VISVESVARAYA TECHNOLOGICAL UNIVERSITY

BELGAUM

DESIGN OF RC STRUCTURAL ELEMENTS

(Subject Code: 21CV53)

LECTURE NOTES

(MODULE-1)

V-SEMESTER

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Module - 1

INTRODUCTION TO WORKING STRESS AND LIMIT STATE DESIGN

1.1 REINFORCED CONCRETE

Concrete may be remarkably strong in compression, but it is equally remarkably weak in tension [Fig.1.1)]. Its tensile 'strength' is approximately one-tenth of its compressive 'strength'. Hence, the use of plain concrete as a structural material is limited to situations where significant tensile stresses and strains do not develop

Concrete would not have gained its present status as a principal building material, but for the invention of **reinforced concrete,** which is concrete with steel bars embedded in it. The idea of reinforcing concrete with steel has resulted in a new *composite* material, having the potential of resisting significant tensile stresses, which was previously impossible. Thus, the construction of load-bearing flexural members, such as beams and slabs, became viable with this new material. The steel bars (which are embedded in the tension zone of the concrete) compensate for the concrete's incapacity for tensile resistance, effectively taking up all the tension, without separating from the concrete [Fig. 1.1(b)]. The *bond* between steel and the surrounding concrete ensures *strain compatibility*, i.e., the strain at any point in the steel is equal to that in the adjoining concrete. Moreover, the reinforcing steel imparts *ductility* to a material that is otherwise brittle. In practical terms, this implies that if a properly reinforced beam were to fail in tension, then such a failure would, fortunately, be preceded by large deflections caused by the yielding of steel, thereby giving ample warning of the impending collapse [Fig. 1.1(c)].

Tensile stresses occur either directly, as in direct tension or flexural tension, or indirectly, as in shear, which causes tension along diagonal planes ('diagonal tension'). Temperature and shrinkage effects may also induce tensile stresses. In all such cases, reinforcing steel is essential, and should be appropriately located, in a direction that cuts across the principal tensile planes (i.e., across potential tensile cracks). If insufficient steel is provided, cracks would develop and propagate, and could possibly lead to failure.

Reinforcing steel can also supplement concrete in bearing compressive forces, as in columns provided with longitudinal bars. These bars need to be confined by transverse steel ties [Fig. 1.1(d)], in order to maintain their positions and to prevent their lateral buckling. The lateral

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ties also serve to confine the concrete, thereby enhancing its compression load-bearing capacity.

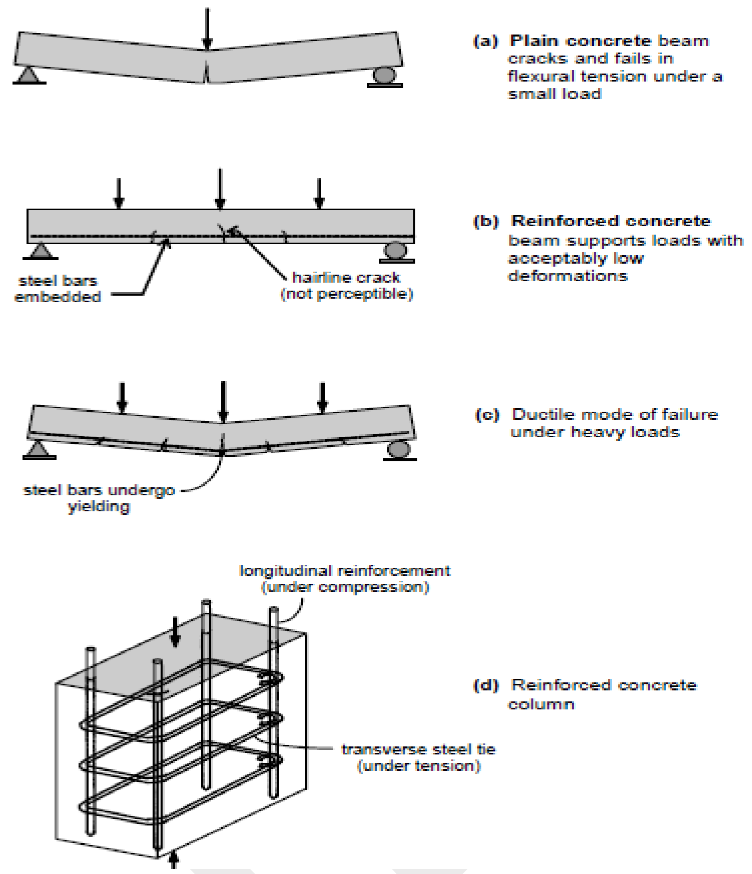

Fig 1.1 Contribution of steel bars in reinforced concrete

It is worth noting that, although these reinforced concrete structures appear to be completely different from one another, the actual principles underlying their design are the same. In the chapters to follow, the focus will be on these fundamental principles.

1.2 OBJECTIVES OF STRUCTURAL DESIGN

The design of a structure must satisfy three basic requirements:

1) ***Stability*** to prevent overturning, sliding or buckling of the structure, or parts of it, under the action of loads;

2) ***Strength*** to resist safely the stresses induced by the loads in the various structural members; and

3) **Serviceability** to ensure satisfactory performance under service load conditions — which implies providing adequate stiffness and reinforcements to contain deflections, crack-widths and vibrations within acceptable limits, and also providing impermeability and durability (including corrosion-resistance), etc.

There are two other considerations that a sensible designer ought to bear in mind, viz., economy and aesthetics. One can always design a massive structure, which has more-than-adequate stability, strength and serviceability, but the ensuing cost of the structure may be excessive, and may be aesthetically won't be appealing. It is indeed a challenge, and a responsibility, for the structural designer to design a structure that is not only appropriate for the architecture, but also strikes the right balance between safety and economy.

1.3 REINFORCED CONCRETE CONSTRUCTION

Reinforced concrete construction is not the outcome of structural design alone. It is a collaborative venture involving the client, the architect, the structural engineer, the construction engineer/project manager and the contractor. Other specialists may also have to be consulted, with regard to soil investigation, water supply, sanitation, fire protection, transportation, heating, ventilation, air-conditioning, acoustics, electrical services, etc. Typically, a construction project involves three phases viz. planning, design (including analysis) and construction.

1. Planning Phase: It is the job of the architect/planner to conceive and plan the architectural layout of the building, to suit the functional requirements of the client, with due regard to aesthetic, environmental and economic considerations. Structural feasibility is also an important consideration, and for this the structural designer has to be consulted.

2. Design Phase: Once the preliminary plans have been approved, the actual details of the project have to be worked out (on paper) by the various consultants. In the case of the structural engineer/consultant, the tasks involved are (i) selection of the most appropriate structural system and initial proportioning of members, (ii) estimation of loads on the structure, (iii) ***structural analysis*** for the determination of the stress resultants (member forces) and displacements induced by various load combinations, (iv) ***structural design*** of the actual proportions (member sizes, reinforcement details) and grades of materials required for

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safety and serviceability under the calculated member forces, and (v) submission of working drawings that are detailed enough to be stamped ‘good for construction’.

3**. Construction Phase**: The plans and designs conceived on paper get translated into concrete reality. A structure may be well-planned and well-designed, but it also has to be well-built, for, the proof of the pudding lies in the eating. And for this, the responsibility lies not only with the contractor who is entrusted with the execution, but also with the construction engineers who undertake supervision on behalf of the consultants. The work calls for proper management of various resources, viz. manpower, materials, machinery, money and time. It also requires familiarity with various construction techniques and specifications. In particular, expertise in concrete technology is essential, to ensure the proper mixing, handling, placing, compaction and curing of concrete.

1.4 STRUCTURAL SYSTEMS

Any structure is made up of structural elements (load-carrying, such as beams and columns) and non-structural elements (such as partitions, false ceilings, doors). The structural elements, put together, constitute the ‘structural system’. Its function is to resist effectively the action of gravitational and environmental loads, and to transmit the resulting forces to the supporting ground, without significantly disturbing the geometry, integrity and serviceability of the structure.

Reinforced concrete buildings consist of several structural components (or members). The basic components of a reinforced concrete building are (see Figure 1.2)

- Floor and roof systems
- Beams
- Column
- Walls
- Foundations

These structural components can be classified into horizontal components (Floors, roofs, and beams) and vertical components (columns and walls). According to another classification, the part of the building above ground is called the superstructure, while the part below ground (including foundations, basemen, and other underground structures) is called the substructure.

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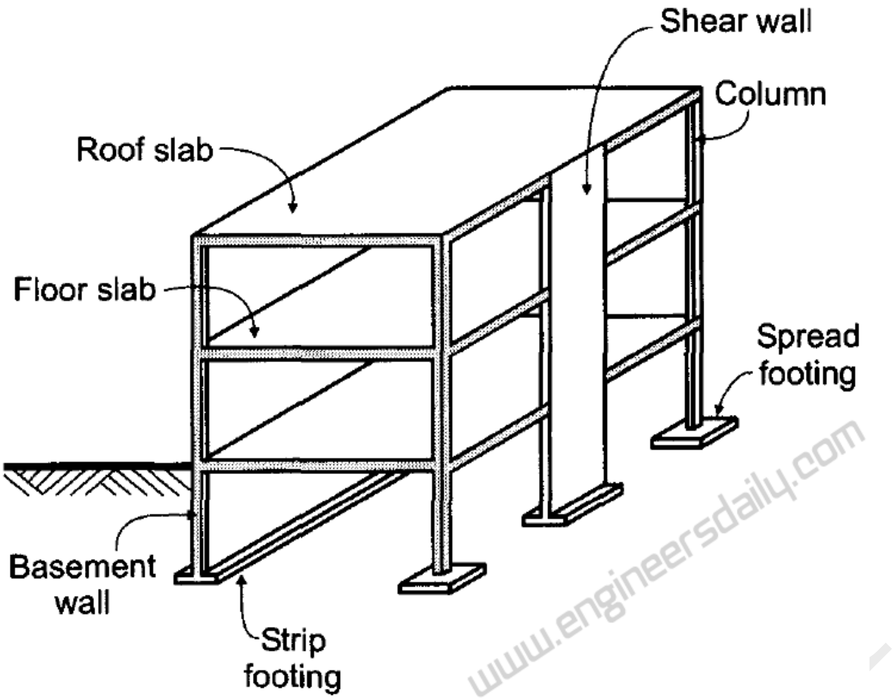

Fig 1.2 Components of a reinforced concrete building.

The floor and roof systems are the main horizontal structural components in a building. They carry gravity loads and transfer them to the vertical components (columns and/or walls), and also act as horizontal diaphragms by transferring the lateral load to the vertical components of a structure.

- *Slabs:*

These are the plate element and carry the loads primarily by flexure. They usually carry the vertical loads.

- *Beams:*

These carry the loads from slabs and also the direct loads as masonry walls and their self-weights. The beams may be supported on the other beams or may be supported by columns forming an integral part of the frame. These are primarily the flexural members.

- *Columns:*

These are the vertical members carrying loads from the beams and from upper columns. The loads carried may be axial or eccentric. Columns are the most important when compared with beams and slabs. This is because, if one beam fails, it will be a local failure of one floor but if one column fails, it can lead to the collapse of the whole structure.

- *Foundation:*

These are the load transmitting members. The loads from the columns and walls are transmitted to the solid ground through the foundations.

1.5 STRUCTURAL ANALYSIS AND DESIGN

It is convenient to separate the work of a structural designer into analysis and design, although a rigid separation is neither possible nor desirable. The purpose of analysis is to determine the stress resultants and displacements in the various members of a structure under any loading (static or dynamic). Structures when subjected to external loads (actions) have internal reactions in the form of bending moment, shear force, axial thrust and torsion in individual members. As a result, the structures develop internal stresses and undergo deformations.

The aim of structural design is to design a structure so that it fulfills its intended purpose during its intended lifetime with adequate safety (in terms of strength, stability and structural integrity), adequate serviceability (in terms of stiffness, durability, etc.) and economy.

Safety implies that the likelihood of (partial or total) collapse of the structure is acceptably low not only under the normal expected loads (service loads), but also under abnormal but probable overloads (such as due to earthquake or extreme wind). Collapse may occur due to various possibilities such as exceeding the load−bearing capacity, overturning, sliding, buckling, fatigue fracture, etc. Another related aspect of safety is structural integrity (see Section 15.1.3). The objective here is to minimize the likelihood of progressive collapse.

Serviceability implies satisfactory performance of the structure under service loads, without discomfort to the user due to excessive deflection, cracking, vibration, etc. Other considerations that come under the purview of serviceability are durability, impermeability, acoustic and thermal insulation, etc. A design that adequately satisfies the 'safety' requirement need not necessarily satisfy the 'serviceability' requirement. For example, a thin reinforced concrete slab can be made safe against collapse (by suitable reinforcement); but if it is too thin, it is likely to result in excessive deflections, crack-widths and permeability (leakage), and the exposed steel becomes vulnerable to corrosion (thereby affecting durability).

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1.6 DESIGN CODES AND HANDBOOKS

National building codes have been formulated in different countries to lay down guidelines for the design and construction of structures. The codes have evolved from the collective wisdom of expert structural engineers, gained over the years. These codes are periodically revised to bring them in line with current research, and often, current trends.

The codes serve at least four distinct functions.

- They ensure adequate structural safety, by specifying certain essential minimum requirements for design.
- They render the task of the designer relatively simple; often, the results of sophisticated analyses are made available in the form of a simple formula or chart.
- The codes ensure a measure of consistency among different designers.
- They have some legal validity; in that they protect the structural designer from any liability due to structural failures that are caused by inadequate supervision and/or faulty material and construction.

Basic Code for Design

The design procedures, described, conform to the following Indian code for reinforced concrete design, published by the Bureau of Indian Standards, New Delhi:

- IS 456: 2000 — Plain and reinforced concrete – Code of practice (fourth revision)

Loading Standards

The loads to be considered for structural design are specified in the following loading standards:

- IS 875 (Parts 1-5): 1987 — Code of practice for design loads (other than earthquake) for buildings and structures (second revision)
- Part 1: Dead loads
- Part 2: Imposed (live) loads
- Part 3: Wind loads
- Part 4: Snow loads
- Part 5: Special loads and load combinations
- IS 1893: 2002 — Criteria for earthquake resistant design of structures (fourth revision).

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1.7 LOADS AND FORCES

The following are the different types of loads and forces acting on the structure. Load values have been assumed based on earlier data and experiences. It is worth mentioning that their assumed values as stipulated in IS 875 have been used successfully.

- **Dead loads**

These are the self-weight of the structure to be designed. Needless to mention that the dimensions of the cross section are to be assumed initially which enable to estimate the dead loads from the known unit weights of the materials of the structure. The accuracy of the estimation thus depends on the assumed values of the initial dimensions of the cross section. The values of unit weights of the materials are specified in Part 1 of IS 875

- **Imposed loads**

They are also known as live loads and consist of all loads other than the dead loads of the structure. The values of the imposed loads depend on the functional requirement of the structure. Residential buildings will have comparatively lower values of the imposed loads than those of school or office buildings. The standard values are stipulated in Part 2 of IS 875.

- **Wind loads**

These loads depend on the velocity of the wind at the location of the structure, permeability of the structure, height of the structure etc. They may be horizontal or inclined forces depending on the angle of inclination of the roof for pitched roof structures. They can even be suction type of forces depending on the angle of inclination of the roof or geometry of the buildings. Wind loads are specified in Part 3 of IS 875.

- **Snow loads**

These are important loads for structures located in areas having snow fall, which gets accumulated in different parts of the structure depending on projections, height, slope etc. of the structure. The standard values of snow loads are specified in Part 4 of IS 875.

- **Earthquake forces**

Earthquake generates waves which move from the origin of its location (epicenter) with velocities depending on the intensity and magnitude of the earthquake. The impact of earthquake on structures depends on the stiffness of the structure, stiffness of the soil media, height and location of the structure etc. Accordingly, the country has been divided into several zones depending on the magnitude of the earthquake. The earthquake forces are prescribed in IS 1893. Designers have adopted equivalent static load approach or spectral method.

- **Shrinkage, creep and temperature effects**

Shrinkage, creep and temperature (high or low) may produce stresses and cause deformations like other loads and forces. Hence, these are also considered as loads which are time dependent. The safety and serviceability of structures are to be checked following the stipulations of cl. 6.2.4, 5 and 6 of IS 456:2000 and Part 5 of IS 875.

- **Other forces and effects**

It is difficult to prepare an exhaustive list of loads, forces and effects coming onto the structures and affecting the safety and serviceability of them. However, IS 456:2000 stipulates the following forces and effects to be taken into account in case they are liable to affect materially the safety and serviceability of the structures. The relevant codes as mentioned therein are also indicated below

  - Foundation movement (IS 1904) (Fig. 1.1.3)
  - Elastic axial shortening
  - Soil and fluid pressures (vide IS 875 - Part 5)
  - Vibration
  - Fatigue
  - Impact (vide IS 875 - Part 5)
  - Erection loads (Please refer to IS 875 - Part 2) (Fig. 1.1.4)
  - Stress concentration effect due to point of application of load and the like.

1.8 DESIGN PHILOSOPHIES

Over the years, various design philosophies have evolved in different parts of the world, with regard to reinforced concrete design. A 'design philosophy' is built up on a few fundamental premises (assumptions), and is reflective of a way of thinking.

The earliest codified design philosophy is the ***working stress method of design (WSM)*****.** Close to a hundred years old, this traditional method of design, based on linear elastic theory, is still surviving in some countries (including India), although it is now sidelined by the modern ***limit states design philosophy (LSM)***. In the recent (2000) revision of the Code (IS 456), the provisions relating to the WSM design procedure have been relegated from the main text of the Code to an Annexure (Annex B) "so as to give greater emphasis to limit state design" (as stated in the 'Foreword').

1.8.1 WORKING STRESS METHOD (WSM)

This was the traditional method of design not only for reinforced concrete, but also for structural steel and timber design. The conceptual basis of WSM is simple. The method basically assumes that the structural material behaves in a linear elastic manner, and that adequate safety can be ensured by suitably restricting the stresses in the material induced by the expected 'working loads' (service loads) on the structure. As the specified permissible ('allowable') stresses are kept well below the material strength (i.e., in the initial phase of the stress-strain curve), the assumption of linear elastic behaviour is considered justifiable. The ratio of the strength of the material to the permissible stress is often referred to as the *factor of safety*.

Factor of safety (FOS) = Strength of material / Allowable stress in the material

The stresses under the applied loads are analyzed by applying the methods of 'strength of materials' such as the simple bending theory. In order to apply such methods to a composite material like reinforced concrete, *strain compatibility* (due to bond) is assumed, whereby the strain in the reinforcing steel is assumed to be equal to that in the adjoining concrete to which it is bonded. Furthermore, as the stresses in concrete and steel are assumed to be linearly related to their respective strains, it follows that the stress in steel is linearly related to that in the adjoining concrete by a constant factor called as the *modular ratio*, which is defined as the ratio of the modulus of elasticity of steel to that of concrete.

Modular ratio = Young's modulus of steel / Young's modulus of concrete

WSM does not provide a realistic measure of the actual factor of safety underlying a design. WSM also fails to discriminate between different types of loads that act simultaneously, but have different degrees of uncertainty. This can, at times, result in very unconservative designs, particularly when two different loads (say, dead loads and wind loads) have counteracting effects. The design usually results in relatively large sections of structural members (compared to ULM and LSM), thereby resulting in better serviceability performance (less deflections, crack-widths, etc.) under the usual working loads.

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1.8.2 ULTIMATE LOAD METHOD (ULM)

With the growing realization of the shortcomings of WSM in reinforced concrete design, and with increased understanding of the behavior of reinforced concrete at ultimate loads, the ultimate load method of design (ULM) evolved and became an alternative to WSM. This method is sometimes also referred to as the load factor method or the ultimate strength method.

The concept of 'modular ratio' and its associated problems are avoided entirely in this method. The safety measure in the design is introduced by an appropriate choice of the *load factor*, defined as the ratio of the ultimate load (design load) to the working load. The ultimate load method makes it possible for different types of loads to be assigned different load factors under combined loading conditions, thereby overcoming the related shortcoming of WSM. This method generally results in more slender sections, and often more economical designs of beams and columns (compared to WSM), particularly when high strength reinforcing steel and concrete are used.

However, the satisfactory 'strength' performance at ultimate loads does not guarantee satisfactory 'serviceability' performance at the normal service loads. The designs sometimes result in excessive deflections and crack-widths under service loads, owing to the slender sections resulting from the use of high strength reinforcing steel and concrete.

Moreover, the use of the non-linear stress-strain behavior for the design of sections becomes truly meaningful only if appropriate non-linear limit analysis is performed on the structure. Unfortunately, such a structural analysis is generally not performed on reinforced concrete structures (except in the yield line theory for slabs), owing to the difficulties in predicting the behavior of 'plastic hinges' in reinforced concrete

1.8.3 LIMIT STATES METHOD (LSM)

The philosophy of the limit states method of design (LSM) represents a definite advancement over the traditional design philosophies. Unlike WSM, which based calculations on service load conditions alone, and unlike ULM, which based calculations on ultimate load conditions alone, LSM aims for a comprehensive and rational solution to the design problem, by considering safety at ultimate loads and serviceability at working loads.

The LSM philosophy uses a multiple safety factor format which attempts to provide *adequate safety* at ultimate loads as well as *adequate serviceability* at service loads, by considering all possible 'limit states'. The selection of the various multiple safety factors is supposed to have a sound probabilistic basis, involving the separate consideration of different kinds of failure, types of materials and types of loads. In this sense, LSM is more than a mere extension of WSM and ULM.

- **Limit States**

A limit state is a state of impending *failure*, beyond which a structure ceases to perform its intended function satisfactorily, in terms of either *safety or serviceability*; i.e., it either collapses or becomes *unserviceable.*

There are two types of limit states:

1. *Ultimate limit states* (or 'limit states of collapse'), deals with the strength and stability of structures subjected to the maximum design loads out of the possible combinations of several types of loads. Therefore, this limit state ensures that neither any part nor the whole structure should collapse or become unstable under any combination of expected overloads.

2. *Serviceability limit* states, deals with deflection and cracking of structures under service loads, durability under working environment during their anticipated exposure conditions during service, stability of structures as a whole, fire resistance etc.

All relevant limit states have to be considered in the design to ensure adequate degree of safety and serviceability. The structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states.

- **Characteristic Strengths and Loads**

The general definition of the characteristic strength of a material (concrete or steel) was given in Section 2.6.1. The general definition of the ***characteristic strength*** of a material (concrete or steel) is defined as the strength below which not more than five per cent of the test results are expected to fall. In the case of reinforcing steel, it refers to the 'specified yield stress' as mentioned in Section 2.14.1.

The ***characteristic load*** is defined as the load that "has a 95 percent probability of not being exceeded during the life of the structure" (Cl. 36.2 of the Code).

- **Partial safety factors**

Partial safety factors of Loads

The characteristic values of loads are based on statistical data. It is assumed that in ninety-five per cent cases the characteristic loads will not be exceeded during the life of the

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structures. However, structures are subjected to overloading also. Hence, structures should be designed with loads obtained by multiplying the characteristic loads with suitable factors of safety depending on the nature of loads or their combinations, and the limit state being considered. These factors of safety for loads are termed as partial safety factors (γ_f) for loads. Thus, the design loads are calculated as

(Design load F_d) = (Characteristic load F) (Partial safety factor for load γ_f)

Respective values of γ_f for loads in the two limit states as given in Table 18 of IS 456 for different combinations of loads are furnished in Table 1.1**.**

Table 1.1 Values of partial safety factor γ_f for loads

Load combinations	Limit state of collapse			Limit state of serviceability (for short term effects only)		
	DL	*IL*	*WL*	*DL*	*IL*	*WL*
DL + IL	1.5		1.0	1.0	1.0	-
DL + WL	1.5 or $0.9^{1)}$	-	1.5	1.0	-	1.0
DL + IL + WL	1.2			1.0	0.8	0.8

<u>Partial safety factors of Material's</u>

The characteristic strength of a material as obtained from the statistical approach is the strength of that material below which not more than five per cent of the test results are expected to fall however, such characteristic strengths may differ from sample to sample also. Accordingly, the design strength is calculated dividing the characteristic strength further by the partial safety factor for the material (γ_m), where γ_m depends on the material and the limit state being considered. Thus,

$$\text{Design strength of the material } f_d = \frac{\text{Characteristic strength of the material } f}{\text{Partial safety factor of the material } \gamma_m}$$

Clause 36.4.2 of IS 456 states that γ_m for concrete and steel should be taken as 1.5 and 1.15, respectively when assessing the strength of the structures or structural members employing limit state of collapse. For serviceability limit states, γ_m= 1.0. A safety factor of unity is appropriate here, because the interest is in estimating the actual deflections and crack-widths under the service loads, and not 'safe' (conservative) values. However, when assessing the deflection, the material properties such as modulus of elasticity should be taken as those associated with the characteristic strength of the material.

Further, in case of concrete the characteristic strength is calculated on the basis of test results on 150 mm standard cubes. But the concrete in the structure has different sizes. To take the size effect into account, it is assumed that the concrete in the structure develops strength of 0.67 times the characteristic strength of cubes. Accordingly, in the calculation of strength employing the limit state of collapse, the characteristic strength (f_{ck}) is first multiplied with 0.67 (size effect) and then divided by 1.5 (γm for concrete) to have 0.446 fck as the maximum strength of concrete in the stress block.

1.9 DESIGN STRESS-STRAIN CURVE FOR CONCRETE

The maximum stress in the 'characteristic' curve is restricted to 0.67. The curve consists of a parabola in the initial region up to a strain of 0.002 (where the slope becomes zero), and a straight line thereafter, at a constant stress level of 0.67 up to an ultimate strain of 0.0035.

For the purpose of limit states design, the appropriate partial safety factor γ_m has to be applied, and γ_m is equal to 1.5 for the consideration of ultimate limit. Thus, the 'design curve' is obtained by simply scaling down the ordinates of the characteristic curve - dividing by γ_m [Fig. 1.3]. Accordingly, the maximum design stress becomes equal to 0.447, and the formula for the design compressive stress f_c corresponding to any strain $\varepsilon \leq 0.0035$ is given by

$$f_c = \begin{cases} 0.447 f_{ck}\left[2\left(\dfrac{\varepsilon}{0.002}\right)-\left(\dfrac{\varepsilon}{0.002}\right)^2\right] & \text{for } \varepsilon < 0.002 \\ 0.447 f_{ck} \quad \text{for } 0.002 \leq \varepsilon \leq 0.0035 \end{cases}$$

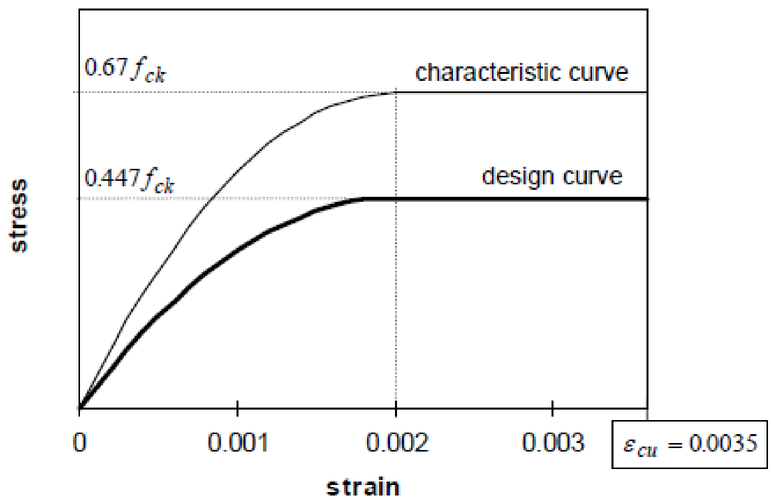


Fig 1.3 Characteristic and design stress-strain curves for concrete in flexural compression

When concrete is subjected to uniform compression, as in the case of a concentrically loaded short column, the ultimate strain is limited to 0.002, and the corresponding maximum design stress is 0.447. When concrete is subject to axial compression combined with flexure, the ultimate strain is limited to a value between 0.002 and 0.0035, depending on the location of the neutral axis. The maximum design stress level remains unchanged at 0.447.

1.10 DESIGN STRESS-STRAIN CURVE FOR REINFORCING STEEL

The characteristic and design stress−strain curves specified by the Code for various grades of reinforcing steel (in tension or compression) are shown in Figs. 1.4 and 1.5. The design yield strength f_{yd} is obtained by dividing the specified yield strength f_y by the partial safety factor γs = 1.15 (for ultimate limit states); accordingly, = 0.870 f_y. In the case of mild steel (Fe 250), which has a well−defined yield point, the behavior is assumed to be perfectly linear-elastic up to a design stress level of 0.87 f_y and a corresponding design yield strain ε_y = 0.87fy/ Es; for larger strains, the design stress level remains constant at 0.87 f_y.

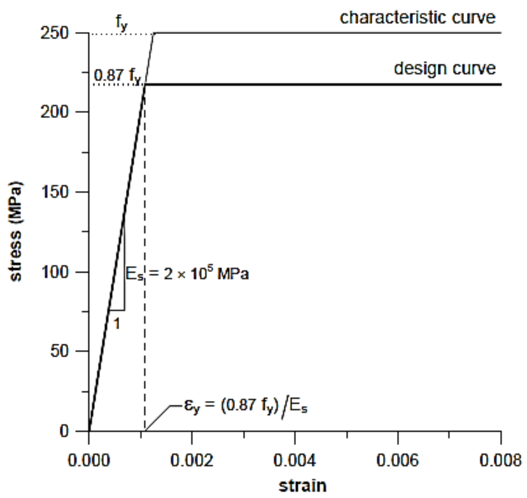


Fig 1.4 Characteristic and design stress-strain curves for Fe 250 grade mild steel

In the case of cold-worked bars (Fe 415 and Fe 500), which have no specific yield point, the transition from linear elastic behavior to nonlinear behavior is assumed to occur at a stress level equal to 0.8 times f_y in the characteristic curve and 0.8 times f_{yd} in the design curve. The full design yield strength 0.87 f_y is assumed to correspond to a 'proof strain' of 0.002; i.e, the design yield strain is to be taken as 0.87 *fy*/ Es+ 0.002, as shown in Fig. 1.5. The coordinates of the salient points of the design stress-strain curve for Fe 415 and Fe 500 are listed in Tables 1.2.

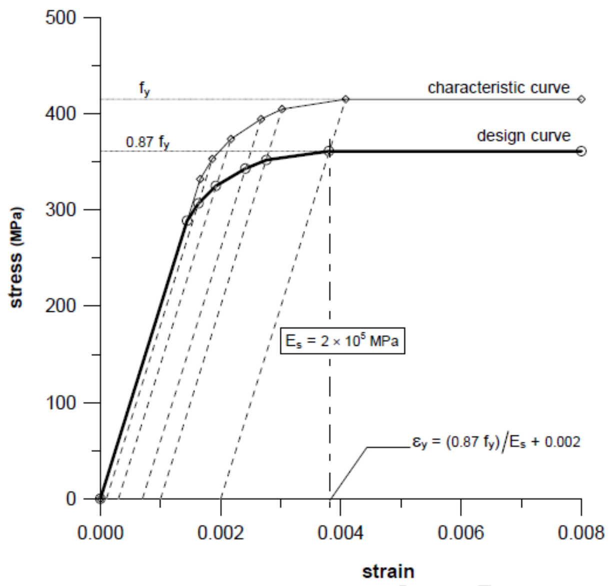


Fig 1.5 Characteristic and design stress-strain curves for Fe 415 grade cold-worked steel

Table 1.3 Salient points on the stress-strain curve for cold-worked steels

Design Stress	$.80\ f_{yd}$	$.85\ f_{yd}$	$.90\ f_{yd}$	$.95\ f_{yd}$	$.975\ f_{yd}$	$1.0\ f_{yd}$
Inelastic Strain	0.0000	0.0001	0.0003	0.0007	0.0010	0.0020

- **Difference between Working stress and Limit State Method of design**

Working Stress Method	Limit State Method
The Stresses in an element is obtained from the working loads and compared with permissible stresses	The stresses are obtained from design loads and compared with design strength.
The method follows linear stress-strain behaviour of both the materials.	In this method, it follows linear strain relationship but not linear stress relationship (one of the major differences between the two methods of design).
Modular ratio can be used to determine allowable stresses.	The ultimate stresses of materials itself are used as allowable stresses.
Material capabilities are under estimated to large extent. Factor of safety are used in working stress method.	The material capabilities are not under estimated as much as they are in working stress method. Partial safety factors are used in limit state method.
The member is considered as working	

stress.	
Ultimate load carrying capacity cannot be predicted accurately.	
The main drawback of this method is that it results in an uneconomical section.	

1.11 Assumptions in the limit state design (Limit state of collapse by flexure)

The following assumptions are made for calculating the ultimate moment of resistance or the strength at limit state of flexural collapse of reinforced concrete beams (IS 456, Clause 38.1):

1. Plane sections remain plane in bending up to the point of failure (i.e. strains are proportional to distance from the neutral axis), see Fig 1.6
2. Ultimate limit state of bending failure is deemed to have been reached when the strain in concrete at the extreme bending compression fibre ε_{cu} reaches 0.0035.
3. The stress distribution across the compression face will correspond to the stress-strain diagram for concrete in compression. Any suitable shape like parabolic, rectangular or any combinations of shapes that give results which are in substantial agreement with tests may be assumed for this compression block. For design purpose, the maximum compressive strength in the structure is assumed as 0.67 times the characteristic laboratory cube strength (i.e., 2/ 3fck). With an additional partial safety factor of γ_m = 1.5 applied to concrete strength, the values of the maximum concrete stress in a beam will be 0.446 fck which can be taken as equal to 0.45 fck for all practical purposes
4. The tensile strength of concrete is neglected as the section is assumed to be cracked up to the neutral axis.
5. The stress in steel will correspond to the corresponding strain in the steel ε_s, and can be read off from the stress-strain diagram of the steel. For design purposes, a partial safety factor of 1.15 is used for strength of steel so that the maximum stress in steel is limited to fy/ 1.15 = 0.87fy.
6. In order to avoid sudden and brittle compression failure in singly reinforced beams, the maximum strain in tension steel at failure (ε_{su}) to be not less than the following:

$$\boldsymbol{\varepsilon_{su} = \frac{f_y}{1.15\,E_s} + 0.002 = \frac{0.87\,f_y}{E_s} + 0.002}$$

where ε_{su} = strain in steel at ultimate failure, fy = characteristic strength of steel, Es = modulus of elasticity of steel = 200 × 10^5 N/ mm^2

Limit state of collapse: Compression

7. The maximum compressive strain in concrete in axial compression is taken as 0.002.
8. The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

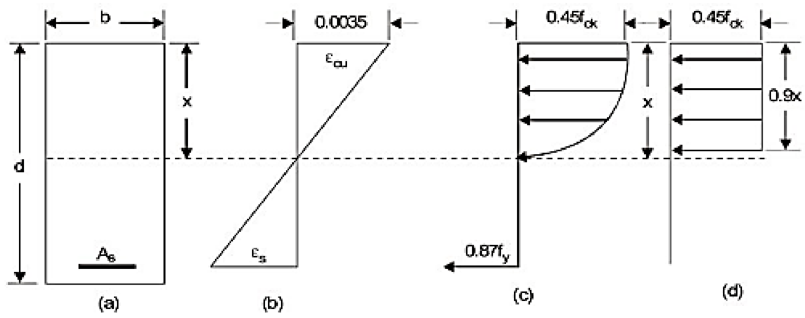


Fig. 1.6 Strain diagram and stress blocks: (a) Section; (b) Strain diagram (plane sections remain plane); (c) Stress block with partial safety factors; and (d) Simple rectangular stress block (BS).

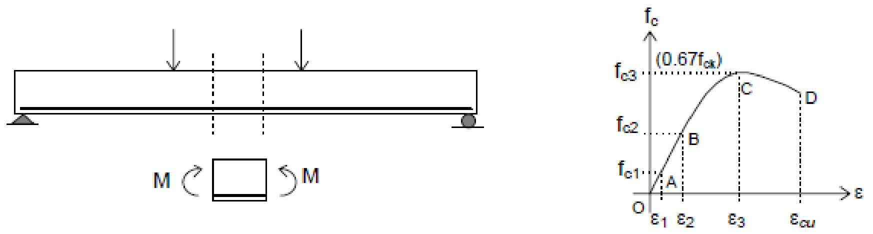
1.12 BEHAVIOR OF REINFORCED CONCRETE BEAM

To understand the behavior of beam under transverse loading, a simply supported beam subjected to two points loading as shown in Fig. 1.7 (a) is considered. This beam is of rectangular cross-section and reinforced at bottom

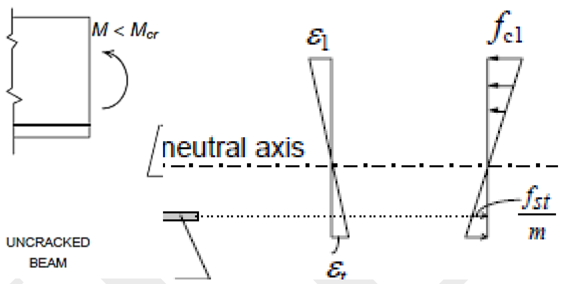
When the load is gradually increased from zero to the ultimate load value, several stages of behavior can be observed. In the early stages of loading, the applied moment (at any section) is less than the cracking moment Mcr and the maximum tensile stress fct in the concrete is less than its flexural tensile strength fcr. This phase is the uncracked phase, wherein the entire section is effective in resisting the moment and is under stress. At this stage, due to bonding tensile stress is also induced in steel bars. The distribution of strains and stresses are as indicated in Fig. 1.7 (C).

The uncracked phase reaches its limit when the applied moment *M* becomes equal to the cracking moment *Mcr*. In the concrete stress-strain curve shown in Fig. 1.7(b), the uncracked phase falls within the initial linear portion OA.

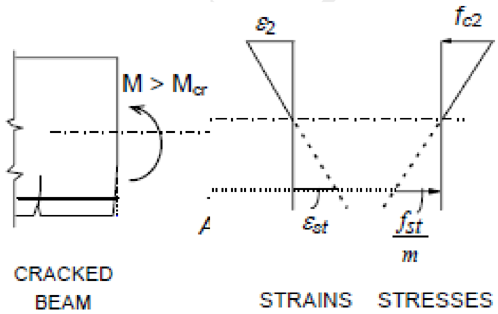
With increase in load moment exceeds *Mcr*, the maximum tensile stress in concrete exceeds the flexural tensile strength of concrete and the section begins to crack on the tension side. The cracks are initiated in the bottom (tensile) fibres of the beam, and with increasing loading, widen and propagate gradually towards the neutral axis.



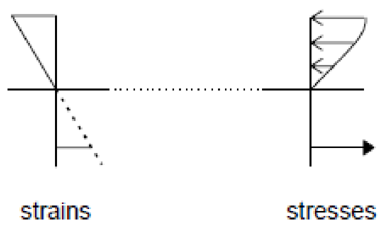
(a) beam with loading **(b)** concrete stress-strain curve



(C) Uncracked Phase



(D) Cracked Phase Linear



(E) Cracked Phase ultimate

Fig. 1.7 Behavior of RC beams

Width of crack is small. Tensile stresses developed are absorbed by steel bars. Stress and strain are proportional till fcr< (f_t/2). Further increase in load, increases strain and stress in the section and are no longer proportional. Fig 1.7d shows the stress distribution at ultimate load.

As the applied moment on the beam section is increased beyond the 'linear elastic cracked phase', the concrete strains and stresses enter the nonlinear range BCD in Fig. 1.7(b). For example, if the strain in the extreme compression fibre reaches a value of ε_3 (equal to 0.002, according to the Code), corresponding to the maximum stress level 0.67 fck, the compressive stress distribution in the cracked section (above the neutral axis) will take the shape of the curve OBC in Fig. 1.7(b), as shown in Fig. 1.7(e).

Reinforcing steel can sustain very high tensile strains, due to the ductile behavior of steel, following 'yielding'; the ultimate strain can be in the range of 0.12 to 0.20. However, the concrete can accommodate compressive strains which are much lower in comparison; the 'ultimate compressive strain' ε_{cu} is in the range of 0.003 to 0.0045.

As will be seen later, the final collapse of a normal beam at the ultimate limit state is caused inevitably by the crushing of concrete in compression, regardless of whether the tension steel has yielded or not. If the tension steel yields at the ultimate limit state, the beam is said to be *under-reinforced;* otherwise, if the steel does not yield, the beam is said to be *over-reinforced*. The terms 'under-' and 'over-' are used with reference to a benchmark condition called the '*balanced' section*. If the area of tension steel provided at a beam section is less than that required for the balanced section condition, the beam is *under-reinforced*; otherwise, if the steel area is in excess, the beam is *over-reinforced*.

1.13 STRESS BLOCK PARAMETERS

Stress blocks adopted by different codes are based on the stress blocks proposed by different investigators. Among them that proposed by Hognestad and Whitney equivalent rectangular block are used by most of the codes.

Stress block of IS456-2000 is shown in Fig 1.8. Code recommends ultimate strain ε_{cu}=0.0035 & strain at which the stress reaches design strength ε_0=0.002. Using similar triangle properties on strain diagram

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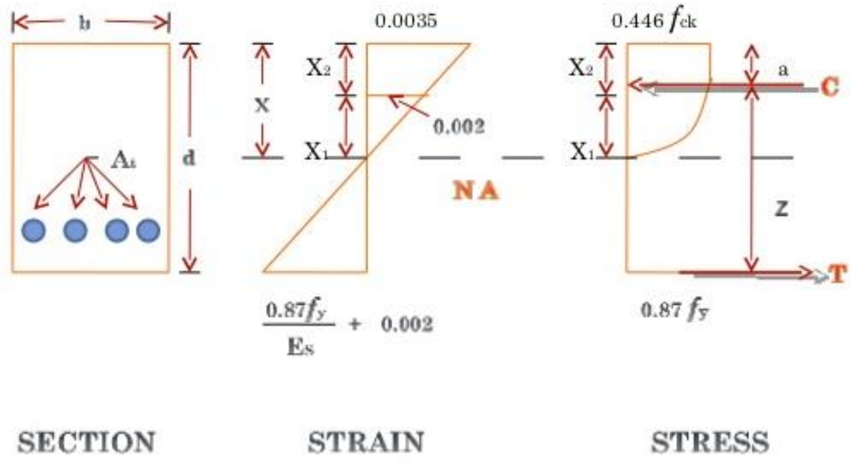

Fig 1.8 Properties of I.S. stress block in compression; and Forces acting on a section subjected to bending.

Similar triangle properties on strain diagram

$$\frac{0.0035}{x} = \frac{0.002}{x_1}$$

x_1 = 0.57 x

and x_2 = x - 0.57x = 0.43 x

Where x = depth of the neutral axis.

The constants k_1, k_2 are called the constants for the stress block. 4.1. The values of k_1 and k_2 for the parabolic rectangular block assumed in IS 456 can be derived as given in the following sub-sections.

Value of k_1

The areas of the stress block can be calculated by assuming that the parabola extends to x_1(where the strain is 0.002) from the neutral axis and the rest of the stress block to the top fibre is a rectangle The value of x_1=0.57x. This geometry of the stress-strain curve is important and useful in solving problems from basic considerations. Let the total compression be equal to C. From Fig. 1.9,

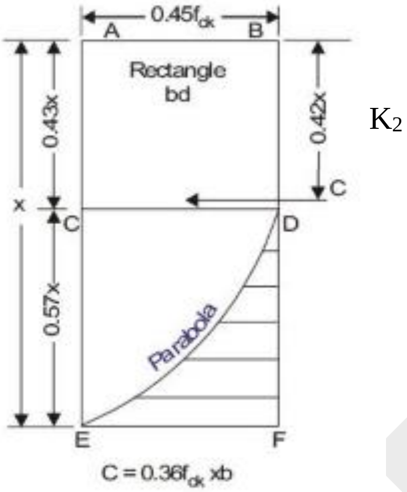

Fig 1.9 Calculation of resisting moment of a section:

$$C = \text{area of (rectangle – outer parabola)} = 0.45f_{ck}\,(x)\,b - 0.45\,f_{ck}\,\frac{0.57}{3}\,(x)\,b$$

$$= 0.45\,f_{ck}\,(x)\,b - 0.086\,f_{ck}\,(x)\,b$$

$$= 0.364\,f_{ck}\,(x)\,b$$

Therefore, $k_1 = 0.364$......................(1.1)

Thus, k_1 is a constant and its value may be assumed as 0.36 for all design calculations (IS 456, Clause 38.1). Thus, we get

$$C = 0.36\,f_{ck}\,(x)\,b \text{......................(1.1a)}$$

It is also sometimes expressed as a function of the area bd as

$$C = 0.36\,f_{ck}\,bd \text{......................(1.1b)}$$

<u>Value of k_2</u>

Let $k_2(x)$ be the distance of the centre of compression in the concrete from the top compression fibre. Taking x_1 as the distance to which the parabola extends, which is equal to 0.57x, and equating the moment of the total forces about the neutral axis to the moment of the separate forces, we obtain

$$[0.36f_{ck}\,(x)\,b]\,[x - k_2(x)] = 0.45f_{ck}\,(x)\,b\,\frac{x}{2} - 0.45\,f_{ck}\;b\,\frac{x_1}{3}\,\frac{3\,x_1}{10}$$

where $x_1 = 0.57x$, the point where $\varepsilon_{co} = 0.002$. Simplifying the equation, we get

$$1 - k_2 = 0.584$$

Hence,

$$k_2 = 0.416 = 0.42 \text{ (approx.) ....(1.2)}$$

Thus, according to IS 456, k_2 will also be a constant of value 0.42, irrespective of the strength of concrete. The position of the centre of compression is thus taken as 0.42 x from the compression edge (see IS 456, notes to Clause 38.1c).

- **General Expression for Depth of Neutral Axis x**

Beams are assumed to fail when the concrete reaches failure compression strain. But in all cases of design, the steel need not have reached its yield point at the same time, unless it is so designed. If the section is designed as a balanced or under reinforced one, the steel also reaches yield as concrete fails. But in over reinforced beams, the steel stress at failure will be below its yield strength. As equilibrium of forces in bending requires that at all times tension be equal to compression, we have

$$\text{Total tension } T = f_{st}\,A_{st}$$

$$\text{Total compression } C = 0.36\, f_{ck}\, b(x)$$

Where fst = actual tension in steel corresponding to the strain in steel. Equating the two expressions, we obtain

$$0.36 \text{ fck } b(x) = \text{fst Ast or}$$

$$x = \frac{\text{fst Ast}}{0.36 \text{ fck b}} \quad \text{.....................(1.3)}$$

For under reinforced beams, steel first reaches yield stress of 0.87fy. Substituting its value and dividing both sides by the effective depth d (IS 456 Annexure G), we get

$$\frac{x_u}{d} = \frac{0.87\text{fy.Ast}}{0.36 \text{ fck b}} \text{.....................(1.3a)}$$

where x_u is the depth of neutral axis at ultimate failure of the beam.

- **Limiting Values of x/ d in IS 456**

The sixth assumption in IS code states that in order to avoid brittle failure, the limiting value of x/ d is to be determined from the condition that the steel strain ε_{su} at failure should not be less than the following:

$$\boldsymbol{\varepsilon\ su = \frac{f_y}{1.15\,E_s} + 0.002 = \frac{0.87\,f_y}{E_s} + 0.002}$$

Assuming Es = 2.0×10^5 N/ mm^2, the yield strain for design purposes for different steels works out to the values given in Table 1.1. From proportionality of strains, we have

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$$\frac{x_u}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{su}} = \frac{0.0035}{0.0035 + \varepsilon_{su}}$$

$$\frac{xumax}{d} = \frac{0.0035}{\frac{0.87fy}{Es} + 0.0055}$$

Substituting the various values of ε su for different grades of steel, the maximum limiting values of x_u/d for various grades of steel are obtained as in Table 1.1.

Table 1.1 Limiting values of x/ d

Type of steel	f_y	Yield strain (ε_{su})	x_u/d
Mild steel	250	0.0031	0.53
High yield strength	415	0.0038	0.48
High yield strength	500	0.0042	0.46

1.14 BALANCED, UNDERREINFORCED AND OVERREINFORCED SECTIONS

Balanced Section

A ‘balanced section’ is one in which the area of tension steel is such that at the ultimate limit state, the two limiting conditions are reached simultaneously; viz., the compressive strain in the extreme fibre of the concrete reaches the ultimate strain ε_{cu}, and the tensile strain at the level of the centroid of the steel reaches the ‘yield strain’ ε_y. The failure of such a section, termed ‘balanced failure’, is expected to occur by the simultaneous initiation of crushing of concrete and yielding of steel.

Under-Reinforced Section

An ‘under-reinforced section’ is one in which the area of tension steel is such that as the ultimate limit state is approached, the yield strain ε_y is reached in the steel before the ultimate compressive strain is reached in the extreme fibre of the concrete. When the reinforcement strain reaches ε_y (and the stress reaches the yield strength f_y), the corresponding maximum concrete strain is less than $\varepsilon_{cu.}$

It should be remembered that yielding of steel does not mean ultimate failure of the beam. When steel yields, there will be excessive deflection and consequent cracking but complete rupture of steel takes place at a much higher strain. It is preferable that a beam be designed as an under reinforced beam, where 'failure' will take place after yielding of steel, with enough warning signals like excessive cracking and deflection taking place before ultimate failure.

Over-Reinforced Section

An 'over-reinforced' section is one in which the area of tension steel is such that at the ultimate limit state, the ultimate compressive strain in concrete is reached, however the tensile strain in the reinforcing steel is less than the yield strain $\varepsilon_{y.}$ The concrete fails in compression before the steel reaches its yield point. Hence, this type of failure is termed compression failure. The failure occurs (often, explosively) without warning. Such designs are not recommended in practice.

Serviceability Limit States: Deflection and Cracking

According to the design philosophy of the limit states method, there are two distinct classes of limit states to be considered: ultimate limit states and serviceability limit states. Whereas the former deals with safety in terms of strength, overturning, sliding, buckling, fatigue fracture, etc., the latter deals with serviceability in terms of deflection, cracking, vibration, durability, etc. The aim of structural design by the LSM philosophy is to ensure both 'safety' and 'serviceability', so that the structure performs its intended function satisfactorily.

Two of the important serviceability conditions are:

1. The member should not undergo excessive deformation (i.e. limit state of deflection).
2. The crack width at the surface in the reinforced concrete member should not be more than that which is normally allowed by codes of practice (i.e. the limit state of cracking).

Codes also specify the partial safety factors for load combinations under which these are to be checked. According to IS 456, Table 18 (Table 2.1 of the text), the combinations of loads for serviceability conditions should be the largest of the following:

1. 1.0 DL + 1.0 LL
2. 1.0 DL + 1.0 WL

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3. 1.0 DL + 0.8 LL + 0.8 WL (EL)

For control of deflection, two methods are usually described in codes of practices:

1. The empirical method of keeping the span-effective depth ratios of the members not more than those specified in the codes.

2. The theoretical method of calculating the actual deflection and checking it with the allowable deflection in codes of practice.

Similarly, for control of crack width, two methods are recommended:

1. The empirical method of detailing the reinforcements according to the provisions of the code regarding spacing of bars, minimum steel ratios, curtailment and anchorage of bars, lapping of bars, etc.

2. The theoretical method of calculating the actual width of cracks and checking whether they satisfy the requirements in the codes for the given environmental conditions.

1.15 DEFLECTION LIMITS

Various factors are involved in prescribing limits to deflection in flexural members, such as:

• Aesthetic/psychological discomfort;

• Crack-width limitation (limiting deflection is an indirect way of limiting crack-widths);

• Effect on attached structural and non-structural elements;

• Ponding in (roof) slabs.

The Code (Cl. 23.2) prescribes the following two limits for flexural members in general:

1. span/250— the final deflection due to all loads (including long-term effects of creep and shrinkage);

2. span/350 or 20 mm (whichever is less) — the deflection (including long-term effects of creep and shrinkage) that occur after the construction of partitions and finishes.

The first limit is based on considerations of crack control and aesthetic/psychological discomfort to occupants, and the second limit is aimed at preventing damage to partitions and finishes

Accurate prediction of deflections in reinforced concrete members is difficult, so that approximations and simplifications are essential in deflection calculations. The calculations are considered in two parts:

(i) ***Immediate or short-term:*** Deflection occurring on application of the load. For calculating the ‘immediate’ deflection, the loading to be considered is the full load (dead plus live).

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(ii) ***Long*-term deflection:** Deflection resulting mostly from differential shrinkage and creep under sustained loading. However, for calculating the long-term deflection due to creep, only the 'permanent' load (dead load plus the sustained part of live load) is to be considered.

1.15.1 Basic Span-depth Ratios

Allowable deflection/ span ratio can be controlled by the span/ depth ratio. This principle is used for specifying the span/ depth ratio for control of deflection in beams and slabs. The empirical procedure for control of deflection is to control the span to effective depth ratio. It assumes that the deflection of beams and slabs will depend on the following factors:

1. The span/ effective depth ratios
2. Type of supports, as to whether simply supported, fixed or continuous
3. Percentage of tension steel or the stress level in the steel level at service loads if more than the necessary steel is provided at the section
4. Percentage of compression steel provided
5. Type of beam (whether it is flanged or rectangular).

IS 456, Clause 23.2 gives the values of the basic span/ effective depth ratios to be used for beams and slabs with spans less than 10 m. Separate values have been given for cantilevered, simply supported and continuous beams and slabs. It may be noted that for ordinary two-way slabs supported on the four sides, the shorter span controls the deflection. The recommended values of the basic span/ depth ratios in IS code are given in Table 1.2 and Fig. 1.10. For simply supported and continuous spans over 10 m, these ratios are to be multiplied by a factor

F = 10 / Span in meters

For cantilevers over 10 m in length, the actual deflection should be estimated by calculations, and the requirements for limit state of deflection checked.

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Table 1.2 Effective Span/ depth ratio

Type of support	Rectangular sections	Flange beams
Cantilever	7	Multiply values for rectangular by factor F_3 (see Fig. 9.4)
Simply supported	20	
Continuous	26	

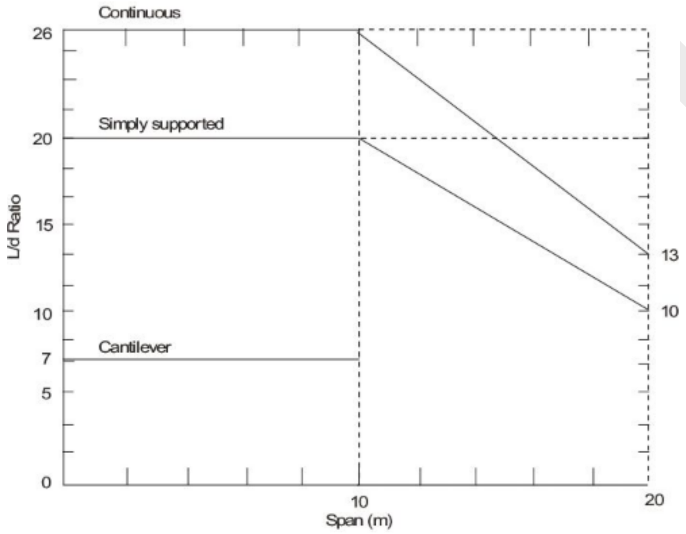

Fig 1.10 Basic span-depth ratio for beams and slabs.

1.15.12 Calculation of Short terms deflection as per IS 456

Short-term deflections, due to the applied service loads, are generally based on the assumption of linear elastic behaviour, and for this purpose, reinforced concrete is treated as a homogeneous material. Expressions for the maximum elastic deflection Δ of a homogeneous beam of effective span *l* and flexural rigidity *EI* (for any loading and support conditions) can be derived using the standard methods of structural analysis. The loads are taken as per limit state of serviceability combination. The flexural rigidity EI, which is the product of the modulus of elasticity of concrete E = Ec, and the second moment of area, I, of the cross-section.

$$\left(E_c = 5000\sqrt{f_{ck}}\right)$$

The second moment of area, I, to be considered in the deflection calculations is influenced by percentage of reinforcement as well as the extent of flexural cracking, which in turn depends on the applied bending moment and the modulus of rupture fcr of concrete.

For the calculation of the short term IS 456 specifies a method in Annexure C in the clause C_2. The short-term deflection may be calculated by the usual methods for elastic deflections using the short-term modulus of elasticity of concrete, E, and an effective moment of inertia I_{eff}, given by the following equation:

$$I_{eff} = \frac{I_r}{1.2 - \frac{M_r}{M}\frac{z}{d}\left(1-\frac{x}{d}\right)\frac{b_w}{b}}; \text{ but}$$

$$I_r \leq I_{eff} \leq I_{gr}$$

Ir = moment of inertia of the cracked section,

Mr = cracking moment = $\frac{\text{fcr Igr}}{y_t}$

f_{cr}, is the modulus of rupture of concrete, Igr is the moment of inertia of the gross section about the centroidal axis, neglecting the reinforcement, and y_t is the distance from centroidal axis of gross section, neglecting the reinforcement, -to extreme fibre in tension

M= maximum moment under service loads,

Z = lever arm,

X = depth of neutral axis

d = effective depth,

b_w = breadth of web, and

b = breadth of compression face.

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

Ir for the Single Reinforced Section is $I_r = \frac{bx^3}{3} + m\,A_{st}\,(d-x)^2$

Where m = $\frac{E_s}{E_c}$

x can be found out by $\frac{bx^2}{2} = m\,A_{st}\,(d-x)$

1.15.3 LONG-TERM DEFLECTION

The deflection of a reinforced concrete flexural member increases with time, mainly due to:

• Differential shrinkage or temperature variation (causing differential strains across the cross-section, resulting in curvature);

• Creep under sustained loading; and

• Temperature effects in statically indeterminate frames

The combined long-term deflection due to shrinkage, creep and temperature effects may be as large as two to three times the short-term deflection due to dead and live loads.

In an unrestrained reinforced concrete member, drying shrinkage of concrete results in shortening of the member. Under sustained loading, compressive strains in concrete keep increasing nonlinearly with time, owing to the phenomenon called creep. Is 456 access these deflections

DEFLECTION DUE TO SHRINKAGE

The deflection due to shrinkage α_{cs} may be computed from the following equation:

$$\alpha_{cs} = k_3\ \Psi cs\ l^2$$

Where k_3 is a constant depending upon the support conditions,

0.5 for cantilevers,

0.125 for simply supported members,

0.086 for members continuous at one end, and

0.063 for fully continuous members.

Ψcs is shrinkage curvature equal to $\frac{k_4 \epsilon_{cs}}{2}$

where ε_{cs} is the ultimate shrinkage strain of concrete

$$k_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}} \leq 1.0 \text{ for } 0.25 \leq P_t - P_c < 1.0$$

$$= 0.65 \times \frac{P_t - P_c}{\sqrt{P_t}} \leq 1.0 \text{ for } P_t - P_c \geq 1.0$$

p_t and p_c are the percentage of tension and compression reinforcement respectively

DEFLECTION DUE TO CREEP

The creep deflection due to permanent may be obtained from the following equation:

$$\alpha_{cc(Perm)} = \alpha_{i,cc(Perm)} - \alpha_{i(Perm)}$$

$\alpha_{i,cc(Perm}$ = initial plus creep deflection due to permanent loads obtained using an elastic analysis with an effective modulus of elasticity, E_{ce}

$$E_{ce} = \frac{E_c}{1+\theta}$$

Θ being the creep coefficient,

$\alpha_{i(Perm)}$ = short-term deflection due to permanent load using E_C.

Problem 1.

A rectangular simply supported beam of span 5 in is 300 mm x 650 mm in cross section. It carries a total load of 30 kN/m over its entire span, out of which 10 kN/m is the live load. The beam is reinforced with 3 bars of 20 mm on tension side at an effective cover of 50 mm. Calculate the deflection at central span due to shrinkage and creep, if

(a) Ultimate shrinkage strain = 0.0003

(b) Creep coefficient = 1.6

Concrete mix of grade M20 and steel of Fe 415 are used.

Solution

Span L = 5 m, b = 300 mm D = 650 mm Effective cover = 50 mm d = 650 - 50 = 600 mm

Ast = 3 x(π/4) x 20^2 = 942.5 mm^2

Pt = $\frac{942.5}{600\ x\ 300}$ x 100= 0.5236

Singly reinforced. Hence Asc = 0 i.e., pc= 0

Total load (includes live load, dead load, self-weight, weight of partitions etc.) = 30 kN/m

Live load = 10 kNim

(a) Deflection Due to Shrinkage

It is given by

$$\alpha_{cs} = k_3\ \Psi cs\ l^2$$

Since it is simply supported beam, k_3 = 0.125 ε_c= 0.0003, D = 650 mm

p_t-p_c= 0.5236 which is between 0.25 and 1

Pt = 0.72

$k_4 = 0.72\ \frac{pt—pc}{\sqrt{Pt}} = 0.72\ \frac{0.5236}{\sqrt{0.5236}} = 0.5210$

$\Psi cs = \frac{k_4 \in_{cs}}{2} = \frac{0.5210\ x\ 0.0003}{2}$ = = 2.4046 x 10^{-7}

α_{cs} = 0.125 x 2.4046 x 10^{-7}x $(5000)^2$ = 0.751 mm

(b) Deflection Due to Creep

Permanent load= Total load — live load = 30 — 10 = 20 kN/m

Due to permanent loads, maximum bending moment at service loads,

$M = \frac{w\ L^2}{8} = \frac{20x\ 5}{8} = 62.5$ kN-m

$\text{Igr} = \frac{\text{b D}^3}{12} = \frac{300\ 650^3}{12} = 6865.6253 \text{ x } 10^6 \text{ mm}^4$

$f_{cr} = 0.7\ \sqrt{f_{ck}} = 0.7\sqrt{20} = 3.1305 \text{ N/mm}^2$

yt = D/2 = 325 mm

Cracking moment $M_{cr} = \frac{\text{fcr Igr}}{\text{y}_\text{t}} = \frac{3.1305 \text{ x } 6865.625 \text{ x} 10^6}{12} = 66.132 \text{ x } 10^6$ N-mm

$\left(E_c = 5000\sqrt{f_{ck}}\right)$

$\text{Ec} = 5000\sqrt{20} = 22360.68 \text{ N/mm}^2$

$$E_{ce} = \frac{E_c}{1+\theta}$$

$\text{Ece} = \frac{2360.68}{1+1.6} = 8600.26$

$\text{Es} = 2 \text{ x } 10^5 \text{ N/mm}^2$

$\text{m} = \frac{\text{E}_\text{S}}{\text{E}_{\text{ce}}} = \frac{2 \text{ x } 10^{\ 5}}{8600.26} = 23.255$

Depth of neutral axis x is given by

$$\frac{bx^2}{2} = m\ A_{st}\ (d - x)$$

$$\frac{300x^2}{2} = 23.255 \text{ x } 942.5\ (600 - x)$$

$$x = 231.91 \text{ mm}$$

$I_r = \frac{bx^3}{3} + m\ A_{st}\ (d - x)^2$

$I_r = \frac{300\ 231.91^{\ 3}}{3} + 23.255\ 942.5\ (600 - 231.91\ )^2 = 6305.479 \text{ x } 10^6 \text{ mm}^4$

bw/b = 1

z = d – (x/3) = 600 – (231.91 /3) = 522.697

$$Ieff = \frac{4216.917\times10^6}{1.2 - \frac{66.132\times10^6}{62.5\times10^6}\times\frac{522.697}{600}\left(1-\frac{231.91}{600}\right)\times1}$$

$I_{eff} = 6646.05 \text{ x } 10^6 \text{ mm}^4$

As per IS code

$$I_r \leq I_{eff} \leq I_{gr}$$

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I_{eff} = 6646.05 x 10^6 mm^4 is OK

$\alpha_{i,cc(Perm)} = \frac{5\,W\,L^4}{384\,EI} = \frac{5\ x\ 20\ x\ 5000^4}{384\ x\ 8600.26\ x\ 6646.05\ x\ 10^6}$ = 2.85 mm

Short Term Deflection Duo to Permanent Load

Ec = $5000\sqrt{20}$ = 22360.68 N/mm^2

Es = 2 x 10^5 N/mm^2

m =8.944

$$\frac{bx^2}{2} = m\,A_{st}\,(d-x)$$

$$\frac{300x^2}{2} = 8.994\ \text{x}\ 942.5\ (600-x)$$

$$x = 157.66\ \text{mm}$$

z = d – (x/3)= 600 –(157.66 /3) = 522.697

Igr = $\frac{\text{b D}^3}{12} = \frac{300\ 650^3}{12}$ = 6865.6253 x 10^6 mm^4

$I_r = \frac{300\ 231.91\ ^3}{3} + 8.944$ x $942.5\ (600-157.66)^2$ = 6305.479 x 10^6 mm^4

$$Ieff = \frac{2041.261\times10^6}{1.2-\frac{66.132\times10^6}{62.5\times10^6}\times\frac{547.45}{600}\left(1-\frac{157.66}{600}\right)\times1}$$

$$I_{eff} = 4180.80 \text{ x } 10^6$$

Which is more than I_r and less than I_{gr}

$$I_{eff} = 4180.80 \text{ x } 10^6$$

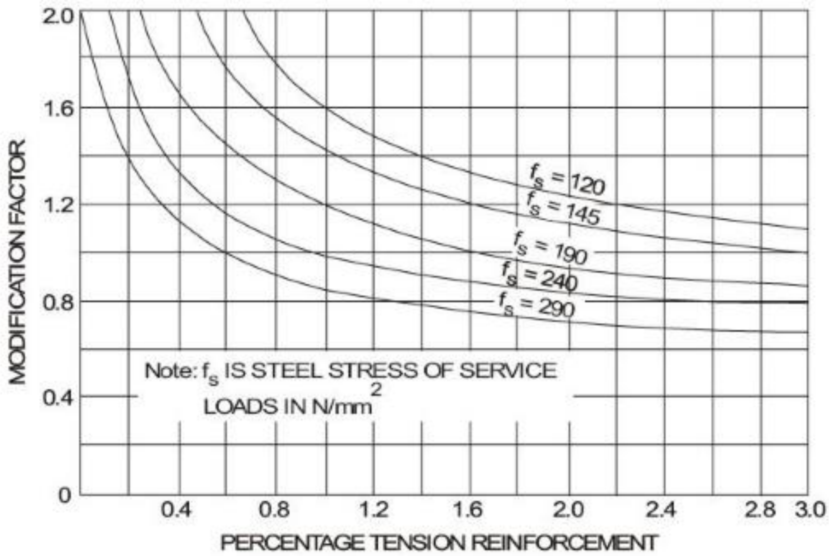
Short term deflection doe to permanent loads = $\frac{5\,W\,L^4}{384\,EI} = \frac{5\ x\ 20\ x\ 5000^4}{384\ x\ 8600.26\ x\ 4180.80\ x\ 10^6}$ = 1.741 mm

Deflection due to creep only = 2.85 -1.741 = 1.11 mm

1.15.4 Alternate Method of ensuring Limit state Requirements of Deflection

Step 1: Depending on condition of supports, choose the basic span/ effective depth ratio from Table 9.1 if the span is 10 m or less. If it is greater than 10 m, reduce the values as indicated in Table 1.2 and Fig. 1.10.

Step 2: Determine modification factor F_1 which depends on the type of steel used (corresponding to the service stress in steel) and the percentage of steel required in the beam at the point of maximum deflection. The modification factor F_1 is to be obtained from IS 456 for different stress levels and percentages of tension steel.



$$f_s = 0.58 f_y \frac{\text{Area of cross-section of steel required}}{\text{Area of cross-section of steel provided}}$$

Fig 1.11 Modification factor $K_1$for Tension reinforcement

Step 3: Determine the modification factor F_2 corresponding to the percentage of compression reinforcement provided at the point of maximum moment. This is given in IS 456, Fig. 5 and Fig. 1.12 of the text.

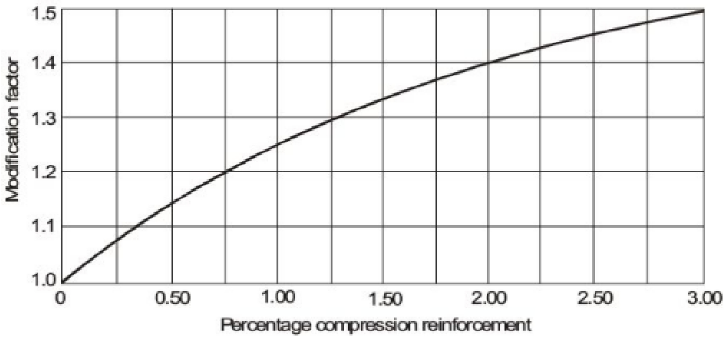


Fig 1.12 Modification factor for compression reinforcement.

Step 4: As the factors F_1 and F_2 for flanged beams are calculated with the effective flange width (b_f), a reduction factor F_3 should be used to allow for the reduced area of concrete in the tension zone.

The reduction factor depends on the ratio of web width (b_w) to effective flange width (bf) as follows:

1. For b_w/ b_f = 0.3 and below, the value of F_3 = 0.8
2. For b_w/ b_f = 1.0, the value of F_3 = 1.0
3. For intermediate values, the value of F_3 is obtained by linear interpolation from Fig. 6 of IS 456 (Fig. 1.13 in this text).

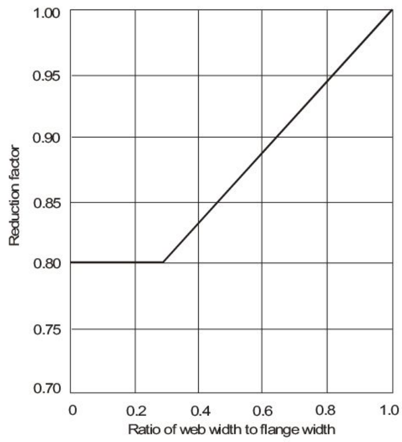


Fig 1.13 Reduction factor for flanged beams. (IS 456 Fig. 6)

Step 5: The final span/ depth ratio allowed is (Basic ratio) (F_1) (F_2) (F_3). The ratio should not exceed this in the designed structure.

Problem 2.

A rectangular beam of size 300 mm x600 mm is reinforced with 6 bars of 20 mm diameters on tension side with Fe 415 steel. The effective cover is 50 mm and the span of the simply supported beam is 7.5m. Check whether depth provided is sufficient from the deflection consideration. Assume - exactly the required amount of steel is provided.

Solution

D= 600 - 50 = 550 mm and b = 300 mm Ast = 1885 mm^2 Span = 7.5 m

The beam is simply supported. Hence basic span to deflection ratio permitted is 20. The steel used is Fe 415 and the amount of steel provided is exactly steel required.

fs = 0.58 x 415 x 1) = 240.7 N/mm^2

% Tension reinforcement = (1885 x 100)/ (300 x 550) = 1. 142 %

From Fig. 5.7 (IS Fig. 4), modification factor for tensile steel provided

F1 = 0.98

F2 = 1.0 Since no compression reinforcement

Since it is rectangular beam, ratio of web width to flange width is 1 and hence F3 = 1

Limiting value of span = F1 x F2 x F3 x basic ratio

=0.98x 1.0 x 1 x20 = 19.6

span /$d_{provided}$ = 7500 /550 =13.636 < 19.6

Hence the depth provided is sufficient from the consideration of deflection.

1.16 CRACKING IN REINFORCED CONCRETE MEMBERS

Cracking of concrete will occur whenever the tensile strength (or the ultimate tensile strain) of concrete is exceeded. As concrete has relatively low tensile strength as well as low failure strain in tension, cracking is usually inevitable in normal reinforced concrete members. However, the degree of cracking (in terms of width and spacing of cracks) can be controlled through proper design. Cracking is considered undesirable, not only for obvious aesthetic reasons, but also because it adversely affects durability (in aggressive environments) and leads to corrosion of the embedded steel. Hence, it is important for the designer to have an understanding of the various causes of cracking, the allowable limits on crack-widths under different situations as well as the methods to achieve crack control

Cracking in reinforced concrete members is attributable to various causes particularly

1. Flexural tensile stress due to bending under applied loads;
2. Diagonal tension due to shear and/or torsion;
3. Direct tensile stress under applied loads (for example, hoop tension in a circular water tank);
4. Lateral tensile strains accompanying high compressive stress/strain due to Poisson effect (as in a compression test) or due to heavy concentrated loads as in a split cylinder test;
5. Restraint against volume changes due to shrinkage and temperature as well as due to creep and chemical effects; and
6. Additional curvatures due to continuity effects, settlement of supports, etc.

Structural cracking in concrete occurs in tension, flexure or a combination of the two effects (eccentric tension). When this happens, splitting of the concrete occurs at the surface, penetrating inwards. Under direct tension, the crack generally runs through the thickness of the member (wall or slab), whereas under flexure, the crack is limited to the flexural tension zone. In all cases, the spacing of cracks as well as width of individual cracks depends not

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only on the magnitude of tensile force acting, but also on the reinforcement detailing, properties of concrete and thickness of section.

Factors Influencing Crack-widths

Crack-widths in RC members subject to flexure, direct tension or eccentric tension, are influenced by a large number of factors, many of which are inter-related. These include:

- Tensile stress in the steel bars;
- thickness of concrete cover;
- Diameter and spacing of bars;
- Depth of member and location of neutral axis; and
- Bond strength and tensile strength of concrete.

1.16.1 Limiting Crack Widths

Under service loads the crack width in concrete should not be excessive. IS 456 Clause 35.3.2 specifies that under normal conditions, crack width at the surface of concrete should not exceed 0.3 mm for the sake of appearance. In "moderate exposure" it should be limited to 0.2 mm and "severe exposure" to 0.1 mm for corrosion resistance. In 'aggressive' conditions of exposure for control of corrosion, the cracks at points nearest to the main reinforcement are generally specified not to be more than 0.004 that of the nominal cover to the reinforcement.

1.16. 2 Method of Crack Control

The empirical method of crack control assumes that cracks will be within the allowable limits, by detailing of steel in the structure according to the rules laid down in codes of practice. The important factors to be considered are the following:

1. Maximum and minimum spacing of reinforcements (bar spacing rules). The bar spacing (horizontally and vertically) is given in IS 456, Clause 26.3.1 and Clause 26.3.1 for beams. 26.3. 2 for slabs

2. Maximum and minimum areas of steel in the member: 26.5.1.1, 26.5.2.1. Clause 26.5.2.1, Clause 23.1.1. 26.5.1.6 IS Clause 26.5.2.2. Of IS 456

3. Curtailment of reinforcement bars

4. Anchorage of reinforcement bars

5. Lapping of steel

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6. Stress level in steel: The sixth criterion given in Section 9.4.3 to control the size of cracks in reinforced members is the stress level (and the strain) in the tension reinforcement.
7. Conformity with the general layout of steel reinforcement in the given structure according to accepted practice.
8. Cover to reinforcement
9. Maximum and minimum sizes of steel to be used for the various types of steel in the member.

1.16.3 Estimation of Flexural Crack-width:

IS 456 Formulation

The IS code in Annex F describes a procedure for predicting flexural crack-widths, which is apparently borrowed from the British Code. The expression for w_{cr} takes the form shown in Eq. below:

$$w_{cr} = \frac{3a_{cr}\varepsilon_m}{1+2(a_{cr}-C_{\min})/(D-x)}$$

α_{cr}=distance from the point considered to the surface of the nearest longitudinal bar,
C_{min} = minimum cover to the longitudinal bar;
ε_m =average steel strain at the level considered,
h= overall depth of the member, and
x =depth of the neutral axis.

$$\varepsilon_m = \varepsilon_1 - \frac{b(h-x)(a-x)}{3E_s A_s(d-x)}$$

As = area of tension reinforcement

b = width of the section at the centroid of the tension steel,
ε_1 = strain at the level considered, calculated ignoring the stiffening of the concrete in the tension zone
a = distance from the compression face to the point at which the crack width is being calculated,
d = effective depth

$$\varepsilon_1 = \frac{f_{st}}{E_s} \times \frac{D-x}{d-x}$$

Problem 3

For a simple supported beam of span 6m and cross section 250x500mm. The average strain is 0.04mm and the depth of the neutral axis is at the center. The distance of the surface from the nearest longitudinal bar is 45 mm. Calculate the crack width.

Given:

α_{cr}= 45 mm

E_m = 0.04

C_{min} = 30 mm for beam

H =500 mm

x=250 mm

$$w_{cr} = \frac{3a_{cr}\varepsilon_m}{1+2(a_{cr}-C_{\min})/(D-x)}$$

$$\text{Wcr} = \frac{3 \, x \, 45 \, x \, 0.04}{1+\frac{2(45-30)}{500-25}} = 1.821 \text{ mm}$$

1.16.4 Side Reinforcement

According to IS 456, Clause 26.5.1.3, if the total depth of the beam is greater than 750 mm, side reinforcements are to be provided along the two faces as shown in Fig. 9.7 at a spacing sb which should not be greater than 300 mm (or web thickness, whichever is less). The total area of such reinforcement shall also be not less than 0.1 per cent of web area. As can be visualized, this steel is a must on the tension part of the beam below the neutral axis.

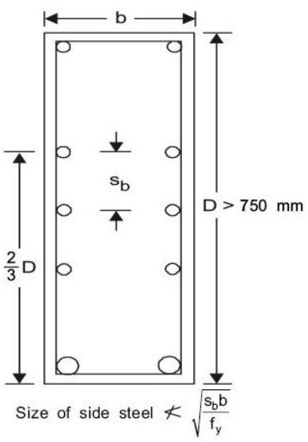

Fig 1.14 Spacing of side reinforcement.

To guard against the bar yielding locally at a crack, in BS 8110, the spacing of the side reinforcement is limited to 250 mm and the diameter of the side reinforcement too should not be less than the value in mm given by the expression

$$f = \sqrt{\frac{s_b b}{f_y}}$$

Where s_b = vertical spacing in mm
b = breadth of the beam in mm
f_y = characteristic strength in N/ mm^2

1.16. 5 SLENDERNESS LIMITS FOR BEAMS FOR STABILITY

It should also be noted that, according to IS 456, Clause 23.3 in the conditions to be satisfied for stability, the compression side of the beam may be likened to a slender column that can buckle sidewards and horizontally. The conditions are:

1. In a simply supported or continuous beam, the clear distance between the lateral restraints should not exceed 60b or $250b^2$/ d, whichever is less.
2. For a cantilever, the distance between the free end of the cantilever and the lateral restraint should not exceed 25b or $100b^2$/ d, whichever is less.

IMPORTANT QUESTIONS

1. Differentiate between working stress method and limit state method of RCC design.
2. Explain the following
   i. Partial safety factor for loads and materials
   ii. Characteristics load
   iii. Characteristic strength
3. Calculate the crack width directly under the bar on tension face at the location of max bending moment in the beam of b=300mm, D=600mm, off cover on comp. side (d') =37.5mm, Reinforcement -3 bars of 20mm dia bars. M=200 kN-m, Ast=1855mm
4. Derive the expression for stress block parameter for compressive force Cu and Tensile force Tu and locate a depth of neutral axis y= 0.42 xu from top
5. Briefly explain under reinforced, over reinforced and balanced sections with sketch. A reinforced concrete beam of cross section 300mmx600mm overall is reinforced with 3

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bars of 20mm HYDS bars of Fe415 grade on tension side with an effective cover of 50mm.compute short term deflection of the beam at mid span, consisting of service load of 20kN/m and concentrated load of 25kN at the center of span. The beam is simply supported over a span of 5m. Use M20 grade concrete and Fe415steel.

6. Explain in short long and short-term deflection of RC beams.

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