VISVESVARAYA TECHNOLOGICAL UNIVERSITY

BELGAUM

DESIGN OF RC STRUCTURAL ELEMENTS

(Subject Code: 21CV53)

LECTURE NOTES

(MODULE-2)

V-SEMESTER

Mrs. Babitha B

Assistant Professor, Dept. of Civil Engineering

AJIET

A J INSTITUTE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

(A unit of Laxmi Memorial Education Trust. (R))
NH - 66, Kottara Chowki, Kodical Cross - 575 006

Module - 2
LIMIT STATE ANALYSIS OF BEAMS

Beam

A structural member that supports transverse (Perpendicular to the axis of the member) load is called a beam. Beams are subjected to bending moment and shear force. Beams are also known as flexural or bending members. In a beam one of the dimensions is very large compared to the other two dimensions.

Beams may be of the following types:

- **Singly or doubly reinforced rectangular beams**

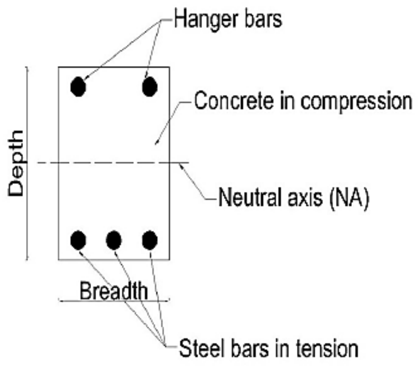


Fig 1: Singly reinforced rectangular beam

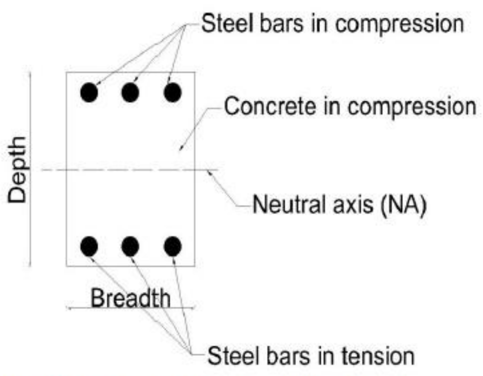


Fig 2: Doubly reinforced rectangular beam

- **Singly or doubly reinforced T-beams**

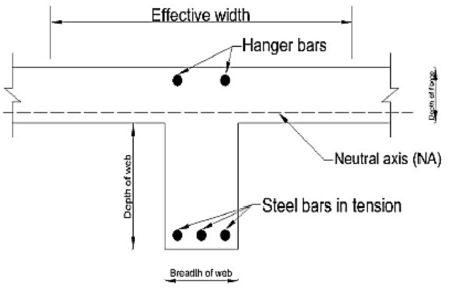


Fig 3: Singly reinforced T beam

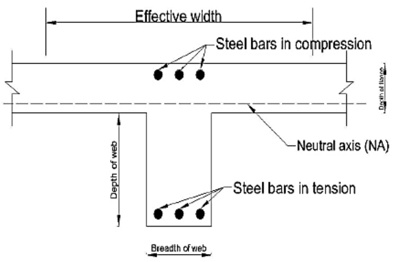
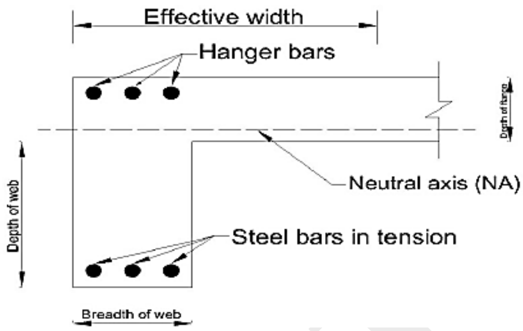


Fig 4: Doubly reinforced T beam

- **<u>Singly or doubly reinforced L-beams</u>**



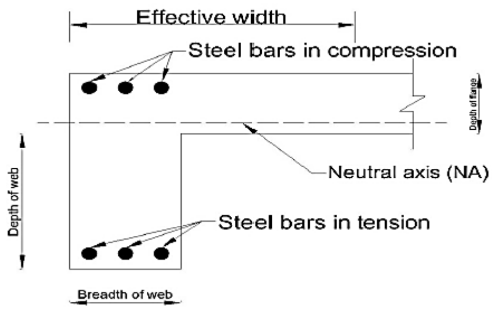


Fig 6: Doubly reinforced L beam

There are two kinds of problems commonly encountered in structural design practice. In the first kind, termed analysis (or 'review') problems, the complete cross-sectional dimensions (including details of reinforcing steel), as well as the material properties of the member are known. It is desired to compute

(1) The stresses in the materials (or deflections, crack-widths, etc.) under given loads or

(2) The allowable or ultimate bending moments (loads) that the member can resist.

The 'analysis' referred to here, which refers to the determination of stress resultants (i.e., bending moments, shear forces, etc.) in an entire structure (such as a frame) or a structural element (such as a beam).

The second type of problem involves design. In this case, the load effects (stress resultants) are known from structural analysis, and it is required to select appropriate grades of materials and to arrive at the required member dimensions and reinforcement details. It is evident that there are many possible solutions to a design problem, whereas the solution to an analysis problem is unique.

2.1 ANALYSIS OF SINGLY REINFORCED RECTANGULAR SECTIONS

The beam that is longitudinally reinforced only in tension zone, it is known as singly reinforced beam. In Such beams, the ultimate bending moment and the tension due to bending are carried by the reinforcement, while the compression is carried by the concrete.

Analysis of a given reinforced concrete section at the ultimate limit state of flexure implies the determination of the ultimate moment of resistance M_{uR} of the section. This is easily obtained from the couple resulting from the flexural stresses.

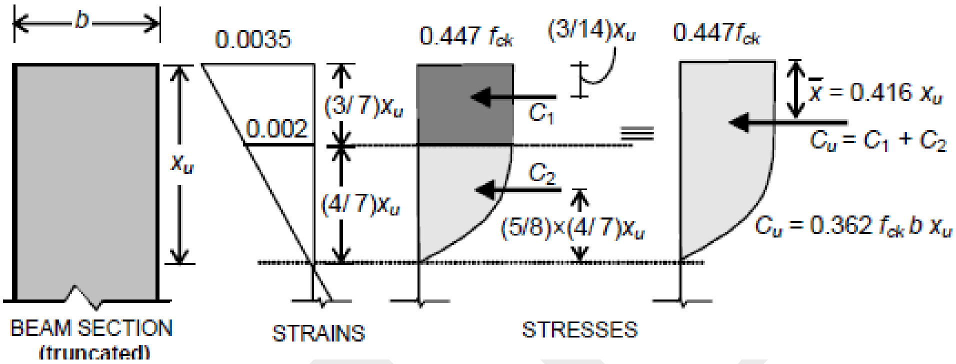


Fig 2.1 Concrete stress-block parameters in compression

$$M_{uR} = C_u . z = T_u . z \qquad \text{------1}$$

Where Cu and Tu are the resultant (ultimate) forces in compression and tension respectively, and *z* is the lever arm. Where

$$T_u = f_{st} . A_{st} \qquad \text{------2}$$

Where fst = 0.87 f_y for $x_u < x_{u,max}$

and the line of action of Tu corresponds to the level of the centroid of the tension steel.

In order to determine the magnitude of Cu and its line of action, it is necessary to analyze the concrete stress block in compression. As ultimate failure of a reinforced concrete beam in flexure occurs by the crushing of concrete, for both under- and over-reinforced beams, the shape of the compressive stress distribution ("stress block") at failure will be, in both cases, as shown in Fig.2.1. The value of Cu can be computed knowing that the compressive stress in concrete is uniform at 0.447 f_{ck} for a depth of $3x_u/7$, and below this it varies parabolically over a depth of $4x_u/7$ to zero at the neutral axis [Fig. 2.1].

For a rectangular section of width b,

$$C_u = 0.447 f_{ck}\, \mathrm{b}\left[\frac{3x_u}{7} + \left(\frac{2}{3}\, x\, \frac{4x_u}{7}\right)\right]$$

Therefore, Cu = 0.361 f_{ck} b x_u ------3

Also, the line of action of Cu is determined by the centroid of the stress block, located at a distance $\bar{x}$ from the concrete fibres subjected to the maximum compressive strain. Accordingly, considering moments of compressive forces C_u, C_1 and C_2 [Fig. 2.1] about the maximum compressive strain location,

$$(0.362 f_{ck} b x_u)\,\mathrm{x}\,\bar{x} = (0.447 f_{ck} b x_u)\left[\left(\frac{3}{7}\right)\left(\frac{1.5x_u}{7}\right) + \left(\frac{2}{3}\,\mathrm{x}\,\frac{4}{7}\right)\left(x_u - \frac{5}{8}\,\mathrm{x}\,\frac{4x_u}{7}\right)\right]$$

Solving $\bar{x}$ = 0.416 xu ------4

2.1.1 Depth of Neutral Axis

For any given section, the depth of the neutral axis should be such that Cu = Tu, satisfying equilibrium of forces. Equating Cu = Tu, with expressions for Cu and Tu given by Eq. (3) and Eq. (2) respectively,

$$x_u = \frac{0.87 f_y A_{st}}{0.361 f_{ck} b}\ \text{, valid only if resulting } x_u \le x_{u,\max} \qquad \text{------5}$$

$$\frac{xu}{d} = 2.41\ \frac{fy}{fck}\ \frac{Ast}{bd} \qquad \text{------5 a}$$

2.1.2 Ultimate Moment of Resistance

The ultimate moment of resistance M_{uR} of a given beam section is obtainable from Eq. (1). The lever arm z, for the case of the singly reinforced rectangular section [Fig. 2.1, Fig. 2.2] is given by

Z = d - 0.416x_u………. (6)

Accordingly, in terms of the concrete compressive strength,

$M_{uR} = 0.361 f_{ck} b x_u (d - 0.416 x_u)$ for all x_u ……………..(7)

Alternatively, in terms of the steel tensile stress,

$M_{uR} =$ fst Ast $(d - 0.416 x_u)$ for all x_u …………….(8)

With $f_{st} = 0.87 f_y$ for $x_u \leq x_{u,max}$

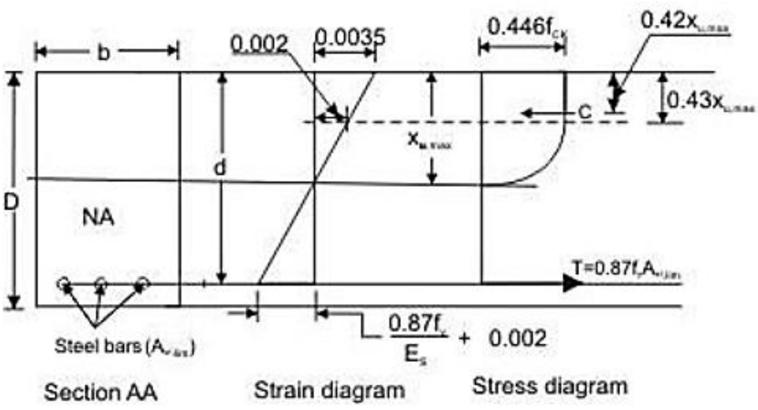


Fig 2.2 Rectangular beam under flexure $x_u = x_{u,max}$

2.1.3 Limiting Moment of Resistance

The limiting moment of resistance of a given (singly reinforced, rectangular) section, according to the Code (Cl. G−1.1), corresponds to the condition, defined by Eq. (9). From Eq. (7), it follows that

$$\frac{x_{u,\max}}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002} \qquad \text{------9}$$

$$M_{u,\lim} = 0.361 f_{ck} b x_{u,\max} (d - 0.416 x_{u,\max}) \qquad \text{------10}$$

$$M_{u,\lim} = 0.361 f_{ck} \left(\frac{x_{u,\max}}{d}\right)\left(1 - \frac{0.416 x_{u,\max}}{d}\right) b d^2 \qquad \text{------10.a}$$

2.1.4 Limiting Percentage Tensile Steel

Corresponding to the limiting moment of resistance $M_{u,lim}$, there is a limiting percentage tensile steel $p_{t,lim} = 100 \times A_{st,lim} / bd$. An expression for $p_{t,lim}$ is obtainable from Eq. (5) with: $x_u = x_{u,max}$

$$\frac{x_{u,\max}}{d} = \frac{0.87 f_y}{0.361 f_{ck}} \times \frac{p_{t,\lim}}{100}$$

$$\Rightarrow p_{t,\lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u,\max}}{d}\right) \qquad \text{------ 11}$$

The values of $p_{t,lim}$ and $\frac{Mu}{bd^2}$ (in MPa units) for, different combinations of steel and concrete grades are listed in Table 2. These values correspond to the so-called "balanced" section for a singly reinforced rectangular section.

Table 2 Limiting values of $p_{t,\lim}$ and $\frac{M_u}{bd^2}$ for singly reinforced rectangular beam sections fc various grades of steel and concrete.

(a) $p_{t,\lim}$ values

	M20	M25	M30	M35	M40
Fe 250	1.769	2.211	2.653	3.095	3.537
Fe 415	0.961	1.201	1.441	1.681	1.921
Fe 500	0.759	0.949	1.138	1.328	1.518

(a) $\frac{M_{u,\lim}}{bd^2}$ values (MPa)

	M20	M25	M30	M35	M40
Fe 250	2.996	3.746	4.495	5.244	5.993
Fe 415	2.777	3.472	4.166	4.860	5.555
Fe 500	2.675	3.444	4.013	4.682	5.350

2.1.5 Safety at Ultimate Limit State in Flexure

The bending moment expected at a beam section at the ultimate limit state due to the factored loads is called the factored moment Mu. For the consideration of various combinations of loads (dead loads, live loads, wind loads, etc.), appropriate load factors should be applied to the specified “characteristic” loads, and the factored moment Mu is determined by structural analysis.’

The beam section will be considered to be “safe”, according to the Code, if its ultimate moment of resistance M_{uR} is greater than or equal to the factored moment Mu. In other words, for such a design, the probability of failure is acceptably low. It is also the intention of the Code to ensure that at ultimate failure in flexure, the type of failure should be a tension (ductile) failure. For this reason, the Code requires the designer to ensure that $x_u \leq x_{u,max}$,whereby it follows that, for a singly reinforced rectangular section, the tensile reinforcement percentage p_t should not exceed $p_{t,lim}$ and the ultimate moment of resistance M_{uR} should not exceed $M_{u,lim}$.

2.1.6 Modes of failure: Types of section

A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:

(a) Equal to failure strain of steel (0.87 f y /E s) + 0.002 corresponding to balanced section.

(b) More than failure strain, corresponding to under reinforced section.

(c) Less than failure strain corresponding to over reinforced section.

Thus, for a given section, the actual value x_u/d of can be determined from Eq. (5). Three cases may arise.

Case-1: x_u/d equal to the limiting value $x_{u,max}/d$: Balanced section.

Case-2: x_u/d less than limiting value $x_{u,max}/d$: under-reinforced section.

Case-3: x_u/d more than limiting value $x_{u,max}/d$: over-reinforced section

Balanced section

In balanced section, the strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is $x_{u} = x_{umax}$.

Under-reinforced section

An under-reinforced section is the one in which steel percentage (pt) is less than critical or limiting percentage ($p_{t,lim}$). Due to this the actual NA is above the balanced NA and xu < $x_{u\ max}$.

Over-reinforced section

In the over reinforced section, the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and $x_u > x_{u\ max}$. Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value.

2.1.7 Computation of Mu

Mu can be obtained by multiplying the tensile force T or the compressive force C with the lever arm. The expressions of C, lever arm and T are given in Eqs. (1) and (2) respectively. Previous section discusses that there are three possible cases depending on the location of xu. The corresponding expressions of Mu are given below for the three cases:

(i) When $x_u < x_{u\ max}$

In this case the concrete reaches 0.0035, steel has started showing ductility (Strain > (0.87 f_y /E_s) + 0.002. So, the computation of Mu is to be done using the tensile force of steel in this case.

Therefore, Mu= T (lever arm) = 0.87 f_y A_{st} (d - 0.42 x_u)

(ii) When $x_u = x_{u\ max}$

In this case steel just reaches the value of 0.87 f_y /E_s) + 0.002 and concrete also reaches its maximum value. The strain of steel can further increase but the reaching of limiting strain of concrete should be taken into consideration to determine the limiting M_u as $M_{u\,lim}$ here. So, we have,

$M_{u\,lim}$ = C (lever arm)

Substituting the expressions of C

$$M_{u,\lim} = 0.36\frac{x_{u,\max}}{d}\left(1-0.42\frac{x_{u,\max}}{d}\right)f_{ck}bd^2$$

(iii) When xu> xu, max

In this case, concrete reaches the strain of 0.0035, tensile strain of steel is much less than (0.87 fy /Es) + 0.002 and any further increase of strain of steel will mean failure of concrete, which is to be avoided. On the other hand, when steel reaches 0.87 fy /Es) + 0.002 the strain of concrete far exceeds 0.0035. Hence, it is not possible. Therefore, such design is avoided and the section should be redesigned.

However, in case of any existing reinforced concrete beam where x_u> $x_{u,\,max}$, the moment of resistance Mu for such existing beam is calculated by restricting x_u to $x_{u,max}$ only and the corresponding Mu will be as per the case when x_u= $x_{u,\,max}$

Numerical Problem

1. Find the moment of resistance of a singly reinforced concrete beam of 200 mm width 400mm effective depth, reinforced with 3-16 mm diameter bars of Fe 415 steel. Take M_{20} grade of concrete.

Solution

$$A_{st} = 3x\frac{\pi}{4}(16)^2 = 603.19\,mm^2$$

$$\%\,p_t = 100x\frac{603.19}{200x400} = 0.754\%$$

$$\frac{x_u}{d} = 2.417p_t\frac{f_y}{f_{ck}} = 2.417x\frac{0.754}{100}x\frac{415}{20} = 0.378$$

Now for Fe 415 grade of steel, $\frac{x_{u,\max}}{d} = 0.479$

Hence the beam is under-reinforced.

The moment of resistance is given by

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$= 0.87x415x603.19x400\left(1 - \frac{415x603.19}{20x200x400}\right)$$

=73.48 KN-m.

2. Find MR of the section with the following details. Width of section: 230mm, Overall depth of section: 500mm, Tensile steel: 3 bars of 16mm dia, Grade of concrete: M25, Type of steel: Fe 415, Environmental condition: severe

<u>Solution:</u>

b= 230mm, D= 500mm, f_{ck} = 25 N/mm^2, f_y= 415 N/mm^2

From table 16 (page 47, IS 456-2000)

Min clear cover (CC) = 45mm

Assume Clear Cover 50mm.

Effective depth = 500 - (50 + 8) = 442mm

$A_{st} = 3 \text{ x } \frac{\pi}{4} \text{ x } 16^2 = 603.18 \text{ mm}^2$

Depth of NA, $x_u/d = \frac{0.87\ fy\ \ Ast}{0.361\ fck\ \ b\ \ d}$

$= \frac{0.87\ x\ 415\ x\ 603.18}{0.361\ x\ 25\ x\ 230\ x\ 442}$

= 0.237

Therefore, x_u = 104.754 mm

For Fe415, $x_{u\ lim}/d$ = 0.48

$x_u < x_{u\ lim}$

Therefore, under-reinforced section

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$= 0.87 \text{ x } 415 \text{ x } 603.18 \text{ x } 442 \left(1 - \frac{415 \ x \ 603.18}{25 \ x \ 230 \ x \ 442}\right)$$

= 86.77 kNm

3. A singly reinforced beam 200mm x 600mm is reinforced with 4 bars of 16mm dia with an effective cover of 50mm, effective span is 4m. Assuming M20 concrete & Fe 250 steel, determine the central concentrated load P that can be carried by the beam in addition to its self-weight.

Solution

$A_{st} = 4\ (\pi/4)\ 16^2 = 804.25\text{mm}^2$

b = 200mm, D = 600mm, d=550mm, f_{ck} = 20Mpa, f_y = 250Mpa.

Step (1) $\frac{x_u}{d} = 2.41 \frac{fy}{fck} \frac{A_{st}}{bd}$

$x_u = 2.41 \times \frac{250}{20} \times \frac{804.25}{200}$

=121.14mm

Step (2)

$\frac{x_{ulim}}{d}$ for Fe 250 us 0.53

$\therefore$ $x_u$lim = 0.53 X 550 = 291.5mm

$x_u < x_u$lim .'. section is under reinforced.

Step – 3

M_u= MR = $0.87 f_y A_{st}[d-0.42x_u]$

$$\frac{0.87 \times 250 \times 804.25\ [550 - 0.42(121.14)]}{10^6}$$

M_u=87.308kNm.

M= Allowable moment = $\frac{M_u}{1.5} = \frac{87.308}{1.5}$ = 58.20kN-m

M=MD+ML

q_d=self-weight of beam

q_d = volume x density

Density = 25kN/m³ for R C C

Volume = b x h x 2 = 200 x 600 x 2 = 2,40,000 m³

q_d= 0.2 x 0.6 x 1 x 25 = 3 kN/m

$q_d = \frac{240000 \text{ X } 25}{1000^2}$

q_d = 6 KN/m

$Md = \frac{q\,d\,l^2}{8\cdot} = \frac{3 \text{ x } 4^2}{8\cdot}$

Md = 6kN.m

M= 58.2 = 6 + ML

ML= 52.2kN-m

ML= PL /4 = $\frac{52.2 \text{ X } 4}{4}$

P=52.2 kN.

2.2 ANALYSIS OF SINGLY REINFORCED FLANGED SECTIONS

Beams effectively having T-sections and L-sections (called T-beams and L-beams) are commonly encountered in beam-supported slab floor systems [refer Figs.2.3]. In such situations, a portion of the slab acts integrally with the beam and bends in the longitudinal direction of the beam. This slab portion is called the flange of the T- or L-beam. The beam portion below the flange is often termed the web, although, technically, the web is the full rectangular portion of the beam other than the overhanging parts of the flange. Indeed, in shear calculations, the web is interpreted in this manner**.**

When the flange is relatively wide, the flexural compressive stress is not uniform over its width. The stress varies from a maximum in the web region to progressively lower values at points farther away from the web. In order to operate within the framework of the theory of flexure, which assumes a uniform stress distribution across the width of the section, it is necessary to define a reduced effective flange.

The ‘effective width of flange’ may be defined as the width of a hypothetical flange that resists in-plane compressive stresses of uniform magnitude equal to the peak stress in the original wide flange, such that the value of the resultant longitudinal compressive force is the same.

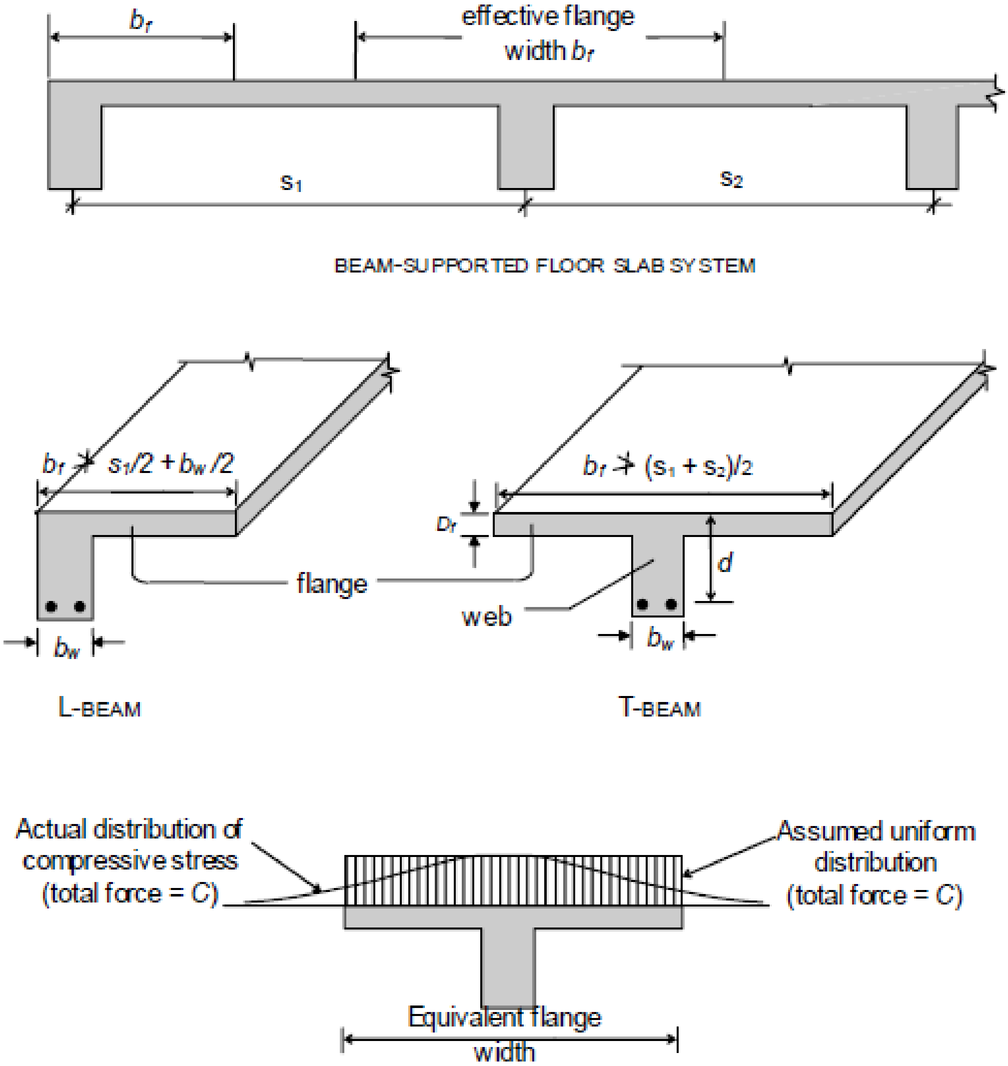


FIG 2. 3 T-beams and L-beams in beam-supported floor slab systems

Approximate formulae for estimating the 'effective width of flange' b_f (Cl. 23.1.2 of Code) are given as follows:

$$b_f = \begin{cases} l_0/6 + b_w + 6D_f & \text{for T-beams} \\ l_0/12 + b_w + 3D_f & \text{for L-beams} \end{cases}$$

For isolated beams, the effective flange width shall be obtained as below but in no case greater than the actual width:

$$\text{T} - \text{beam}, b_f = \frac{l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w$$

$$\text{L} - \text{beam}, b_f = \frac{0.5\ l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w$$

Where b_w is the breadth of the web, D_f is the thickness of the flange [Fig 2.3], and l_0 is the "distance between points of zero moments in the beam" (which may be assumed as 0.7 times the effective span in continuous beams and frames). Obviously, b_f cannot extend beyond the slab portion tributary to a beam, i.e., the actual width of slab available. Hence, the calculated bf should be restricted to a value that does not exceed $(s_1+s_2)/2$ in the case of T−beams, and $s_1/2$ + bw/2 in the case of L−beams, where the spans s_1 and s_2 of the slab are as marked in Fig. 2.3

2.2.1 Analysis of Singly Reinforced Flanged Sections

Moment of resistance of the flanged section depends upon the depth of the neutral axis. The neutral axis of a flanged beam may be either in the flange or in the web depending on the physical dimensions of the effective width of flange b_f, effective width of web b_w, thickness of flange D_f and effective depth of flanged beam *d*.

The basic assumptions used for design of rectangular beams can be used for design of T beams also. The compressive stress remains constant between the strains of 0.002 and 0.0035. It is important to find the depth *h* of the beam where the strain is 0.002.

W.K.T $h = 0.43\ x_u = (3/7)\ x_u$

Bases on the depth of the neutral axis the following three cases arise,

when, $x_u = x_{umax}$ we get

$h = (3/7)\ x_{umax}$ = 0.227d, 0.205 d and 0.197d respectively. In general, we can adopt, say $h/d = 0.2$.

It is now clear that the three values of h are around 0.2 d for the three grades of steel. The maximum value of h may be D_f, at the bottom of the flange where the strain will be 0.002, if $D_f/d = 0.2$. This reveals that the thickness of the flange may be considered small if D_f/d does not exceed 0.2 and in that case, the position of the fibre of 0.002 strain will be in the web and the entire flange will be under a constant compressive stress of 0.446 f_{ck} .

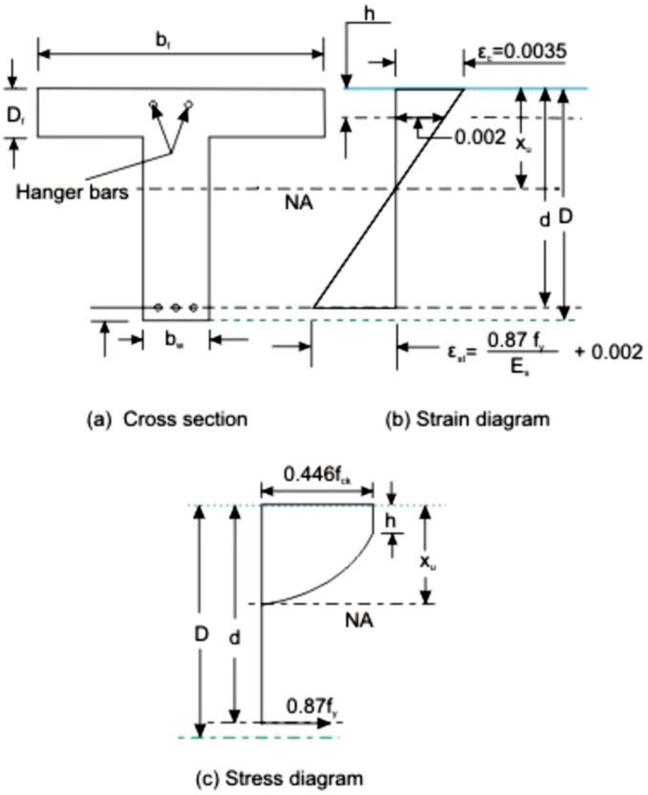


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FIG 2. 4 A typical T beam section

On the other hand, if D_f is > 0.2 d, the position of the fibre of 0.002 strain will be in the flange. In that case, a part of the slab will have the constant stress of 0.446 f_{ck} where the strain will be more than 0.002.

Thus, in the balanced and over-reinforced flanged beams (when $x_u = x_{umax}$), the ratio of D_f /d is important to determine if the rectangular stress block is for the full depth of the flange (when D_f /d does not exceed 0.2) or for a part of the flange (when D_f /d > 0.2). Similarly, for the under-reinforced flanged beams, the ratio of D_f /x_u is considered in place of D_f /d. If D_f /x_u does not exceed 0.43 the constant stress block is for the full depth of the flange. If D_f /x_u > 0.43, the constant stress block is for a part of the depth of the flange

Based on the above discussion, the four cases of flanged beams are as follows:

Case (i): When the neutral axis is in the flange ($x_u < D_f$),

Concrete below the neutral axis is in tension and is ignored. The steel reinforcement takes the tensile force (Fig. 2.5). Therefore, T and L-beams are considered as rectangular beams of width b_f and effective depth d. All the equations of singly and doubly reinforced rectangular beams are also applicable here.

For a singly reinforced beam

$C = 0.36 f_{ck} b_f x_u$

$T = 0.87\ f_y\ A_{st}$

C=T

$x_u = \frac{0.87\ x\ f_y\ A_{st}}{0.36\ f_{ck}\ b_f}$

M u = C x LA

$= 0.36\ f_{ck}\ b_f\ x_u\ (d - 0.42 x_u)$

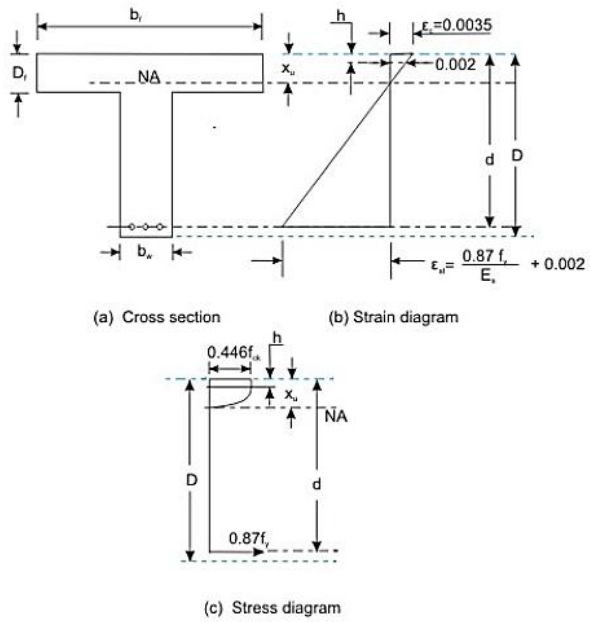


FIG 2. 5 $x_u < D_f$

Case (ii): When the neutral axis is in the web and the section is balanced ($x_{u,max} > D_f$),

(a) When D_f/d does not exceed 0.2

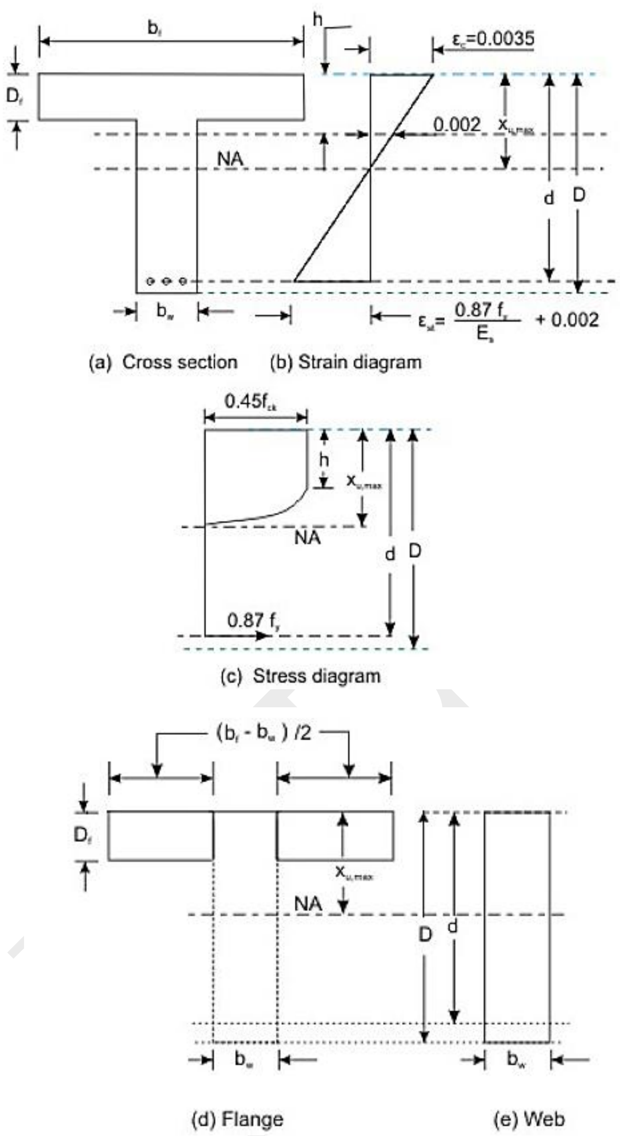


Fig 2.6 When D_f/d <= 2 and balanced $X_{umax} > D_f$

The depth of the rectangular portion of the stress block (of constant stress = 0.446 fck) in this case is greater than D_f (Figs. 2.6 a, b and c). The section is split into two parts: (i) rectangular web of width bw and effective depth d, and (ii) flange of width (bf - bw) and depth D_f (Figs. 2.6 d and e).

Total compressive force = Compressive force of rectangular beam of width b_w and depth d + Compressive force of rectangular flange of width (b_f - b_w) and depth D_f.

Thus, total compressive force

$$C = 0.36 f_{ck} \; b_w \; x_{u,\,max} + 0.45 \; f_{ck} \, (b_f - b_w) \, D_f$$

(Assuming the constant stress of concrete in the flange as 0.45 fck in place of 0.446 fck, as per G-2.2 of IS 456), and the tensile force

$T = 0.87\, f_y\, A_{st}$

The lever arm of the rectangular beam (web part) is (d - 0.42 $x_{u,\,max}$) and the same for the flanged part is (d - 0.5 D_f).

So, the total moment = Moment due to rectangular web part + Moment due to rectangular flange part

or $M_u = 0.36\, f_{ck}\, b_w\, x_{u,\,max}\, (d - 0.42\, x_{u,\,max}) + 0.45\, f_{ck}\, (b_f - b_w)\, D_f\, (d - D_f/2)$

or $M_u = 0.36(x_{u,\,max}/d)\, \{1 - 0.42(xu_{,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\, (b_f - b_w)\, D_f\, (d - D_f/2)$ ……2.2

Equation 2.2 is given in G-2.2 of IS 456.

(b) When D_f /d > 0.2

In this case, the depth of rectangular portion of stress block is within the flange (Figs. 2.7 a, b and c). It is assumed that this depth of constant stress (0.45 f_{ck}) is y_f, where

$y_f = 0.15\, x_u$, max + 0.65 D_f, but not greater than D_f (Refer G 2.2.1)

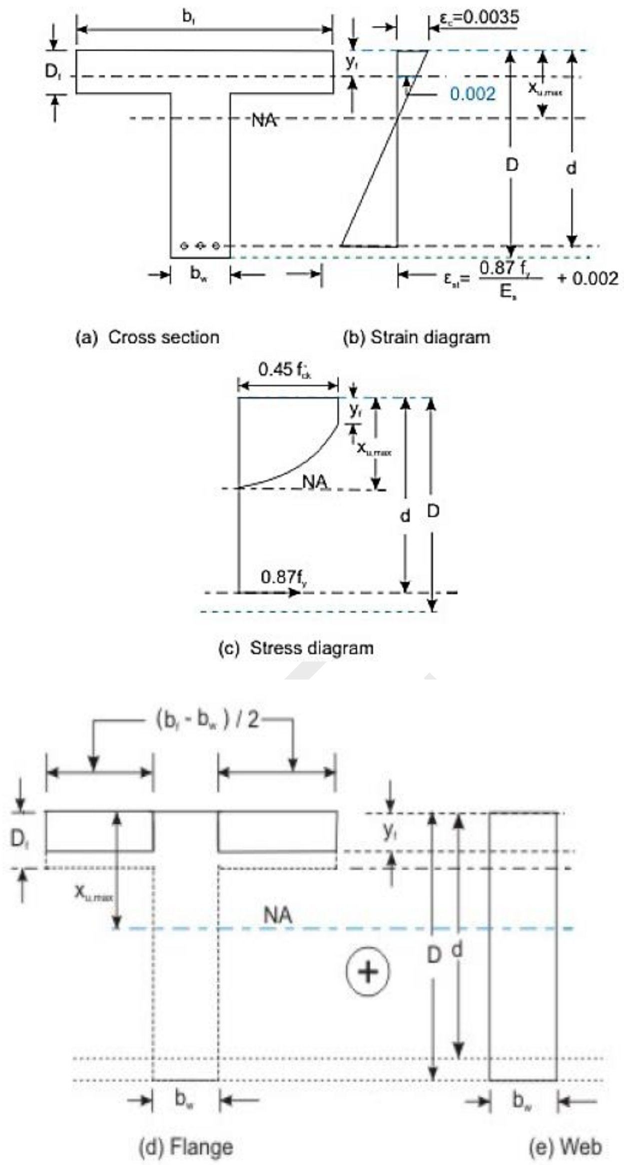
As in the previous case (ii a), when D_f /d does not exceed 0.2, equations of C, T and Mu are obtained from Eqs. In case 2 by changing D_f to y_f. Thus, we have (Figs. 2.7 d and e)

$C = 0.36\, f_{ck}\, b_w\, x_{u,\,max} + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$ ……2.3

$T = 0.87\, f_y\, A_{st}$

The lever arm of the rectangular beam (web part) is (d - 0.42 xu, max) and the same for the flange part is (d - 0.5 y_f). Accordingly, the expression of Mu is as follows:

$Mu = 0.36(x_{u,\,max}/d)\, \{1 - 0.42(xu_{,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f(d - y_f/2)$…2.4



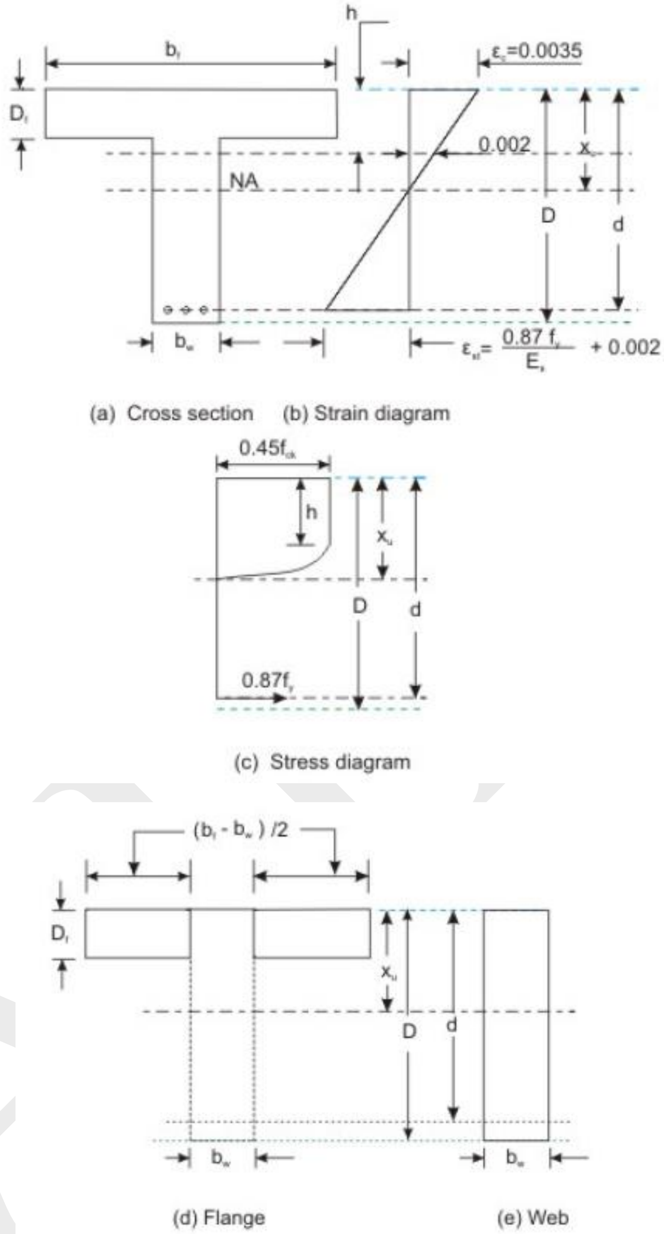
Figs. 2.7 When $D_f/d > 2$ and balanced $X_{umax} > D_f$

Case (iii): When the neutral axis is in the web and the section is under-reinforced ($x_u > D_f$)

(a) When D_f / x_u does not exceed 0.43

Since D_f does not exceed 0.43 x_u and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the entire flange will be under a constant stress of 0.45 f_{ck} (Figs. 2.8 a, b and c). The equations of C, T and M_u can be written in the same manner as in case (ii a).

The final forms of the equations are obtained from Eqs. case (ii a) by replacing $x_{u,\,max}$ by x_u. Thus, we have (Figs. 2.8 d and e)



Figs. 2.8 D_f / x_u does not exceed 0.43, under-reinforced ($x_u > D_f$)

$$C = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, D_f$$

$$T = 0.87\, f_y\, A_{st}$$

$$M_u = 0.36(x_u/d)\,\{1 - 0.42(xu/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\, (b_f - b_w)\, D_f\,(d - D_f/2)$$

b) When D_f / x_u > 0.43,

Since $D_f > 0.43\, x_u$ and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the part of the flange having the constant stress of 0.45 f_{ck} is assumed as y_f (Fig. 2.9). The expressions of y_f, C, T and M_u can be written from Eqs. in case (ii b), by replacing $x_{u,max}$ by x_u. Thus, we have (Fig. 2.9 d and e)

$$y_f = 0.15\, x_u + 0.65\, D_f, \text{ but not greater than } D_f$$

$$C = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$$

$$T = 0.87\, f_y\, A_{st}$$

$$M_u = 0.36(x_u/d)\{1 - 0.42(xu/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2)$$

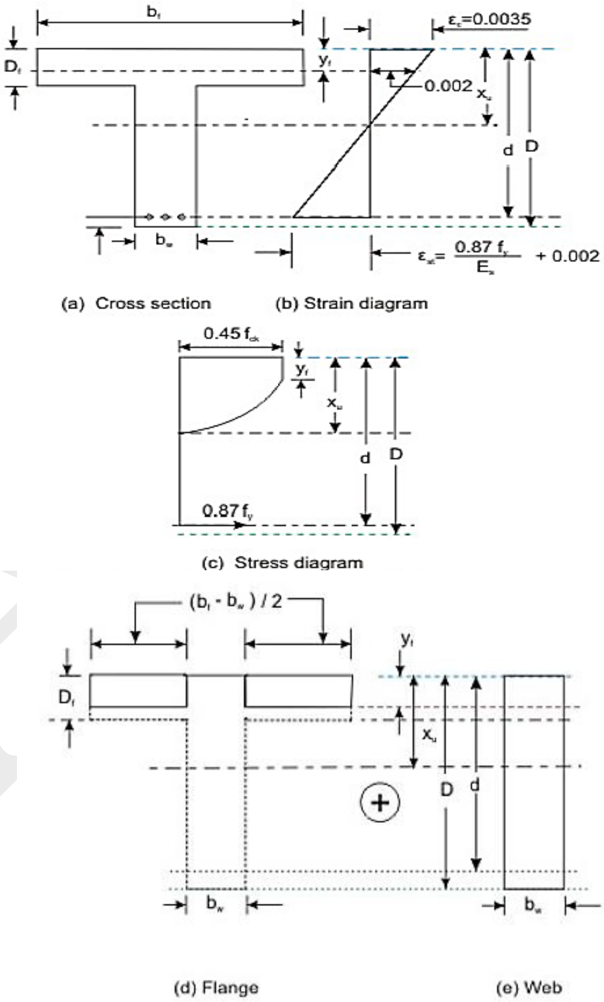


Fig 2.9 D_f / x_u > 0.43, under-reinforced ($x_u > D_f$)

Case (iv): When the neutral axis is in the web and the section is over-reinforced (xu > Df),

For the over-reinforced beam, the depth of neutral axis x_u is more than $x_{u, max}$ as in rectangular beams. However, x_u is restricted up to $x_{u,max}$. Therefore, the corresponding expressions of *C*, *T* and M_u for the two situations (a) when D_f / d does not exceed 0.2 and (b) when $D_f / d > 0.2$ are written from Eqs. in Case ii(a) and Case ii(b), respectively.

(a) When D_f/d does not exceed 0.2 (Fig Case ii a)

The equations are:

$$C = 0.36\, f_{ck}\, b_w\, x_{u, max} + 0.45\, f_{ck}\, (b_f - b_w)\, D_f$$

$$T = 0.87\, f_y\, A_{st}$$

$$M_u = 0.36(x_{u, max}/d)\, \{1 - 0.42(\mathrm{x_u}{}_{max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\, (b_f - b_w)\, D_f\, (d - D_f/2)$$

(b) When Df /d > 0.2 (Case ii b)

$$y_f = 0.15\, x_{u, max} + 0.65\, D_f, \text{ but not greater than } D_f$$

$$C = 0.36\, f_{ck}\, b_w\, x_{u, max} + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$$

$$T = 0.87\, f_y\, A_{st}$$

$$M_u = 0.36(x_{u\, max}/d)\, \{1 - 0.42(\mathrm{x_u}{}_{max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2)$$

It is clear from the above that the over-reinforced beam will not have additional moment of resistance beyond that of the balanced one. Moreover, it will prevent steel failure. It is, therefore, recommended either to re-design or to go for doubly reinforced flanged beam than designing over-reinforced flanged beam

2.2.2 Limiting Moment of Resistance

The limiting moment of resistance $M_{u\,lim}$ is obtained for the condition $x_u = x_{u,max}$, where $x_{u,max}$ takes the values of 0.531d, 0.479d and 0.456d for Fe 250, Fe 415 and Fe 500 grades of tensile steel reinforcement . The condition ($x_u / D_f \geq 7/3$) in Eq. 13, for the typical case of Fe 415, works out, for $x_u = x_{u,max}$, as $0.479d/D_f \geq 7/3$, i.e., $D_f/d \leq 0.205$. The Code (Cl. G−2.2) suggests a simplified condition of $D_f/d \leq 0.2$. for all grades of steel — to represent the condition $x_u / D_f \geq 7/3$.

Eq. 13 and Eq. 16 take the following forms:

$$M_{u,\lim} = 0.362 f_{ck} b_w x_{u,\max}\left(d - 0.416 x_{u,\max}\right) + 0.447 f_{ck}\left(b_f - b_w\right) y_f\left(d - y_f/2\right) \quad \text{for } x_{u,max} > D_f \qquad \ldots\ldots 17$$

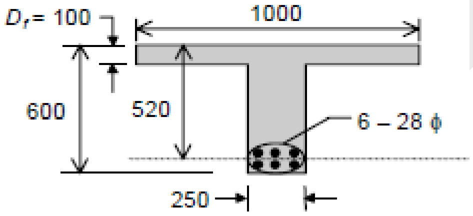
$$y_f = \begin{cases} 0.15 x_{u,\max} + 0.65 D_f & \text{for } D_f/d > 0.2 \\ D_f & \text{for } D_f/d \le 0.2 \end{cases} \qquad \ldots\ldots 18$$

for $x_{u\ max} \le D_f$ (i.e., neutral axis within the flange),

$$M_{u,\lim} = 0.362 f_{ck} b_f x_{u,\max}\left(d - 0.416 x_{u,\max}\right) \quad \text{for } x_{u,max} \le D_f \qquad \ldots\ldots 19$$

Numerical Problem

1. An isolated T-beam, having a span of 6 m and cross-sectional dimensions shown in Fig. below, is subjected to a service load moment of 200 kNm. Compute the maximum stresses in concrete and steel, assuming M 20 concrete and Fe 250 steel.



Solution

Given: b = 1000 mm, D_f = 100 mm, b_w = 250 mm, d = 520 mm, A_{st} = 3695 mm^2, span = 6m

f_y = 250 MPa and f_{ck} = 20 MPa

$$\text{T-beam}, b_f = \frac{l_0}{\left(\frac{l_0}{b}\right)+4} + b_w$$

$$b_f = \frac{6000}{\left(\frac{6000}{1000}\right)+4} + 250 = 850\text{mm}$$

$x_{u\ max}/d$ = 0.531 for Fe 250

$x_{u,max}$ = 0 .531 x 520 = 276.12 mm

First assuming $x_u \le D_f$ and $x_u \le x_{u\ max}$, and considering force equilibrium

$C_u = T_u \Rightarrow 0.362\ f_{ck}\ b_f\ x_u = 0.87\ f_y\ A_{st}$

$$x_u = \frac{0.87 \times 250 \times 3695}{0.362 \times 20 \times 850} = 130.6 \text{ mm} > D_f = 100 \text{ mm}$$

Hence, this calculated value of x_u is not correct, as $x_u > D_f$.

As, $x_u > D_f$ the compression in the 'web' is given by

$C_{uw} = 0.362\ f_{ck}\ b_w\ x_u$

$= 0.362 \times 20 \times 250 \times x_u = (1810)\ x_u$ N

Assuming $x_u \geq 7/3\ D_f = 233.3$ mm, the compression in the 'flange' is given by

$$C_{uf} = 0.447 f_{ck}\left(b_f - b_w\right)D_f$$

$C_{uf} = 0.447 \times 20\ (850\text{-}250) \times 100$

Also assuming $x_u \leq x_{u,max} = 276.1$ mm

$T_u = 0.87 \times 250 \times 3695 = 803662$ N.

Applying the force equilibrium condition $(C_{uw} + C_{uf}) = T_u$

$1810\ x_u + 536400 = 803662. \Rightarrow x_u = 147.7\ \text{mm} < (7/3)\ D_f = 233.3$ mm

Hence, this calculated value of xu is also not correct.

As $D_f < x_u < (7/3)\ D_f$, the depth $y_f \leq D_f$ of the equivalent concrete stress block is obtained as:

$$y_f = 0.15x_u + 0.65D_f = \left(0.15x_u + 65\right)\ \text{mm.}$$

$$\Rightarrow C_{uf} = 536400 \times \left({}^{y_f}\!/_{D_f}\right) = \left(804.6x_u + 348660\right)\ \text{N}.$$

$C_{uw} + C_{uf} = T_u$

$1810\ x_u + (804.6\ xu + 348660) = 803662$

$x_u = 174\ \text{mm} < x_{u,max}$

hence, the assumption $f_{st} = 0.87\ f_y$ is ok

$y_f = (0.15 \times 174) + 65 = 91.1$ mm

Taking moments of C_{uw} and C_{uf} about the centroid of tension steel

$$M_{uR} = C_{uw}\left(d - 0.416x_u\right) + C_{uf}\left(d - y_f/2\right)$$

$= 1810 \times 174 \times (520\text{-}\ 0.416 \times 174) + (804.6 \times 174 + 348660 \times (520\text{-}\ 91.1/2)$

$= 372.8 \times 10^6$ Nmm

$= 373$ kNm

2.3 ANALYSIS OF DOUBLY REINFORCED SECTIONS

When compression reinforcement is provided in addition to tension reinforcement in beams, such beams are termed doubly reinforced beams. Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension. However, these beams have their respective limiting moments of resistance with specified width, depth and grades of concrete and steel. The amount of steel reinforcement needed is known as $A_{st,lim}$. Problem will arise, therefore, if such a section is subjected to bending moment greater than its limiting moment of resistance as a singly reinforced section.

There are two ways to solve the problem. First, we may increase the depth of the beam, which may not be feasible in many situations. In those cases, it is possible to increase both the compressive and tensile forces of the beam by providing steel reinforcement in compression face and additional reinforcement in tension face of the beam without increasing the depth. The total compressive force of such beams comprises (i) force due to concrete in compression and (ii) force due to steel in compression. The tensile force also has two components: (i) the first provided by $A_{st,lim}$ which is equal to the compressive force of concrete in compression. The second part is due to the additional steel in tension - its force will be equal to the compressive force of steel in compression. Such reinforced concrete beams having steel reinforcement both on tensile and compressive faces are known as doubly reinforced beams.

Doubly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams. However, other than in doubly reinforced beams compression steel reinforcement is provided when:

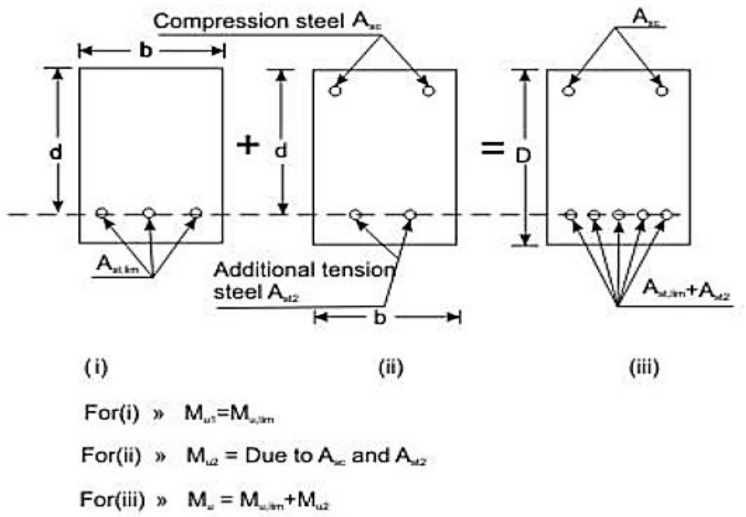


Fig 2.5 Doubly reinforced beam

(i) Some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.

(ii) The ductility requirement has to be followed.

(iii) The reduction of long-term deflection is needed

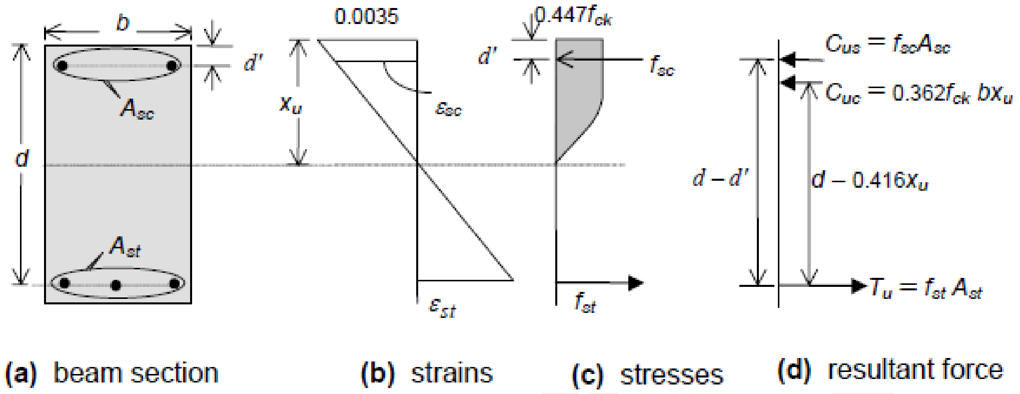


Fig 2.6 Behaviour of doubly reinforced rectangular beam at ultimate limit state

The distribution of stress and strain in a doubly reinforced section (Fig 2.6) are similar to those obtained in a 'singly reinforced' section except that there is a stress f_{sc} in the compression steel (area A_{sc}) which also needs to be accounted for. This stress f_{sc} may or may not reach the design yield stress $0.87f_y$, depending on the strain ε_{sc} in the compression steel. An expression for ε_{sc} can be easily obtained from strain compatibility [Fig. 2.6(b)]:

$$\varepsilon_{sc} = 0.0035 \times \left(1 - {d'}/{x_u}\right)$$

Where d' is the distance between compression steel and the extreme compression fibre in the concrete. In practice the ratio d′/d is found to vary in the range 0.05 to 0.20. It can be shown that the compression steel will, in most cases, attain the design yield stress (f_{sc} = 0.87fy) in the case of Fe 250 grade steel, but is generally unlikely to do so in the case of Fe 415 and Fe 500 (because of their higher strains at yield). Values of the stress fsc (corresponding to $x_u = x_{u\,max}$) for various grades of steel and ratios of d'/d are listed in Table 3.

Table 3 Value of f_{sc} (in MPa units) at $x_u = x_{u\,max}$ — for various d'/d ratios and different grades of compression steel

Grade of steel	d'/d			
	0.05	0.10	0.15	0.20
Fe 250	217.5	217.5	217.5	217.5
Fe 415	355.1	351.9	342.4	329.2
Fe 500	423.9	411.3	395.1	370.3

Applying the condition of force equilibrium (Fig 2.6 d)

$$C_{uc} + C_{us} = T_u \quad \ldots\ldots 1$$

Were C_{uc} and C_{us} denote, respectively, the resultant compressive force in the concrete and the compression steel. For convenience, the full area of the concrete under compression ($b \times x_u$) is assumed to be effective in estimating C_{uc}. The force in concrete area displaced by steel (equal to A_{sc}, stressed to a level that is exactly or nearly equal to 0.447 f_{ck}, the stress in concrete) already included in Cuc, is accounted for in the estimation of C_{us} as follows:

$$C_{uc} = 0.362 f_{ck} b x_u \quad \ldots..2$$

$$C_{us} = \left(f_{sc} - 0.447 f_{ck}\right) A_{sc} \quad \ldots.3$$

$T_u = f_{st} A_{st}$ where $f_{st} = 0.87\, f_y$ if $x_u \leq x_{u\,max}$. Accordingly, the depth of the neutral axis x_u is obtained from Eq 1 as

$$x_u = \frac{f_{st} A_{st} - \left(f_{sc} - 0.447 f_{ck}\right) A_{sc}}{0.362 f_{ck} b}$$

This equation provides a closed form solution to x_u only if $f_{st} = 0.87\, f_y$ and $f_{sc} = 0.87\, f_y$ otherwise f_{st} and f_{sc} will depends on x_u. Initially the values of f_{sc} and f_{st} may be taken as 0.087 f_y and then revised if necessary, employing the strain compatibility.

Having determined f_{sc} and x_u, the ultimate moment of resistance can be calculated by considering moments of C_{uc} and C_{us} about the centroid of the tension steel as follows:

$$M_{uR} = C_{uc}\left(d - 0.416 x_u\right) + C_{us}\left(d - d'\right)$$

2.3.1 Limiting Moment of Resistance

The limiting value of M_{uR} obtained from condition $x_u = x_{u\,max}$ is given by the following expression

$$M_{u,\lim} = 0.362 f_{ck} b x_{u,\max}\left(d - 0.416 x_{u,\max}\right) + \left(f_{sc} - 0.447 f_{ck}\right) A_{sc}(d - d')$$

where the value of f_{sc} depends on ε_{sc}. For convenience, the value of the stress for various grade of steel and d'/d ratio is given in table 3.

Numerical Problem

1. Determine the moment of resistance of an existing beam having the following data: b=350 mm; d=900mm; d'=50mm. Tension reinforcement: 5-20mm HYSD bars (Fe 415); compression reinforcement 2-20 HYSD bars (Fe 415); grade of concrete M15.

Solution:

$$A_{st} = 5x\frac{\pi}{4}(20)^2 = 1570.8mm^2;$$

$$A_{sc} = 2x\frac{\pi}{4}(20)^2 = 628.3mm^2$$

$T = 0.87 f_y A_{st} = 0.87x415x1570.8 = 567120N$

$C_u = 0.36 f_{ck} x_u b + f_{sc} A_{sc} - 0.446 f_{ck} A_{sc}$

$= 1890 x_u + 628.3 f_{sc} - 4203$

Let assume x_u =230mm; hence (3/7) x_u =98.6mm > d'

$$\varepsilon_{sc} = \frac{0.0035(x_u - d')}{x_u} = \frac{0.0035(230-50)}{230} = 0.00274$$

C_u = 1890x230 + (628.3x351)-4203=651030 N

This is much more than T_u = 567120 N. Hence take x_u = 190 mm.

$$\varepsilon_{sc} = \frac{0.0035(x_u - d')}{x_u} = \frac{0.0035(190-50)}{190} = 0.00258$$

Hence f_{sc} =347 N/mm^2

C_u=572919 N ≈T

Therefore, M_u = 1890x190x(900-0.416x190) + (628.3x347-4203) x (900-50)

= 476.5 KN-m

2.4 Analysis of beams shear

In order to gain an insight into the causes of flexural shear failure in reinforced concrete, the stress distribution in a homogeneous elastic beam of rectangular section is reviewed here. In such a beam, loaded as shown in Fig. 2.7(a), any transverse section (marked 'XX'), in general, is subjected to a bending moment *M* and a transverse shear force *V*. From basic mechanics of materials, it is known that the flexural (normal) stress f_x and the shear stress τ at any point in the section, located at a distance y from the neutral axis, are given by:

$$f_x = \frac{M\,y}{I}$$
$$\tau = \frac{VQ}{Ib}$$

It may be noted that the variation of shear stress is parabolic, with a maximum value at the neutral axis and zero values at the top and bottom of the section. The distributions of f_x and τ are depicted in Fig. 6.1(b). Considering an element at a distance y from the NA [Fig. 2.7(c)], and neglecting any possible vertical normal stress f_y caused by the surface loads, the combined flexural and shear stresses can be resolved into equivalent principal stresses f_1 and f_2 acting on orthogonal planes, inclined at an angle α to the beam axis (as shown):

In general, the stress f_1 is tensile (say = f_t) and f_2 is compressive (say = f_c). The relative magnitudes of f_t and f_c and their directions depend on the relative values of f_x and τ [Eq. 1, 2]. In particular, at the top and bottom fibers where shear stress τ is zero, it follows from Eq. 2 that α = 0, indicating that one of the principal stresses is in a direction parallel to the surface, and the other perpendicular to it, the latter being zero in the present case. Thus, along the top face, the nonzero stress parallel to the beam axis is f_c, and along the bottom face, it is f_t. On the other hand, a condition of 'pure shear' occurs for elements located at the neutral axis (where τ is maximum and f_x = 0), whereby $f_t = f_c = \tau_{max}$ and α = 45°. The stress pattern is indicated in Fig. 2.7(d), which depicts the principal stress trajectories in the beam.

$$f_{1,2} = \frac{1}{2} f_x \pm \sqrt{\left(\frac{1}{2} f_x\right)^2 + \tau^2} \qquad 1$$

$$\tan\, 2\alpha = \frac{2\tau}{f_x} \qquad 2$$

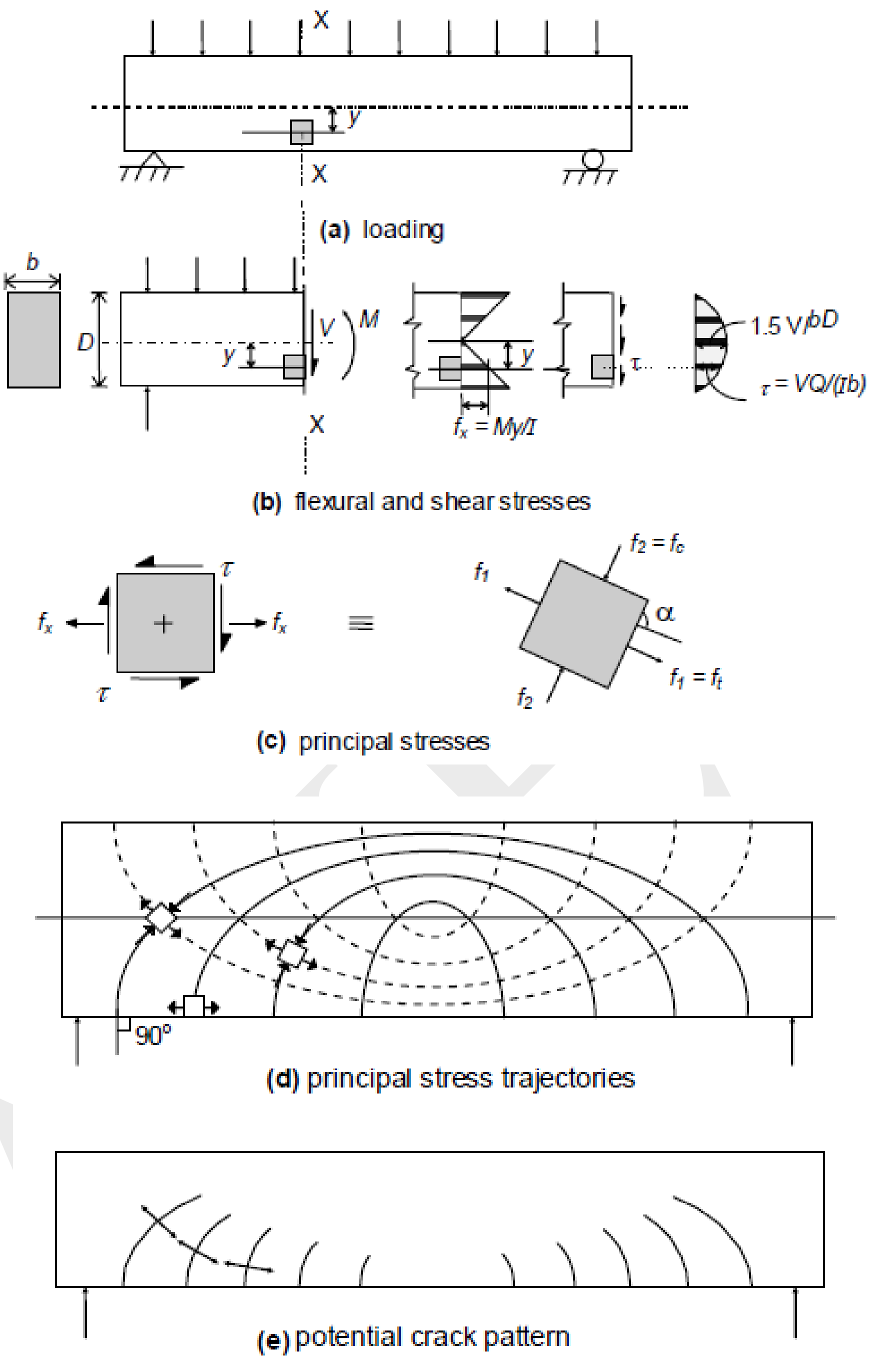


Fig 2.7 Stress distribution in homogeneous beams of rectangular section

In reinforced concrete beams of usual proportions, subjected to relatively high flexural stresses f_x and low shear stresses τ, the maximum principal tensile stress is invariably given by the flexural stress f_x max in the outer fibre (bottom face of the beam in Fig. 2.7) at the peak moment locations; the resulting cracks are termed flexural cracks [Fig. 2.8(a)]. These are controlled by the tension bars. On the other hand, in short−span beams which are relatively deep and have thin webs (as in I-sections) and are subjected to high shear stresses τ (due to heavy concentrated loads) and relatively low flexural stresses f_x, it is likely that the maximum principal tensile stress is located at the neutral axis level at an inclination $\alpha = 45^o$ (to the longitudinal axis of the beam); the resulting cracks (which generally occur near the supports, where shear force is maximum) are termed web shear cracks or diagonal tension cracks [Fig. 2.8(b)].

The so-called 'diagonal tension cracks' can be expected to occur in reinforced concrete beams in general, and appropriate shear reinforcement is required to prevent the propagation of these cracks. When a 'flexural crack' occurs in combination with a 'diagonal tension crack' (as is usually the case), the crack is sometimes referred to as a flexure-shear crack [Fig. 2.8(c)]. In such a case, it is the flexural crack that usually forms first, and due to the increased shear stresses at the tip of the crack, this flexural crack extends into a diagonal tension crack.

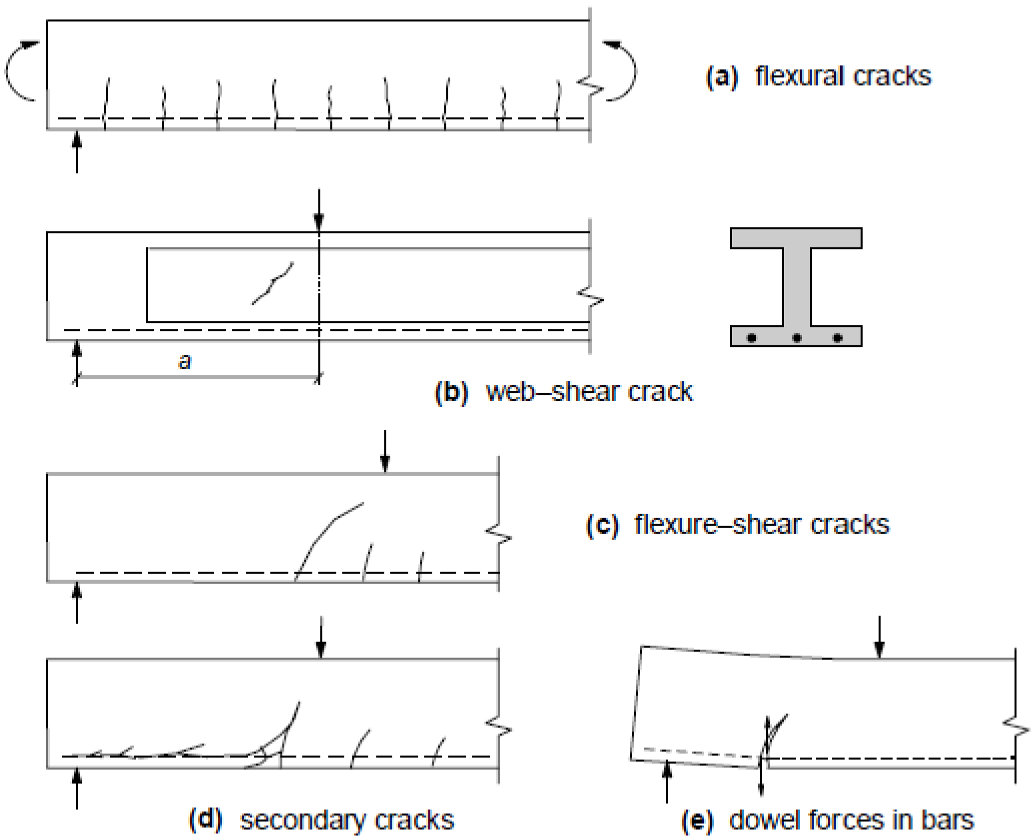


Fig. 2.8 Modes of cracking

Sometimes, the inclined crack propagates along the tension reinforcement towards the support [Fig. 2.8(d)]. Such cracks are referred to as secondary cracks or splitting cracks.

These are attributed to the wedging action of the tension bar deformations and to the transverse 'dowel forces' introduced by the tension bars functioning as dowels across the crack, resisting relative transverse displacements between the two segments of the beam (dowel action) [Fig 2.8(e)].

Shear Transfer Mechanisms

There are several mechanisms by which shear are transmitted between two adjacent planes in a reinforced concrete beam. The prominent among these are identified in Fig. 2.9, which shows the free body of one segment of a reinforced concrete beam separated by a flexure-shear crack.

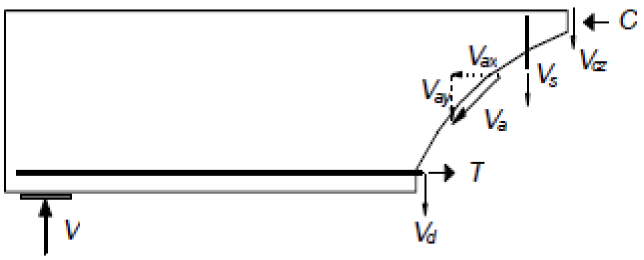


Fig. 2.9 Internal forces acting at a flexural-shear crack

The transverse (external) shear force is denoted as V (and has a maximum value near the support, equal to the support reaction). It is resisted by various mechanisms, the major ones [Fig. 2.9] being:

1. shear resistance V_{cz} of the uncracked portion of concrete;
2. vertical component V_{ay} of the 'interface shear' (*aggregate interlock*) force V_a;
3. dowel force V_d in the tension reinforcement (due to dowel action); and
4. shear resistance V_s carried by the shear (transverse) reinforcement, if any.

The interface shear V_a is a tangential force transmitted along the inclined plane of the crack, resulting from the friction against relative slip between the interlocking surfaces of the crack. Its contribution can be significant, if the crack-width is limited.

$$V = V_{cz} + V_{ay} + V_d + V_s$$

The relative contribution of the various mechanisms depends on the loading stage, the extent of cracking and the material and geometric properties of the beam. Prior to flexural cracking, the applied shear is resisted almost entirely by the uncracked V(V≈Vcz) concrete . At the commencement of flexural cracking, there is a redistribution of stresses, and some interface shear Va and dowel action Vd develop. At the stage of diagonal tension cracking, the shear reinforcement (hitherto practically unstressed) that intercepts the crack undergoes a sudden increase in tensile strain and stress. All the four major mechanisms are effective at this stage. The subsequent behaviour, including the failure mode and the ultimate strength in shear, depends on how the mechanisms of shear transfer break down and how successfully the shear resisting forces are redistributed.

Beams without Shear Reinforcement

In beams without shear reinforcement, the component V_s is absent altogether. Moreover, in the absence of *stirrups* enclosing the longitudinal bars, there is little restraint against splitting failure, and the dowel force V_d is small. Furthermore, the crack propagation is unrestrained, and hence, fairly rapid, resulting in a fall in the aggregate interface force V_a and also a reduction in the area of the uncracked concrete (in the limited compression zone) which contributes to V_{cz}.

Thus, in beams without shear reinforcement, the breakdown of any of the shear transfer mechanisms may cause immediate failure, as there is little scope for redistribution. Further, owing to the uncertainties associated with all the above effects, it is difficult to predict precisely the behaviour and the strength beyond the stage of diagonal cracking in beams without shear reinforcement.

The objective of shear design is to avoid premature brittle shear failures, such as those displayed by beams without web reinforcement, before the attainment of the full flexural strength. Members should be designed so that the shear capacity is high enough to ensure a ductile flexural failure.

Beams with Shear Reinforcement

In beams with moderate amounts of shear reinforcement, shear resistance continues to increase even after inclined cracking, until the shear reinforcement yields in tension, and the force V_s cannot exceed its ultimate value V_{us}. Any additional shear V has to be resisted by increments in V_{cz}, V_d and/or V_{ay}. With progressively widening crack-width (now accelerated by the yielding of shear reinforcement), V_{ay} decreases (instead of increasing), thereby forcing

V_{cz} and V_d to increase at a faster rate until either a splitting (dowel) failure occurs or the concrete in the compression zone gets crushed under the combined effects of flexural compressive stress and shear stress.

Owing to the pronounced yielding of the shear reinforcement, the failure of shear reinforced beams is gradual and ductile in nature – unlike beams without shear reinforcement, whose failure in shear is sudden and brittle in nature. However, if excessive shear reinforcement is provided, it is likely that the 'shear-compression' mode of failure will occur first, and this is undesirable, as such a failure will occur suddenly, without warning.

2.4.1 Types of Cracks

The types and formation of cracks depends on the span-to-depth ratio of the beam and loading. These variables influence the moment and shear along the length of the beam. For a simply supported beam under uniformly distributed load, without prestressing, three types of cracks are identified.

1) Flexural cracks: These cracks form at the bottom near the mid span and propagate upwards.

2) Web shear cracks: These cracks form near the neutral axis close to the support and propagate inclined to the beam axis.

3) Flexure shear cracks: These cracks form at the bottom due to flexure and propagate due to both flexure and shear.

In the following figure, the formation of cracks for a beam with large span-to-depth ratio and uniformly distributed loading is shown Fig 2.10.

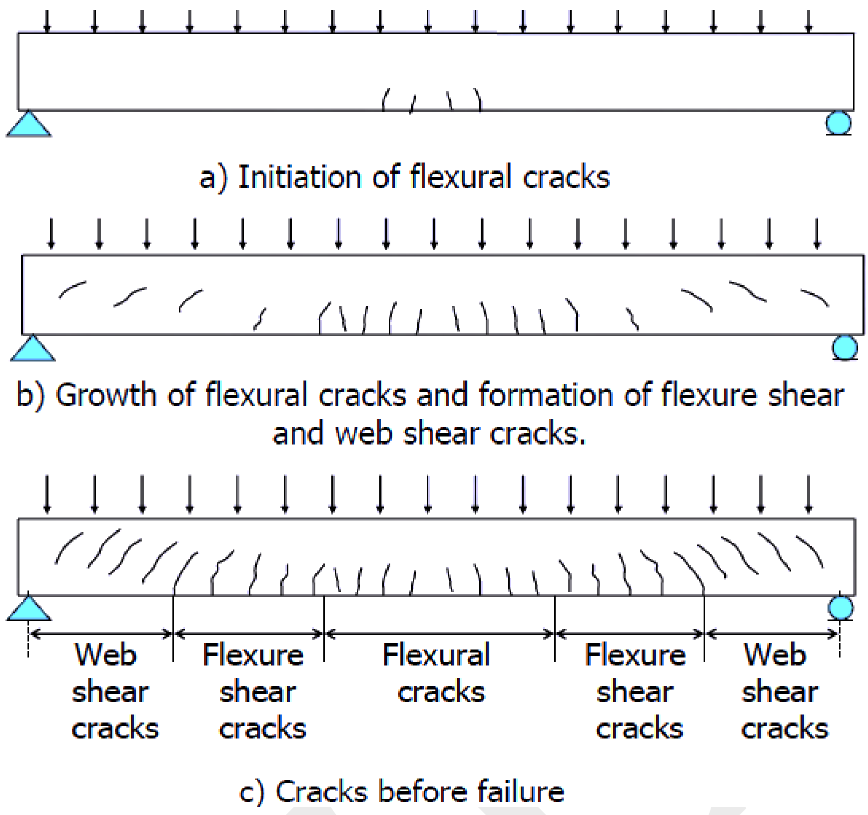


Figure 2.10 Formation of cracks in a reinforced concrete beam

2.4.5 Modes of Failure

For beams with low span-to-depth ratio or inadequate shear reinforcement, the failure can be due to shear. A failure due to shear is sudden as compared to a failure due to flexure. The following five modes of failure due to shear are identified.

1) Diagonal tension failure

2) Shear compression failure

3) Shear tension failure

4) Web crushing failure

5) Arch rib failure

The occurrence of a mode of failure depends on the span-to-depth ratio, loading, cross section of the beam, amount and anchorage of reinforcement.

Diagonal tension failure

In this mode, an inclined crack propagates rapidly due to inadequate shear reinforcement.

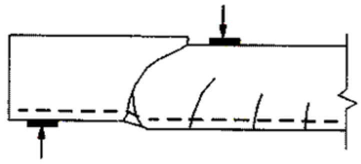


Figure 2.11 a Diagonal tension failure

Shear compression failure

There is crushing of the concrete near the compression flange above the tip of the inclined crack

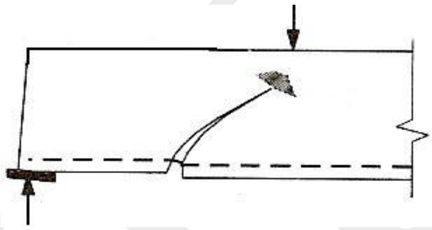


Figure 2.11 b Shear compression failure

Web crushing failure

The concrete in the web crushes due to inadequate web thickness

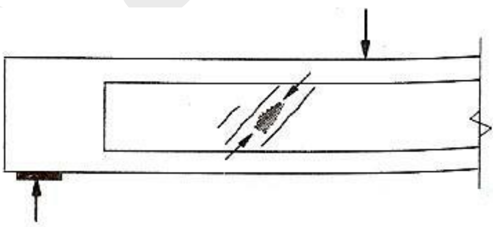


Figure 2.11 c Shear tension failure

Shear tension failure

Due to inadequate anchorage of the longitudinal bars, the diagonal cracks propagate horizontally along the bars

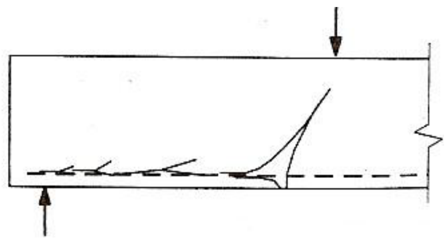


Figure 2.11 d Shear tension failure

Arch rib failure

For deep beams, the web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing

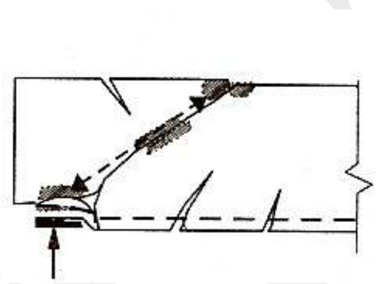


Figure 2.11 d Arch rib failure

Shear Stress

The distribution of shear stress in reinforced concrete rectangular, T and L-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):

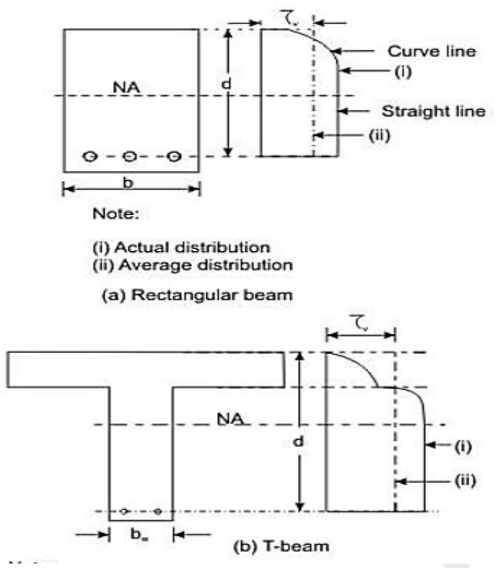


Figure 2.12 : Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs. 2.12 and b):

$$\lambda v = V_u / bd$$

where V_u= shear force due to design loads,

b = breadth of rectangular beams and breadth of the web bw for flanged beams, and

d = effective depth

(ii) In beams of varying depth:

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd}$$

where λv, V_u, b or b_w and d are the same as in (i),

Mu = bending moment at the section, and

β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment Mu decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment Mu increases numerically in the same direction as the effective depth increases.

2.4.6 Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value τ_{cmax} depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

2.4.6.1 Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete (f_{ck}) with a wide range of percentages of positive tensile steel reinforcement $p_t = 100A_{st}/(bd)$. The values of τ_c given in the Code (Table 19) are based on the following empirical formula.

$$\tau_c = 0.85\sqrt{(0.8 f_{ck})}\left(\sqrt{1+5\beta}-1\right)\Big/(6\beta)$$

$$\text{where} \qquad \beta \equiv \begin{cases} (0.8 f_{ck})/(6.89 p_t) \\ 1 \end{cases} \quad \text{whichever is greater}$$

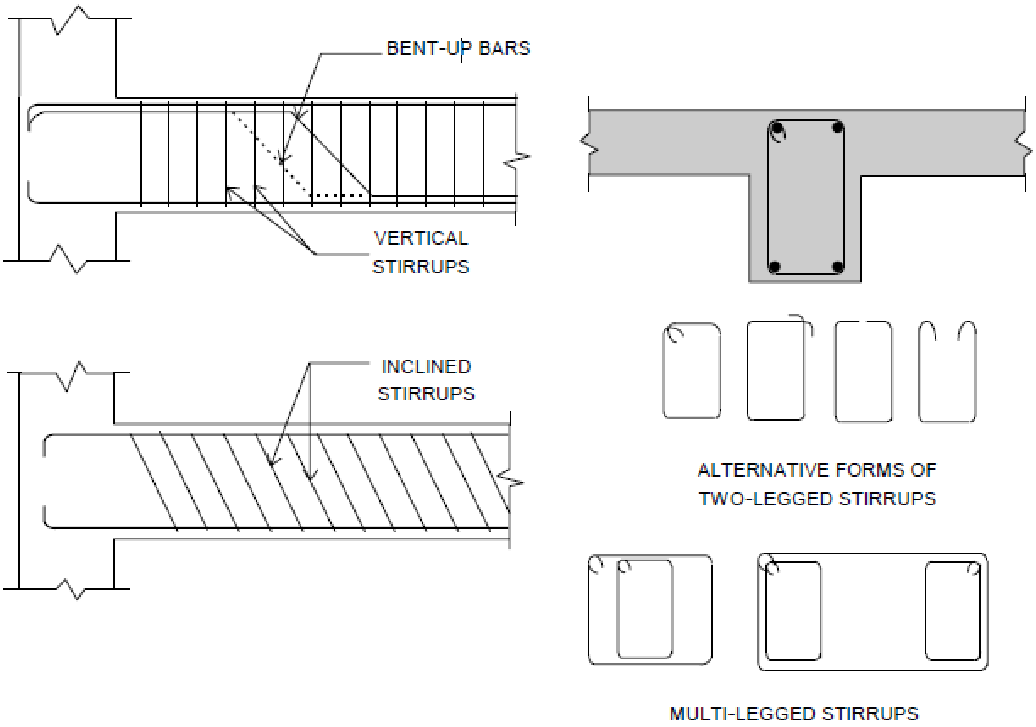
It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c .

2.4.6.2 Design shear strength with shear reinforcement

➢ **Types of Shear Reinforcement**

Shear reinforcement, also known as web reinforcement may consist of any one of the following systems (Cl. 40.4 of the Code)

- Stirrups perpendicular to the beam axis;
- Stirrups inclined (at 45° or more) to the beam axis; and
- Longitudinal bars bent-up (usually, not more than two at a time) at 45° to 60° to the beam axis, combined with stirrups.



2.11 Types of shear reinforcement

By far, the most common type of shear reinforcement is the two-legged stirrup, comprising a closed or open loop, with its ends anchored properly around longitudinal bars/stirrup holders (to develop the yield strength in tension). It is placed perpendicular to the member axis ('vertical† stirrup'), and may or may not be combined with bent-up bars, as shown in Fig. 2.11. The direction of bending up of the tension bar (or the direction of the inclined stirrup) should be such that it intercepts the potential inclined (diagonal tension) crack, nearly at right angles, thereby most effectively restraining the opening up and propagation of crack.

➢ **Factors Contributing to Ultimate Shear Resistance**

If V_{uc} and V_{us} denote respectively the ultimate shear resistance of the concrete and the shear reinforcement, then the total ultimate shear resistance V_{uR} at any section of the beams is given by

$$V_{uR} = V_{uc} + V_{us}$$

The shear resistance V_{uc} of concrete is made up of all the components V_{cz}, V_{ay} and V_{d}. V_{uc} is assumed to be constant, and obtainable from the design strength of concrete τ_c as

$$V_{uc} = \tau_c bd$$

V_{uc} denotes the shear resistance at the stage of initiation of diagonal cracking in flexural members, regardless of whether or not shear reinforcement is provided. From a design viewpoint, suitable shear reinforcement has to be designed if the factored shear V_u exceeds V_{uc} (i.e., τ_v exceeds τ_c), and the shear resistance required from the web reinforcement is given by

$$V_{us} \geq V_u - V_{uc} = (\tau_v - \tau_c)\, bd$$

Expressions for V_{us} for different cases are given as below:

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) Vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}(\sin\alpha + \cos\alpha)$$

(c) For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87 f_y A_{sv} d \sin\alpha$$

where A_{sv} = total cross-sectional area of stirrup legs or bent-up bars within a distance sv,

s_v = spacing of stirrups or bent-up bars along the length of the member,

λ_v = nominal shear stress,

λ_c = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web b_w,

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm^2,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45^o, and

d = effective depth.

The following two points are to be noted:

(i) The total shear resistance shall be computed as the sum of the resistance for the various types separately where more than one type of shear reinforcement is used.

(ii) The area of stirrups shall not be less than the minimum specified in cl. 26.5.1.6.

Maximum shear stress τ_{cmax} with shear reinforcement (cls. 40.2.3, 40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress of reinforced concrete in beams τ_{cmax} as given below in Table 6.2. Under no circumstances, the nominal shear stress in beams τ_v shall exceed τ_{cmax}

Code Recommendations

The Code (Cl. 26.5.1.5) also limits the value of the spacing sv to 0.75 d for 'vertical' stirrups and d for inclined stirrups with α = 45°. This is done to ensure that every potential diagonal crack is intercepted by at least one stirrup. Further, the Code specifies that "in no case shall the spacing exceed 300 mm".

For the purpose of design for a given factored shear force Vu, the web reinforcement is to be designed for a design shear force of (Vu − $\tau_c$bd), provided $\tau_v \leq, \tau_{cmax}$ (i.e., Vu < $V_{uR,lim}$).

Minimum Stirrup Reinforcement

The Code (Cl. 26.5.1.6) specifies a minimum shear reinforcement to be provided in the form of stirrups in all beams where the calculated nominal shear stress τv exceeds $0.5\tau_c$:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y} \quad \text{when } \tau_v > 0.5\ \tau_c$$

$$\Rightarrow s_v < \frac{2.175 f_y A_{sv}}{b}$$

The maximum spacing of stirrups should also comply with the requirements described earlier. For normal 'vertical' stirrups, the requirement is

$$s_v \leq \begin{cases} 0.75d \\ 300 \text{ mm} \end{cases} \text{whichever is less}$$

The Code objective in recommending such minimum shear reinforcement is to prevent the sudden formation of an inclined crack in an unreinforced (or very lightly reinforced) web, possibly leading to an abrupt failure.

2.4.7 Critical Section for Shear

The maximum shear force usually occurs in a flexural member at the face of the support, and progressively reduces with increasing distance from the support. When concentrated loads are involved, the shear force remains high in the span between the support and the first concentrated load.

When a support reaction introduces transverse compression in the end region of the member, the shear strength of this region is enhanced, and inclined cracks do not develop near the face of the support (which is usually the location of maximum shear). In such a case, the Code (Cl. 22.6.2.1) allows a section located at a distance d (effective depth) from the face of the support to be treated as the critical section. As the shear force at this critical section will be less than (or equal to) the value at the face of the support, the Code recommendation will usually result in a more favourable (less) value of τ_v than otherwise.

For beams generally subjected to uniformly distributed loads or where the principal load is located further than 2d from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in Figs. 2.12 a, b and c and are mentioned below.

However, when a heavy concentrated load is introduced within the distance 2d from the face of the support, then the face of the support becomes the critical section [Fig. 2.12(b)], as inclined cracks can develop within this region if the shear strength is exceeded. In such cases, closely spaced stirrups should be designed and provided in the region between the concentrated load and the support face.

Also, when the favourable effect of transverse compression from the reaction is absent - as in a suspended beam [Fig. 2.12(c)], or a beam (or bracket) connected to the side of another supporting beam [Fig. 2.12(d)] - the critical section for shear should be taken at the face of the support. In the latter case [Fig. 2.12(d)], special shear reinforcement detailing is called for — to ensure that effective shear transfer takes place between the supported beam .

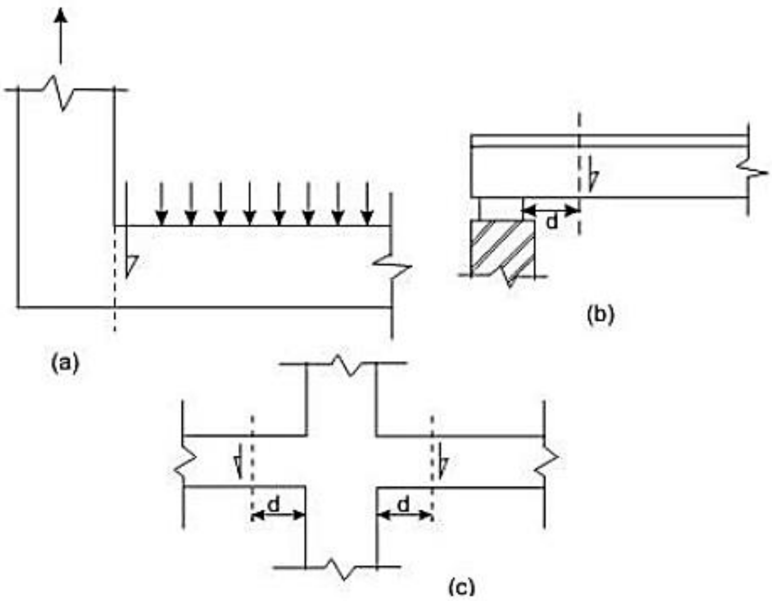


Fig 2.12 Support condition for locating factored shear force

Numerical Problem

1. Examine the following rectangular beam section for their shear. B=250mm, s=500mm, P_t=1.25, Vu=200kN, M20 concrete & Fe 415 steel

Step 1: Check for shear stress

Nominal Shear Stress τ_v = Vu/ bd = (200 x 10^3)/ (250x500) = 1.6 N/mm^2

From table 19, p=73, t_c = 0.67N/mm^2.

From table 20, p=73, t_{cmax} = 2.8N/mm^2.

$\tau_c < \tau_v < \tau_{cmax}$.

The depth is satisfactory & shear reinforcement is required.

2. Examine the shear strength of T-beam with following data: flange width = 2000mm. Thickness of flange = 150mm, overall depth = 750mm, effective cover = 50mm, longitudinal steel = 4 bars of 25mm dia, web width = 300mm simply supported span=6m, loading =50kN/m, UDL throughout span. Adopt M20 concrete & Fe 415 steel.

Step -1 Shear stress

V= 50 x 6 /2 = 150 KN

Vu= 1.5 X 150 = 225KN

τ_v = Vu/ bw d = 225 x 10^3 / (300 x 700)

Ast = 4 × 491 = 1964mm^2.

Pt == 0.93

From table 19, By interpolation

τc = 0.6N/mm^2.

From table 20, τcmax = 2.8

τc < τv < τcmax = 2.8

Design of shear reinforcement is required

IMPORTANT QUESTIONS

1. Define Singly reinforced beam and doubly reinforced beam. List the situation which requires the adoption of the same
2. A singly reinforced beam 250mmx450mm deep up to center of reinforcement Effective cover 50mm Effective span 6m using M20 concrete and Fe500 steel.

Determine the central point load that can be supported in addition to self-weight. When i)3-16mm dia bars ii) 3-20mm dia bars are used as reinforcement.

3. Determine the moment of resistance of a T-beam for the following data Breadth of the flange=740mm; Effective depth=400mm; Breadth of web=240mm; Tensile reinforcement =5-20Φ; Depth of flange =110mm: Adopt M20 grade concrete and Fe415 grade steel.
4. A doubly reinforced beam section is 250mm wide and 450mm deep to the center of the tensile reinforcement. It is reinforced with 2#16Φ as compression reinforcement at an effective cover of 50mm and 4#25Φ as tensile steel. Using M15 concrete and Fe250 steel. Calculate the ultimate moment of resistance of the beam section.
5. Define Doubly reinforced beam and doubly reinforced beam. List the situation which requires the adoption of the same.