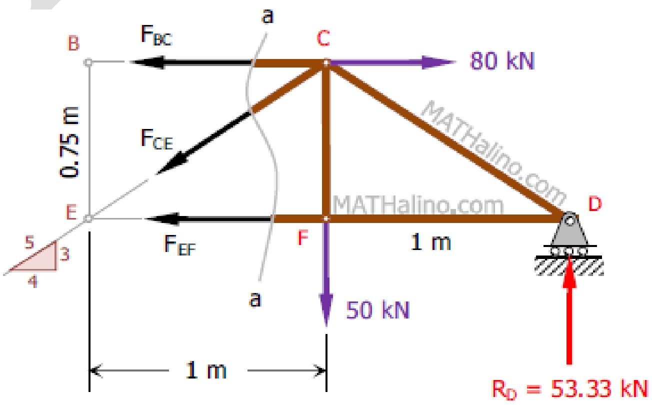
Figure T-01

Solution



$\Sigma M_A=0$
$3R_D=50(2)+80(0.75)$
$R_D=53.33$ kN

From the FBD of the section through a-a
$\Sigma M_E=0$
$0.75F_{BC}+2R_D=0.75(80)+1(50)$
$0.75F_{BC}+2(53.33)=60+50$
$F_{BC}=4.45$ kN tension *answer*



Section to the right of a-a

$\Sigma M_C=0$

$0.75F_{EF}=1(R_D)$

$0.75F_{EF}=53.33$

$F_{EF}=71.11$ kN tension *answer*

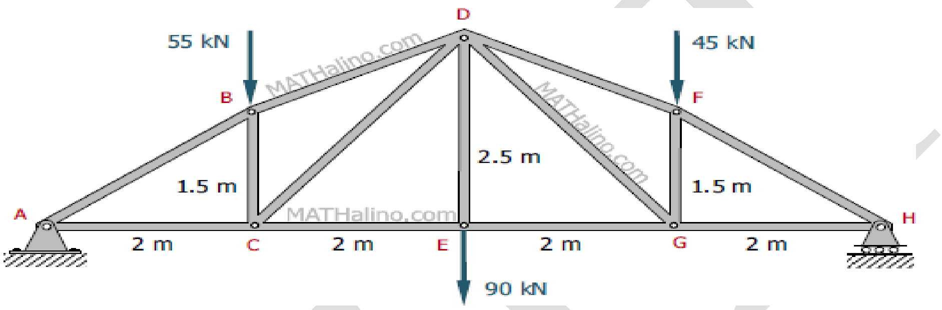
$\Sigma F_V=0$

$3 / 5\ F_{CE}+50=RD$

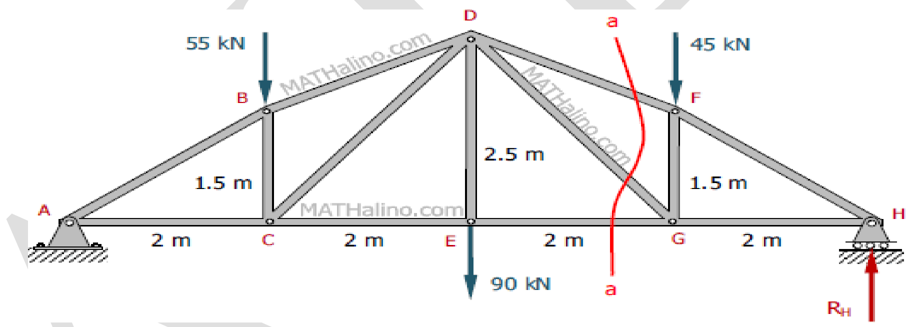
$3 / 5\ F_{CE}+50= 53.33$

$F_{CE}=5.55$ kN tension *answer*

2. **The roof truss shown in Fig. T-03 is pinned at point A, and supported by a roller at point H. Determine the force in member DG.**



<u>Solution</u>



$\Sigma M_A=0$

$8R_H=2(55) +4(90)+6(45)$

$R_H=92.5$ kN

From section to the right of a-a

$x+2 / (1.5) = (x+4) / 2.5$

$2.5x+5=1.5\ x+6$

$x=1$ m

$\Sigma M_O=0$

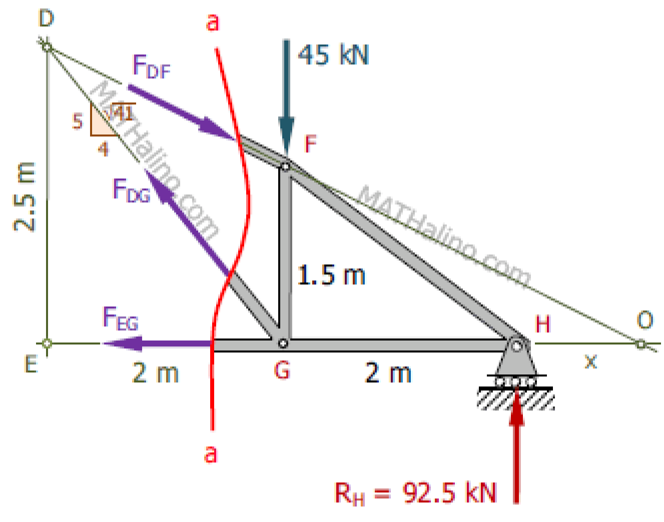
$(x+2)(5 /\sqrt{41} * F_{DG})+x\ R_H= (x+2)(45)$

$(1+2)\ (5 / \sqrt{41} * F_{DG})+ 1(92.5) = (1+2)(45)$

$(15 / \sqrt{41})\ F_{DG}+92.5=135$

$(15 / \sqrt{41})\ F_{DG}=42.5$

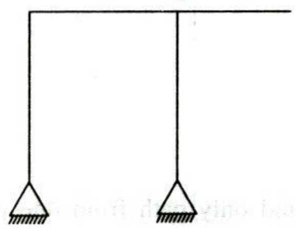
$F_{DG}=18.14$ kN tension *answer*



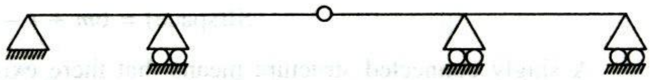
Section to the right of a-a

REVIEW QUESTIONS

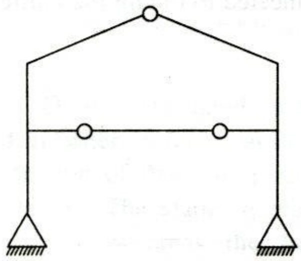
1. Determine Degrees of indeterminacy and classify the structures.

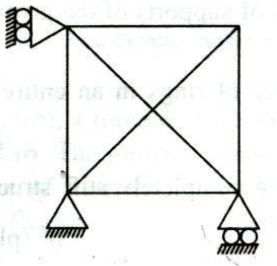


P.3.1

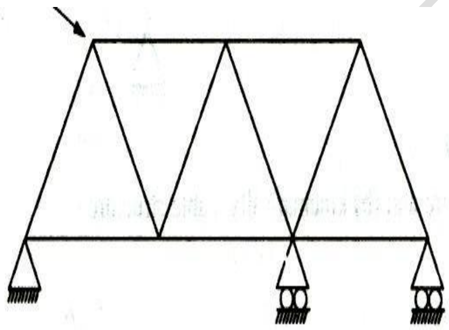


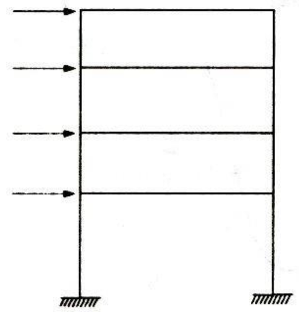
P.3.2

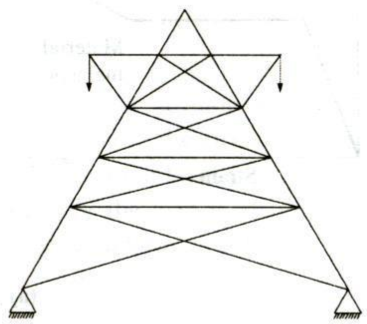


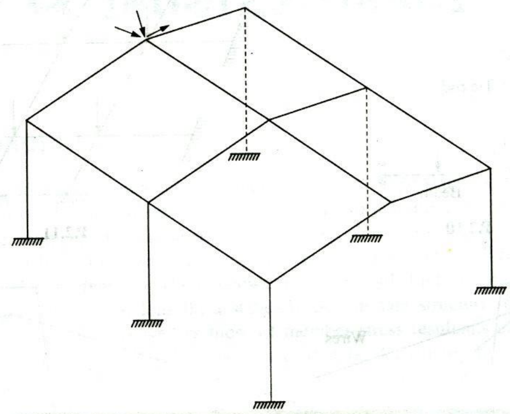


2. Determine Degrees of Kinematic indeterminacy

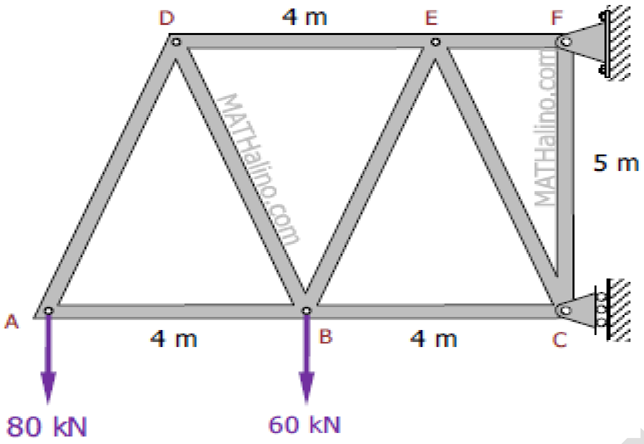




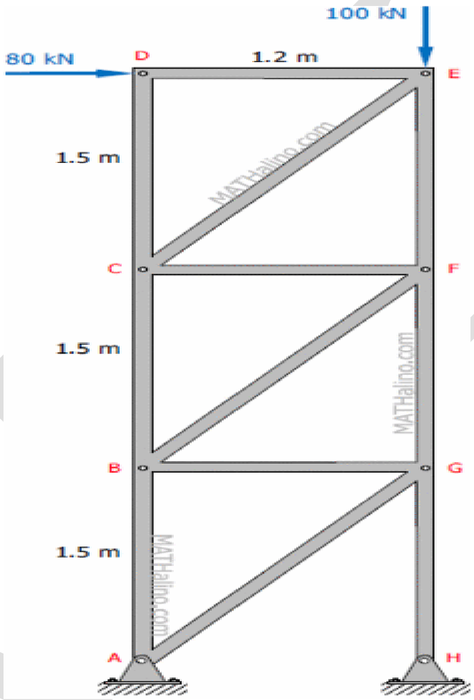




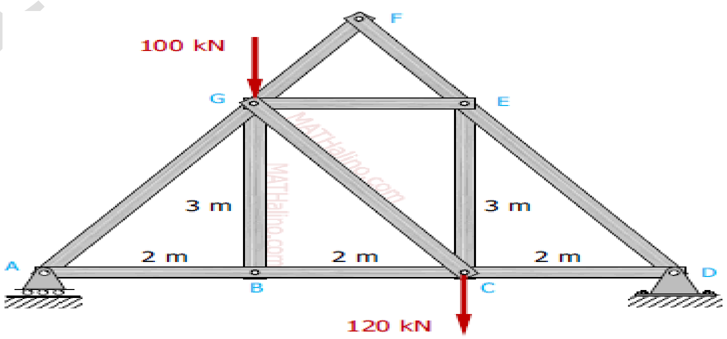
3. The truss in Fig. T-03 is pinned to the wall at point F, and supported by a roller at point C. Calculate the force (tension or compression) in members BC, BE, and DE using method of sections.



4. The structure shown in Figure is pinned to the floor at A and H. Determine the magnitude of all the support forces acting on the structure and find the force in member BF.



5. The truss pinned to the floor at D, and supported by a roller at point A is loaded as shown in Fig. Determine the force in member CG. use method of joints



6. Compute the force in all members of the truss shown in Fig. using method of joints.