

VISVESVARAYA TECHNOLOGICAL UNIVERSITY

BELGAUM



DESIGN OF RC STRUCTURAL ELEMENTS

(Subject Code: 21CV53)

LECTURE NOTES

(MODULE-3)

V-SEMESTER

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Module – 3

LIMIT STATE DESIGN OF BEAMS

Design problem is somewhat the reverse of the analysis problem. The external loads (or load effects), material properties and the skeletal dimensions of the beam are given, and it is required to arrive at suitable cross-sectional dimensions and details of the reinforcing steel, which would give adequate safety and serviceability. In designing for flexure, the distribution of bending moments along the length of the beam must be known from structural analysis. For this, the initial cross-sectional dimensions have to be assumed in order to estimate dead loads; this is also required for the analysis of indeterminate structures (such as continuous beams). The adequacy of the assumed dimensions should be verified and suitable changes made, if required.

A complete design of a beam involves considerations of safety under the ultimate limit states in flexure, shear, torsion and bond, as well as considerations of the serviceability limit states of deflection, crack-width, durability etc.

3.1 DESIGN TYPE OF PROBLEMS

The designer has to make preliminary plan lay out including location of the beam, its span and spacing, estimate the imposed and other loads from the given functional requirement of the structure. The dead loads of the beam are estimated assuming the dimensions b and d initially. The bending moment, shear force and axial thrust are determined after estimating the different loads. Therefore, the problem is such that the designer has to start with some initial dimensions and subsequently revise them, if needed. The following guidelines are helpful to assume the design parameters initially.

(i) Selection of breadth of the beam b

Normally, the breadth of the beam b is governed by: (i) proper housing of reinforcing bars and (ii) architectural considerations. It is desirable that the width of the beam should be less than or equal to the width of its supporting structure like column width, or width of the wall etc. Practical aspects should also be kept in mind. It has been found that most of the

requirements are satisfied with b as 150, 200, 230, 250 and 300 mm. Again, width to overall depth ratio is normally kept between 0.5 and 0.67.

(ii) Selection of depths of the beam d and D

The effective depth has the major role to play in satisfying (i) the strength requirements of bending moment and shear force, and (ii) deflection of the beam. The initial effective depth of the beam, however, is assumed to satisfy the deflection requirement depending on the span and type of the reinforcement. IS 456 stipulates the basic ratios of span to effective depth of beams for span up to 10 m as (Clause 23.2.1)

- Cantilever 7
- Simply supported 20
- Continuous 26

For spans above 10 m, the above values may be multiplied with $10/\text{span}$ in metres, except for cantilevers where the deflection calculations should be made. Further, these ratios are to be multiplied with the modification factor depending on reinforcement percentage and type. Figures 4 and 5 of IS 456 give the different values of modification factors. The total depth D can be determined by adding 40 to 80 mm to the effective depth.

(iii) Selection of the amount of steel reinforcement A_{st}

The amount of steel reinforcement should provide the required tensile force T to resist the factored moment M_u of the beam. Further, it should satisfy the minimum and maximum percentages of reinforcement requirements also. The minimum reinforcement A_s is provided for creep, shrinkage, thermal and other environmental requirements irrespective of the strength requirement. The minimum reinforcement A_s to be provided in a beam depends on the f_y of steel and it follows the relation: (cl. 26.5.1.1a of IS 456).

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

The maximum tension reinforcement should not exceed 0.04 bD (cl. 26.5.1.1b of IS 456), where D is the total depth.

Besides satisfying the minimum and maximum reinforcement, the amount of reinforcement of the singly reinforced beam should normally be 75 to 80% of $p_{t, \text{lim}}$. This will ensure that strain in steel will be more than $(0.87 f_y/E_s + 0.002)$ as the design stress in steel will be $0.87 f_y$. Moreover, in many cases, the depth required for deflection becomes more than the limiting depth required to resist $M_{u, \text{lim}}$. Thus, it is almost obligatory to provide more depth. Providing more depth also helps in the amount of the steel which is less than that required for $M_{u, \text{lim}}$. This helps to ensure ductile failure. Such beams are designated as under-reinforced beams.

(iv) Selection of diameters of bar of tension reinforcement

Reinforcement bars are available in different diameters such as 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36 and 40 mm. Some of these bars are less available. The selection of the diameter of bars depends on its availability, minimum stiffness to resist while persons walk over them during construction, bond requirement etc. Normally, the diameters of main tensile bars are chosen from 12, 16, 20, 22, 25 and 32 mm

(v) Selection of grade of concrete

Besides strength and deflection, durability is a major factor to decide on the grade of concrete. Table 5 of IS 456 recommends M 20 as the minimum grade under mild environmental exposure and other grades of concrete under different environmental exposures also.

(vi) Selection of grade of steel

Normally, Fe 250, 415 and 500 are in used in reinforced concrete work. Mild steel (Fe 250) is more ductile and is preferred for structures in earthquake zones or where there are possibilities of vibration, impact, blast etc.

(vii) Concrete Cover

Clear cover is the distance measured from the exposed concrete surface (without plaster and other finishes) to the nearest surface of the reinforcing bar. The Code (Cl. 26.4.1) defines the term nominal cover as “the design depth of concrete cover to all steel reinforcements, including links”. This cover is required to protect the reinforcing bars from corrosion and fire, and also to give the reinforcing bars sufficient embedment to enable them to be stressed without ‘slipping’ (losing bond with the concrete). As mentioned earlier, the recent revision in the Code with its emphasis on increased durability, has incorporated increased cover requirements, based on the severity of the environmental exposure conditions (refer Table 2.1). The ‘nominal cover’ to meet durability requirements, depending on exposure condition, are summarised in Table 3.1

Table 2.1 Nominal cover requirements based on exposure conditions

Exposure Condition	Minimum Grade	Nominal Cover (mm)	Allowance permitted
Mild	M 20	20	Can be reduced by 5mm for main rebars less than 12mm dia
Moderate	M 25	30	} Can be reduced by 5mm if concrete grade is M35 or higher
Severe	M 30	45	
Very severe	M 35	50	
Extreme	M 40	75	

3.2 Spacing of Reinforcing Bars

The Code specifies minimum and maximum limits for the spacing between parallel reinforcing bars in a layer. The minimum limits are necessary to ensure that the concrete can be placed easily in between and around the bars during the placement of fresh concrete. The maximum limits are specified for bars in tension for the purpose of controlling crack-widths and improving bond.

The requirements for placement of flexural reinforcement are described in Cl. 26.3 of the Code. The salient features of these specifications are summarised in Fig. 3.1. The requirements for singly reinforced beams and doubly reinforced beams are depicted in parts (a), and (c) respectively of Fig. 3.1.

In addition to the requirements indicated in Fig. 3.1, the Code specifies limits to the maximum spacing of tension reinforcing bars for crack control [refer Table 15 of the Code].

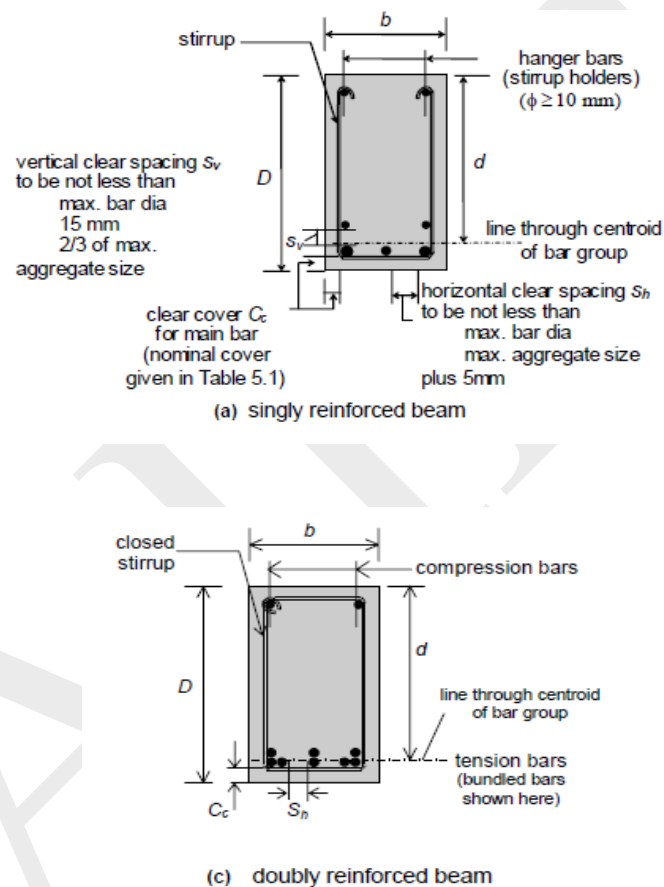


Fig. 3.1 Code requirements for flexural reinforcement placement

3.2.1 Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table 3 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam b_w

, and f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value. The minimum shear reinforcement is provided for the following:

- (i) Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.
- (ii) Brittle shear failure is arrested which would have occurred without shear reinforcement.
- (iii) Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.
- (iv) To hold the reinforcement in place when concrete is poured.
- (v) Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than $0.75 d$ for vertical stirrups and

d for inclined stirrups at 45° , where d is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

3.2 DESIGN OF SINGLY REINFORCED RECTANGULAR SECTIONS

The design problem is generally one of determining the cross-sectional dimensions of a beam, viz. b and D (including d), and the area of tension steel A_{st} required to resist a known factored moment M_u . The material properties f_{ck} and f_y are generally prescribed/selected on the basis of exposure conditions, availability and economy. For normal applications, Fe 415 grade steel is used, and either M 20 or M 25 grade concrete is used (for exposures rated 'severe', 'very severe' and 'extreme', the minimum concrete grades specified are M 30, M 35 and M 40 respectively).

The basic requirement for safety at the 'ultimate limit state of flexure' is that the factored moment M_u should not exceed the ultimate moment of resistance M_{uR} , and that the failure at the limit state should be ductile. Accordingly, the design equation for flexure is given by:

$$M_u \leq M_{uR} \quad \text{with } x_u \leq x_{u,max}$$

3.2.1 Procedure for Design in Singly Reinforced Section

Design procedure

The procedure for the design of beam may be summarized as follows:

1. Estimation of loads
2. Analysis
3. Design

1. Estimation of loads

The loads that get realized on the beams consists of the following:

- a. Self-weight of the beam.
- b. Weight of the wall constructed on the beam
- c. The portion of the slab loads which gets transferred to the beams. These slab loads are due to live loads that are acting on the slab dead loads such as self-weight of the slab, floor finishes, partitions, false ceiling and some special fixed loads

2. Analysis

For the loads that are acting on the beams, the analysis is done by any standard method to obtain the shear forces and bending moments

3. Design

- i. Selection of the Grades of steel and concrete
- ii. **Calculation of the Effective span L_{eff} :** Clause 22.2(a) of IS 456 recommends that the effective span is the lower of (i) clear span plus effective depth and (ii) centre to centre distance between two supports.
- iii. **Fix up dimension of the beam:** Fix up dimension of the beam using thumb rule guidance, which are based on controlling the deflections. If the same beam acts as rectangular section in some portion and flanged beam in some other section, overall dimension is to be based on consideration rectangular section and same is maintained throughout.

As a thumb rule Take the depth as (1/12) to (1/15)th span (higher value for heavier loads). Round it off to nearest multiple of 50 mm. (ii) Take breadth as (1/3) rd to (1/2) the depth, minimum being 200 mm Round it off to multiple 50mm. However, 230 mm is also permitted.

- iv. **Calculation of critical moments and shears.**

The moment and shear that exists at the critical sections are considered for the design. Critical sections are the sections where the values are maximum. Critical section for the moment in a simply supported beam is at the point where the shear force is zero. For continuous beams the critical section for the +ve bending moment is in the span and –ve bending moment is at the support. The critical section for the shear is at the support

- v. **Determine design moment and design shear.**
- vi. **Checking of effective depth d :** It is desirable to design the beam as under-reinforced so that the ductility is ensured with steel stress reaching the design value. Let us now determine the limiting effective depth when $x_u = x_{u, \text{max}}$ and the factored moment $M_u = M_{u, \text{lim}}$

$$M_{u, \text{lim}} = 0.36 \frac{x_{u, \text{max}}}{d} \left\{ 1 - 0.42 \frac{x_{u, \text{max}}}{d} \right\} b d^2 f_{ck}$$

Refer Annexure G of IS 456. For $X_{u, \text{max}}/d$ for various grade of steel Refer Section 38.1 Page No: 70. If the value of d assumed is less than the d value found from the above equation. Revise the depth of the section.

Considering the section to be nearly balanced section and using the equation Annexure G, IS 456-2000 obtain the value of the required depth d_{required} . If the assumed depth “d” is greater than the “ d_{required} ”, it satisfies the depth criteria based on flexure. If the assumed section is less than the “ d_{required} ”, revise the section.

vii. Area of Steel A_{st}

The area of steel is to be calculated from the moment equation (Annexure 1.1.(b) of IS 456), when steel is ensured to reach the design stress $f_d = 0.87 (415) = 361.05 \text{ N/mm}^2$.

$$M_u = 0.87 f_y A_{st} d \left\{ 1 - \frac{A_{st} f_y}{f_{ck} b d} \right\}$$

Here, all but A_{st} are known. However, this will give a quadratic equation of A_{st} and one of the values, the lower one, will be provided in the beam

viii. Design for shear consists of the following:

- Determine nominal shear $\lambda_v = V_u / bd$
- Find design shear strength of concrete λ_c using Table no. 4.1 (Table 19 in IS -456).
- If $\lambda_v < \lambda_c$ provide nominal shear reinforcement consisting of 2-legged 8 mm (or 6 mm) bars at not more than 0.75 d or 300 mm whichever is less.
- If $\lambda_c < \lambda_v < \lambda_c \text{ max}$, design shear reinforcement.
- If $\lambda_c > \lambda_c \text{ max}$, revise the section.

ix. Check for deflection.

x. Check for $A_{st \text{ min}}$, $A_{st \text{ max}}$ and distance between the two bars.

Reinforcement requirements

- Reinforcing steel of same type and grade shall be used as main reinforcement
- At least two bars should be used as tension steel and not more than six bars should be used in one layer in beam.
- The diameter of the bars should not be less than 10mm.
- Minimum reinforcement:**

The minimum area of tension reinforcement shall be not less than that given by the following:

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

where, A_s = minimum area of tension reinforcement.

b = breadth of beam or the breadth of the web of T-beam,

d = effective depth, and

f_y = characteristic strength of reinforcement in N/mm^2

- **Maximum reinforcement** – The maximum area of tension reinforcement shall not exceed $0.04bD$

Compression reinforcement:

The maximum area of compression reinforcement shall not exceed $0.04bD$. Compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint.

Shear Reinforcement

- This type of reinforcement shall be taken around the outermost tension and compression reinforcement.
- If no compression steel is used, two suspenders bars of minimum 10 mm dia should be used to support shear reinforcement.
- The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed $0.75d$ for vertical stirrups and d' for inclined stirrups at 45°. In no case the spacing shall not exceed 300mm.
- The Code (Cl. 26.5.1.6) specifies a minimum shear reinforcement to be provided in the form of stirrups in all beams where the calculated nominal shear stress τ_v exceeds $0.5\tau_c$:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87f_y} \quad \text{when } \tau_v > 0.5 \tau_c$$

$$\Rightarrow s_v < \frac{2.175f_y A_{sv}}{b}$$

- Shear reinforcement of diameter 6, 8, 10 or 12mm may be used. It may be mild steel or Fe415 grade used.
- Two-legged shear reinforcement are commonly used. If spacing comes out too close, leading to problem in concreting, shear reinforcement of 4 legged or 6 legged also may be used.

Side Face Reinforcement

- When the depth of the web of beam exceeds 750 mm side face reinforcement shall be provided along the two parts. The total area of reinforcement shall not exceed 0.1 percent of the web area and shall be distributed equally in two faces at spacing not exceeding 300mm or web thickness whichever is less.

Problems:

1. Design a singly reinforced SSB of clear span 5m to support a working live load of 25 kN/m run. Use Fe 415 steel and M 20 grad concrete. Assume the support thickness as 230 mm.

Step 1 (a): Fixing up the depth of the section.

Taking

$$L/d = 20, \text{ for SSB [Refer 23.2.1, pg 37]}$$

$$d = L/20 = 5/20 = 0.25 \text{ m} = 250 \text{ mm}$$

$$\text{Providing a cover of 25 mm, overall depth } D = 250 + 25 = 275 \text{ mm}$$

Dimensions of the section.

$$\text{Width } b = 230 \text{ mm}$$

$$\text{depth } d = 250 \text{ mm}$$

Step 1 (b): Check for lateral stability/lateral buckling

Refer page 39, clause 23.3

$$\text{Allowable } l = 60b \text{ or } \frac{250 b^2}{d}$$

$$\text{Allowable } l = 60b = 13800 \text{ mm} = 13.8 \text{ m}$$

$$\text{Or } \frac{250 b^2}{d} = 52900 \text{ mm} = 52.9 \text{ m}$$

$$\text{Allowable } l = \text{Lesser of the two values} = 13.8 \text{ m}$$

Actual l of the beam (5m) < Allowable value of l . Hence ok

Step 2: Effective span

Referring class 22.2 page 34,

$$\text{Effective span } l_e = \text{clear span} + d$$

$$\text{Or } l_e = \text{clear span} + (1/2) \text{ support thickness} + (1/2) \text{ support thickness}$$

$$= \text{clear span} + t_s/2 + t_s/2$$

Whichever is lesser.

$$l_e = 5000 + 250 \text{ mm} = 5250 \text{ mm}$$

$$\text{Or } l_e = 5000 + (230/2) + (230/2) = 5230 \text{ mm}$$

Therefore $l_e = 5230 \text{ mm}$

Step 3: Calculation of loads:

Consider 1m length of the beam

a. Dead load = $(0.23 \times 0.275 \times 1 \text{ m} \times 25 \text{ kN/m}^3) = 1.58 \text{ kN/m}$

b. Live load = 25 kN/m

Total working load $w = 26.58 \text{ kN/m}$

Factored load = 26.58×1.5

$W_u = 39.87 \text{ kN/m} \approx 40 \text{ kN/m}$

$$\text{Factored moment } M_u = \frac{W_u l_e^2}{8} = \frac{40 \times 5.23^2}{8} = 136.76 \text{ kN-m}$$

$$\text{Factored shear} = \frac{40 \times 5.23}{2} = 104.6 \text{ kN}$$

Step 4: Check for depth based on flexure or bending moment consideration

Assuming the section to be nearly balanced, and equating M_u to $M_{u\text{lim}}$,

$$M_u = M_{u\text{lim}} = 136.76 \text{ kN-m}$$

Using the equation G 1.1 (c), Annexure G IS 456-2000

$$M_{u\text{lim}} = 0.36 \frac{x_{u\text{max}}}{d} \left(1 - 0.42 \frac{x_{u\text{max}}}{d} \right) b d^2 f_{ck}$$

$$136.76 \times 10^6 = 0.36 \times 0.48 (1 - 0.42 \times 0.48) 230 \times d^2 \times 20$$

$$d = 464.21 \text{ mm}$$

Assumed depth d is less than the required depth of 464mm. Hence revise the section

Assume

$$d = 500 \text{ mm}$$

$$b = 230 \text{ mm}$$

Loads:

Dead load = $0.23 \times 0.525 \times 1 \times 25 = 2.875 \text{ kN/m}$

Live load = 25 kN/m

Total working load = 27.875 kN/m

Factored load $W_u = 27.875 \times 1.5 = 41.8 \approx 42 \text{ kN/m}$

$$\text{Factored moment } M_u = \frac{Wu l e^2}{8} = \frac{42 \times 5.23^2}{8} = 143.6 \text{ kN-m}$$

$$\text{Factored shear } V_u = \frac{Wu l e}{2} = \frac{42 \times 5.23}{2} = 109.83 \text{ kN}$$

Check for depth based on flexure

$$M_u = M_{u \text{ lim}} = 143.6 \text{ kN-m}$$

Using the equation G 1.1 (c), Annexure G IS 456-2000

$$M_{u \text{ lim}} = 0.36 \frac{x_{u \text{ max}}}{d} \left(1 - 0.42 \frac{x_{u \text{ max}}}{d} \right) b d^2 f_{ck}$$

$$143.6 \times 10^6 = 0.36 \times 0.48 (1 - 0.42 \times 0.48) 230 \times d^2 \times 20$$

$$d = 475.68 \text{ mm}$$

Assumed depth is greater than the required depth of 475.68 mm

Required 'd' = 476 mm and Assumed 'd' = 500 mm

Hence OK

Therefore, we shall continue with d = 500 mm and D = 525 mm

Check whether the section is under reinforced

Actual moment acting $M_u = 143.6 \text{ kN-m}$

Using equation G 1.1 (c)

$$M_{u \text{ lim}} = 0.36 \frac{x_{u \text{ max}}}{d} \left(1 - 0.42 \frac{x_{u \text{ max}}}{d} \right) b d^2 f_{ck}$$

$$\begin{aligned} M_{u \text{ lim}} &= 0.36 \times 0.48 (1 - 0.42 \times 0.48) 230 \times 500^2 \times 20 \\ &= 158.66 \text{ kN-m} \end{aligned}$$

$$M_u < M_{u \text{ lim}}$$

Hence the section is under reinforced

Step 5: Calculation of steel:

Since the section is under reinforced, we have,

Using equation G 1.1 (b)

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$143.6 \times 10^6 = 0.87 \times 415 \times A_{st} \times 500 \left(1 - \frac{A_{st} \times 415}{230 \times 500 \times 20} \right)$$

Solving the quadratic equation, $A_{st} = 960.33 \text{ mm}^2 \approx 960 \text{ mm}^2$

Choosing 8 mm diameter bars,

$$\text{Area of 1 bar} = \left(\frac{\pi}{4}\right) \times 8^2 = 50.265 \text{ mm}^2$$

Therefore, number of bars of 8mm required = $19.10 = 20$ bars

Distance between any two bars

Minimum distance between two bars is greater of the following:

a. Size of the aggregate + 5 mm

$$20 \text{ mm} + 5 \text{ mm}$$

b. Size of the bar (whichever is greater)

Therefore, minimum distance = 25 mm

$$\text{Distance between bars} = \left(\frac{230 - 2 \times 25 - 2 \times 8}{19}\right) = 8.63 \text{ mm}$$

$8.63 < 25$. Therefore 8 mm dia bars cannot be provided.

Let us choose 16 mm dia bars.

$$\text{Area of 1 bar} = \left(\frac{\pi}{4}\right) \times 16^2 = 201.06 \text{ mm}^2$$

Therefore, number of bars of 16 mm required = $4.77 = 5$ bars

$$\text{Distance between bars} = \left(\frac{230 - 2 \times 25 - 2 \times 8 - 5 \times 16}{4}\right) = 21 \text{ mm}$$

Minimum distance required = 25 mm

Therefore 16 mm dia cannot be used.

Let us choose 25 mm dia bars.

$$\text{Area of 1 bar} = \left(\frac{\pi}{4}\right) \times 25^2 = 490.890 \text{ mm}^2$$

Therefore, number of bars of 25mm required = $1.95 = 2$ bars

$$\text{Distance between the bars} = \left(\frac{230 - 2 \times 25 - 2 \times 8 - 2 \times 25}{1}\right) = 114 \text{ mm}$$

Check for $A_{st \text{ min}}$

$$A_{st \min} = \frac{0.85bd}{0.87f_y}$$

$$A_{st \min} = \frac{0.85 \times 230 \times 500}{0.87 \times 415} = 270.7 \text{ mm}^2$$

Check for $A_{st \max}$

$$A_{st \max} = 0.04 \times b \times D = 4830 \text{ mm}^2$$

$$A_{st \text{ provided}} = 982 \text{ mm}^2$$

$$A_{st \min} < A_{st} < A_{st \max}$$

Hence ok.

Check for shear

Factored load $W_u = 42 \text{ kN/m}$

$$\text{Support reaction } V_u = \frac{Wul}{2} = \frac{42 \times 5}{2} = 105 \text{ kN}$$

$$V_u = 105 \text{ kN}$$

$$\tau_v = \frac{v_u}{bd} = 0.913 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{bd} = \frac{100 \times 982}{230 \times 500} = 0.8539$$

From table 19, IS 456-2000 page 73

$$\tau_c = 0.58 \text{ N/mm}^2$$

From table 20, IS 456-2000 page 73

$$\tau_{c \max} = 2.8 \text{ N/mm}^2$$

$$\tau_c < \tau_v < \tau_{c \max}$$

Hence design of shear reinforcement is required

Selecting 2 leg vertical stirrups of 8 mm diameter, Fe 415 steel,

$$A_{sv} = 2 \times \left(\frac{\pi}{4}\right) \times 8^2 = 100 \text{ mm}^2$$

V_c = Shear force taken up by the concrete

$$= \frac{\tau_c bd}{1000} = \frac{0.58 \times 230 \times 500}{1000} = 66.7 \text{ kN}$$

$$V_u = 105 \text{ kN}$$

$$V_{us} = V_u - V_c$$

$$= 105 - 66.7$$

$$= 38.3 \text{ kN}$$

$$V_{us} = \frac{0.87 \times f_y \times A_{sv} \times d}{S_v} \text{ from clause 40.4}$$

$$38.3 \times 10^3 = \frac{0.87 \times 415 \times 100 \times 500}{S_v}$$

$$S_v = 471.3 \text{ mm}$$

Check for maximum spacing

Maximum spacing = 0.75d or 300mm whichever is lesser

Maximum spacing = 375 or 300mm

Therefore, maximum spacing allowed = 300mm

Let us provide 8 mm dia 2-leg vertical stirrups at a spacing of 300 mm.

Check for A_{sv} min:

$$A_{sv \text{ provided}} = 100 \text{ mm}^2$$

$$A_{sv \text{ min}} = \frac{0.4 b S_v}{0.87 \times f_y} = \frac{0.4 \times 230 \times 300}{0.87 \times 415} = 76.44 \text{ mm}^2$$

$$A_{sv \text{ provided}} > A_{sv \text{ min}}$$

Hence ok.

Check for deflection:

$$\text{Allowable } \frac{l}{d} = \text{Basic } \frac{l}{d} \times M_t \times M_c \times M_f$$

Basic (l/d) = 20 as the beam is simply supported

To determine M_t

$$f_s = 0.58 \times f_y \times \frac{\text{Area of } A_{st \text{ required}}}{A_{st \text{ provided}}}$$

$$f_s = 0.58 \times 415 \times (960/982) = 235.3 \text{ N/mm}^2$$

from fig 4, $M_t = 1$

To determine M_c

From fig 5, $M_c = 1$ [since there is no compression reinforcement]

To determine $M_f = b_w / b_f = 1$ [since it is rectangular section $b_w = b_f$]

Therefore, allowable $l/d = 20 \times 1 \times 1 \times 1 = 20$

Actual $l/d = (5230/500) = 10.46 < \text{Allowable } l/d.$

Hence ok.

3.3 DESIGN OF DOUBLY REINFORCED BEAM

In the design type of problems, the given data are b , d , D , grades of concrete and steel. The designer has to determine A_{sc} and A_{st} of the beam from the given factored moment.

Step 1: To determine $M_{u,lim}$ and $A_{st,lim}$ from Eqs. 1 and 2, respectively.

$$M_{u,lim} = 0.36 \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d} \right) f_{ck} b d^2 \quad \text{Eq (1)}$$

$$A_{st1} = p_{t,lim} \frac{b d}{100} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})} \quad \text{Eq (2)}$$

Step 2: To determine M_{u2} , A_{sc} , A_{st2} and A_{st} from Eqs. 3, 4, 5 and 6, respectively.

$$M_u = M_{u,lim} + M_{u2} \quad \text{Eq (3)}$$

$$M_{u,lim} = 0.87 A_{st,lim} f_y (d - 0.416 x_{u,max}) \quad \text{Eq (4)}$$

$$A_{sc} (f_{sc} - f_{cc}) = A_{st2} (0.87 f_y) \quad \text{Eq (5)}$$

$$A_{st} = A_{st1} + A_{st2} \quad \text{Eq (6)}$$

Step 3: To select the number and diameter of bars from known values of A_{sc} and A_{st} .

(b) Use of SP table: Tables 45 to 56 present the p_t and p_c of doubly reinforced sections for $d'/d = 0.05, 0.10, 0.15$ and 0.2 for different f_{ck} and f_y values against M_u/bd^2 . The values of p_t and p_c are obtained directly selecting the proper table with known values of M_u/bd^2 and d'/d .

Numerical Problem

1. Design a reinforced concrete beam of rectangular section using the following data:

Effective span = 5 m

Dept. of Civil Engineering

Width of beam = 250 mm

Overall depth = 500 mm

Service load (DL+LL) = 40 kN/m

Effective cover = 50 mm

Materials: M20 grade concrete and Fe 415 steel

a. Data

$b = 250$ mm, $f_{ck} = 20$ N/mm², $D = 500$ mm, $f_y = 415$ N/mm², $d = 450$ mm, $E_s = 2 \times 10^5$ N/mm²

$d' = 50$ mm, $l_e = 5$ m, $w = 40$ kN/m and $W_u = 40 \times 1.5 = 60$ kN/m

b. Ultimate moments and shear forces

$M_u = W_u \times l_e^2 / 8 = 60 \times 5^2 / 8 = 187.5$ kN-m

$V_u = \text{Factored shear} = \frac{v_u l_e}{2} = 150$ kN

c. Determination of M_{ulim} and f_{sc}

$$M_{ulim} = 0.36 \frac{x_{umax}}{d} \left(1 - 0.42 \frac{x_{umax}}{d} \right) b d^2 f_{ck}$$

$M_{ulim} = 0.36 \times 0.48 (1 - 0.42 \times 0.48) \times 250 \times 450^2 \times 20 = 140$ kN-m

Since $M_u > M_{ulim}$, design a doubly reinforced section

$(M_u - M_{ulim}) = 187.5 - 140 = 47.5$ kN-m

$$f_{sc} = \epsilon_{sc} \times E_s$$

$$\text{where, } \epsilon_{sc} = 0.0035 \left(\frac{(0.48 \times 450) - 50}{(0.48 \times 450)} \right)$$

$$f_{sc} = 0.0035 \left(\frac{(0.48 \times 450) - 50}{(0.48 \times 450)} \right) 2 \times 10^5 = 538 \text{ N/mm}^2$$

But $f_{sc} \not\geq 0.87 f_y = (0.87 \times 415) = 361$ N/mm²

Therefore $f_{sc} = 361$ N/mm²

$$\text{steel } A_{sc} = \left[\frac{(M_u - M_{ulim})}{f_{sc} (d - d')} \right]$$

$$= \frac{47.5 \times 10^6}{361 \times 400}$$

$$= 329 \text{ mm}^2$$

Provide 2 bars of 16mm diameter ($A_{sc} = 402 \text{ mm}^2$)

$$A_{st2} = \left(\frac{A_{sc} f_{sc}}{0.87 f_y} \right)$$

$$= \frac{329 \times 361}{0.87 \times 415} = 329 \text{ mm}^2$$

$$A_{st1} = \left[\frac{0.36 f_{ck} b x_u \text{lim}}{0.87 f_y} \right]$$

$$= \frac{0.36 \times 20 \times 250 \times 0.48 \times 450}{0.87 \times 415} = 1077 \text{ mm}^2$$

$$\text{Total tension reinforcement } A_{st} = (A_{st1} + A_{st2})$$

$$= (1077 + 329)$$

$$= 1406 \text{ mm}^2$$

Provide 3 bars of 25mm diameter ($A_{st} = 1473 \text{ mm}^2$)

d. Shear reinforcements

$$\tau_v = V_u / bd = (150 \times 10^3) / (250 \times 450) = 1.33 \text{ N/mm}^2$$

$$P_t = \frac{(100 A_s)}{bd}$$

$$= \frac{100 \times 1473}{250 \times 450}$$

$$= 1.3$$

Referring table 19 of IS: 456 – 2000,

$$\tau_c = 0.68 \text{ N/mm}^2$$

$\tau_{c \text{ max}} = 2.8 \text{ N/mm}^2$ for M20 concrete from table 20 of IS 456-2000

Since $\tau_c < \tau_v < \tau_{c \text{ max}}$, shear reinforcements are required.

$$V_{us} = [V_u - (\tau_c bd)]$$

$$V_{us} = [V_u - (\tau_c bd/1000)]$$

$$= [150 - (0.68 \times 250 \times 450/1000)]$$

$$= 73.5 \text{ kN}$$

Using 8 mm diameter 2 legged stirrups,

$$S_v = \frac{0.87 \times f_y \times A_{sv} \times d}{V_{us}}$$

$$S_v = \frac{0.87 \times 415 \times 2 \times 50 \times 450}{73.5 \times 10^3} = 221 \text{ mm}$$

Maximum spacing is 0.75d or 300 mm whichever is less

$$S_v > 0.75d > 0.75 \times 450 = 337.5 \text{ mm}$$

Adopt a spacing of 200 mm near supports gradually increasing to 300 mm towards the center of the span.

e. Check for deflection control

$$(l/d)_{\text{actual}} = (5000/450) = 11.1$$

$$(l/d)_{\text{allowable}} = [(l/d)_{\text{basic}} \times M_t \times M_c \times M_f]$$

$$P_t = 1.3 \text{ and } P_c = [(100 \times 402) / (250 \times 450)] = 0.35$$

Refer Fig 4, $M_t = 0.93$

Fig 5, $M_c = 1.10$

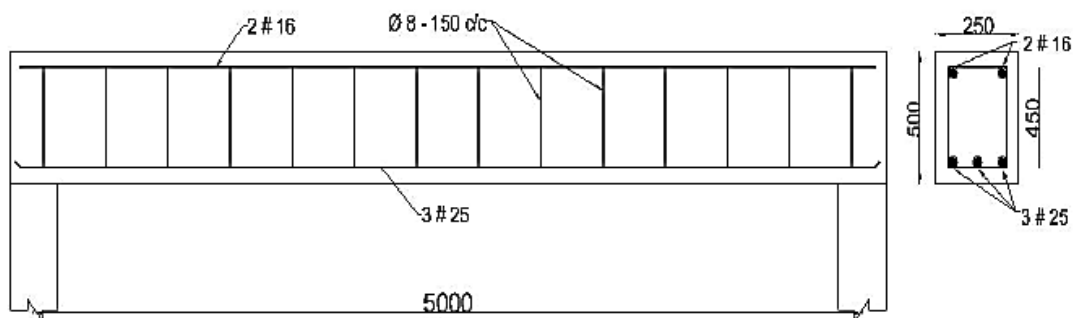
Fig 6, $M_f = 1.0$

$$(l/d)_{\text{allowable}} = [(20 \times 0.93 \times 1.10 \times 1)] = 20.46$$

$$(l/d)_{\text{actual}} < (l/d)_{\text{allowable}}$$

Hence deflection control is satisfied.

f. Reinforcement details



3.4 DESIGN PROCEDURE FOR FLANGED BEAM

In the case of a continuous flanged beam, the negative moment at the face of the support generally exceeds the maximum positive moment (at or near the midspan), and hence governs the proportioning of the beam cross-section. In such cases of negative moment, if the slab is located on top of the beam (as is usually the case), the flange is under flexural tension and hence the concrete in the flange is rendered ineffective. The beam section at the support is therefore to be designed as a rectangular section for the factored negative moment. Towards

the midspan of the beam, however, the beam behaves as a proper flanged beam (with the flange under flexural compression). As the width of the web b_w and the overall depth D are already fixed from design considerations at the support, all that remains to be determined is the area of reinforcing steel; the effective width of flange is determined as suggested by the Code. The determination of the actual reinforcement in a flanged beam depends on the location of the neutral axis x_u , which, of course, should be limited to $x_{u,max}$. If M_u exceeds $M_{u,lim}$ for a singly reinforced flange section, the depth of the section should be suitably increased; otherwise, a doubly reinforced section is to be designed.

3.4.1 Neutral Axis within Flange ($x_u \leq D_f$):

This is, by far, the most common situation encountered in building design. Because of the very large compressive concrete area contributed by the flange in T-beam and L-beams of usual proportions, the neutral axis lies within the flange ($x_u \leq D_f$), whereby the section behaves like a rectangular section having width b_f and effective depth d . A simple way of first checking $x_u \leq D_f$ is by verifying $M_u \leq M_{uR, x_u = D_f}$, where ($M_{uR, x_u = D_f}$) is the limiting ultimate moment of resistance for the condition $x_u = D_f$ and is given by

$$(M_{uR})_{x_u = D_f} = 0.361 f_{ck} b_f D_f (d - 0.416 D_f) \quad \text{Eq (7)}$$

It may be noted that the above equation is meaningful only if $X_{u,max} \geq D_f$. In rare situations involving very thick flanges and relatively shallow beams $X_{u,max}$, may be less than D_f . In such cases, $M_{u,lim}$ is obtained by substituting $x_{u,max}$ in place of D_f in Eq. (7).

3.4.2 Neutral Axis within Web ($x_u > D_f$):

When ($M_u > M_{uR, x_u = D_f}$), it follows that $x_u > D_f$. The accurate determination of x_u can be laborious. The contributions of the compressive forces C_{uw} and C_{uf} in the 'web' and 'flange' may be accounted for separately as follows:

$$M_{uR} = C_{uw} (d - 0.416 x_u) + C_{uf} (d - y_f / 2)$$

$$C_{uw} = 0.361 f_{ck} b_w x_u$$

$$C_{uf} = 0.447 f_{ck} (b_f - b_w) y_f$$

and the equivalent flange thickness y_f is equal to or less than D_f depending on whether x_u exceeds $7D_f/3$ or not.

For $x_{u,max} \geq 7D_f/3$, the value of the ultimate moment of resistance corresponding to and may be first computed. If the factored moment $M_{uR_{xu}} = D_f$, it follows that and. Otherwise, for and $x_u = 7D_f/3$ and $y_f = D_f$ may be first computed. If the factored moment $M_u > M_{uR_{xu}} = 7D_f/3$ it follows that $x_u = 7D_f/3$ and $y_f = D_f$. Otherwise $D_f < x_u > 7D_f/3$ for $M_{uR_{xu}} = D_f < M_u < M_{uR_{xu}} = 7D_f/3$

$$y_f = 0.15x_u + 0.65D_f$$

Inserting the appropriate value — D_f or the expression for y_f in Eq. (11), in Eq. (8), the resulting quadratic equation (in terms of the unknown x_u) can be solved to yield the correct value of x_u . Corresponding to this value of x_u , the values of C_{uw} and C_{uf} can be computed [Eq. (9), (10)] and the required A_{st} obtained by solving the force equilibrium equation.

$$T_u = 0.87 f_f A_{st} = C_{uw} + C_{uf}$$

$$\Rightarrow (A_{st})_{required} = \frac{C_{uw} + C_{uf}}{0.87 f_y}$$

Numerical Problem

A continuous T-beam has the cross-sectional dimensions shown in figure below. The web dimensions have been determined from the consideration of negative moment at support and shear strength requirements. The span is 10 m and the design moment at midspan under factored loads is 800 kNm. Determine the flexural reinforcement requirement at midspan. Consider Fe 415 steel. Assume that the beam is subjected to moderate exposure conditions.

Solution:

Determining approximate A_{st}

Effective flange width b_f

Actual flange width provided = 1500mm; $D_f=100$ mm; $b_w=300$ mm

Maximum width permitted = $(0.7 \times 10000)/6 + 300 + (6 \times 100) = 2067$ mm > 1500 mm

Therefore, $b_f = 1500$ mm

Assuming $d=650$ mm and a lever arm z equal to larger of $0.9d = 585$ mm

And $d - D_f/2 = 600$ mm i.e., $z = 600$ mm

$$A_{st \text{ required}} = \frac{800 \times 10^6}{0.87 \times 415 \times 600} = 3693 \text{ mm}^2$$

- Providing 4 bars, $\phi_{reqd} = \sqrt{\frac{3693/4}{\pi/4}} = 34.3 \text{ mm}$, i.e., 36 mm.

As 4–36 ϕ bars cannot be accommodated in one layer within the width $b_w = 300$ mm, two layers are required.

Assuming a reduced $d \approx 625$ mm, $z \approx 625 - 100/2 = 575$ mm.

$$\Rightarrow (A_{st})_{reqd} \approx 3693 \times \frac{600}{575} = 3854 \text{ mm}^2.$$

- Provide 5–32 ϕ [$A_{st} = 804 \times 5 = 4020 \text{ mm}^2$] with 3 bars in the lower layer plus 2 bars in the upper layer, with a clear vertical separation of 32 mm — as shown in Fig. 5.11(b). Assuming 8 mm stirrups and a clear 32 mm cover to stirrups,

$$\begin{aligned} \Rightarrow d &= 700 - 32 - 8 - \frac{1}{5} [(3 \times 16) + 2 \times (32 + 32 + 16)] \\ &= 700 - 40 - 41.6 = 618 \text{ mm} \end{aligned}$$

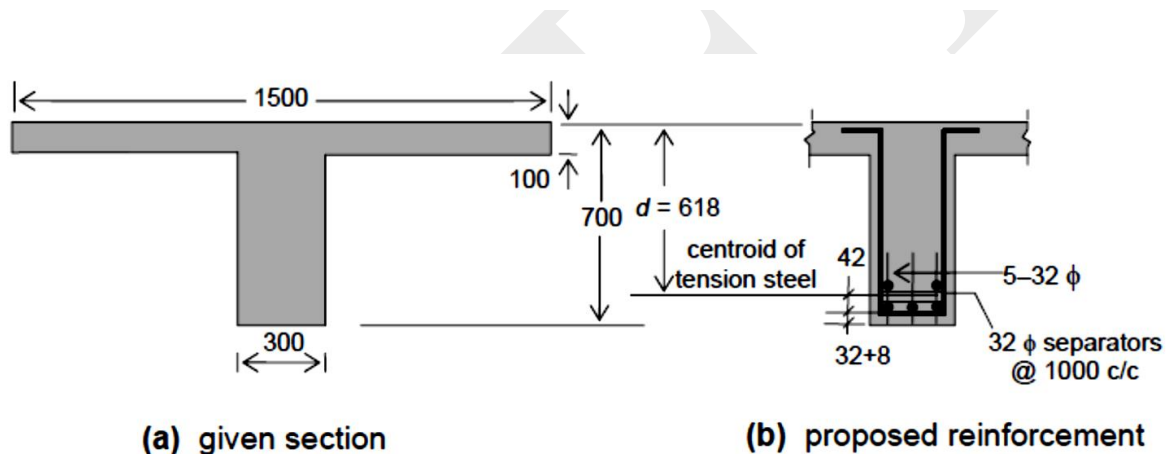


Figure: Reinforcement of T-beam of Example Problem

Determining actual A_{st}

$$x_{u,max} = 0.479 \times 618 = 296 \text{ mm}$$

As $> D_f = 100 \text{ mm}$, the condition $x_u = D_f$ satisfies, $\max x_u < x_{u,max}$

- Assuming M 25 concrete, $f_{ck} = 25 \text{ MPa}$

$$M_{uR_{xu=D_f}} = 0.362 \times 25 \times 1500 \times 100 \times (618 - 0.416 \times 100)$$

$$= 782.5 \times 10^6 \text{ Nmm} < M_u = 800 \text{ kNm}$$

$$x_u > D_f \text{ and } M_u = C_{uw} (d - 0.416 x_u) + C_{uf} (d - y_f/2)$$

where $C_{uw} = 0.362 f_{ck} b_w u = 0.362 \times 25 \times 300 x_u = (2715 x_u) \text{ N}$

$$\text{and } C_{uf} = 0.447f_c k (b_f - b_w) y_f = 0.447 \times 25 \times (1500 - 300) y_f = (13410 y_f)$$

$$\text{Considering } x_u = 7D_f/3 = 233 \text{ mm } (< x_{u,max} = 296 \text{ mm}), y_f = D_f = 100 \text{ mm}$$

$$M_{uR_{xu=7D_f/3}} = (2715 \times 233) (618 - 0.416 \times 233) + (13410 \times 100) \times (618 - 100/2) \\ = 1091.3 \times 10^6 \text{ Nmm} > M_u = 800 \text{ KNm}$$

$$\text{Evidently } D_f < x_u < (7/3) D_f \text{ for which } y_f = 0.15x_u + 0.65D_f$$

$$C_{uf} = 13410(0.15x_u + 0.65 \times 100) = (2011.5x_u + 871650) \text{ N}$$

$$M_u = 800 \times 10^6 = (2715x_u) (618 - 0.416x_u) + (2011.5x_u + 871650) \times [618 - (0.15x_u + 65)/2] \\ = -1280.3x_u^2 + 2790229.5 x_u + 510.35 \times 10^6$$

Solving this quadratic equation,

$$x_u = 109.3 \text{ mm} < x_{u,max} = 296 \text{ mm}$$

$$\Rightarrow y_f = 0.15x_u + 65 = 81.4 \text{ mm}$$

$$\text{Applying } T_u = 0.87f_y A_{st} = C_{uw} + C_{uf}$$

$$(A_{st})_{required} = \frac{(2715 \times 109.3) + (13410 \times 81.4)}{0.87 \times 415} = 3845 \text{ mm}^2$$

The reinforcement (5-32 Φ ; $A_{st} = 4020 \text{ mm}^2$, based on appropriate estimate of A_{st} [Fig.] is evidently adequate and appropriate

3.5 Design for combined bending and torsion as per IS-456

On several situations beams and slabs are subjected to torsion in addition to bending moment and shear force. Loads acting normal to the plane of bending will cause bending moment and shear force. However, loads away from the plane of bending will induce torsional moment along with bending moment and shear. Space frames (Fig.3.6.1a), inverted L-beams as in supporting sunshades and canopies (Fig.3.6.1b), beams curved in plan (Fig.3.6.1c), edge beams of slabs (Fig.3.6.1d) are some of the examples where torsional moments are also present.

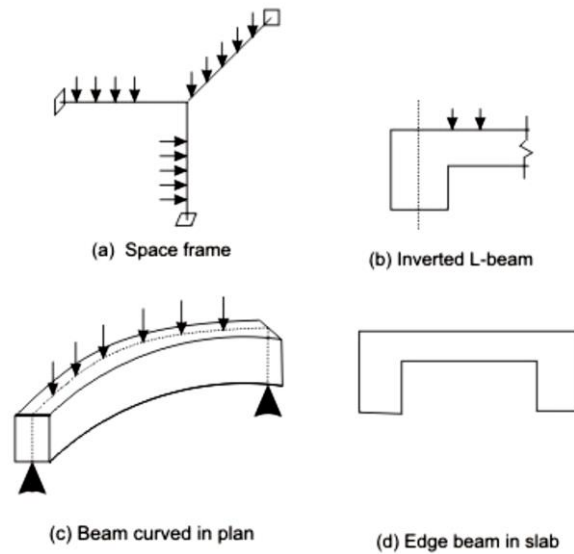


Fig 3.6.1 Beams under combined bending, shear and torsion

Clause 41 of IS 456 stipulates the above stating that, "In structures, where torsion is required to maintain equilibrium, members shall be designed for torsion in accordance with 41.2, 41.3 and 41.4. However, for such indeterminate structures where torsion can be eliminated by releasing redundant restraints, no specific design for torsion is necessary, provided torsional stiffness is neglected in the calculation of internal forces. Adequate control of any torsional cracking is provided by the shear reinforcement as per cl. 40".

3.5.1 Approach of Design for Combined Bending, Shear and Torsion as per IS 456

As per the stipulations of IS 456, the longitudinal and transverse reinforcements are determined taking into account the combined effects of bending moment, shear force and torsional moment. Two empirical relations of equivalent shear and equivalent bending moment are given. These fictitious shear forces and bending moment, designated as equivalent shear and equivalent bending moment, are separate functions of actual shear and torsion, and actual bending moment and torsion, respectively. The total vertical reinforcement is designed to resist the equivalent shear V_e and the longitudinal reinforcement is designed to resist the equivalent bending moment M_{e1} and M_{e2} . These design rules are applicable to beams of solid rectangular cross-section. However, they may be applied to flanged beams by substituting b_w for b . IS 456 further suggests to refer to specialist literature for the flanged beams as the design adopting the code procedure is generally conservative.

1 Critical Section (cl. 41.2 of IS 456)

As per cl. 41.2 of IS 456, sections located less than a distance d from the face of the support is to be designed for the same torsion as computed at a distance d , where d is the effective depth of the beam.

2 Shear and Torsion

- (a) The equivalent shear, a function of the actual shear and torsional moment is determined from the following empirical relation:

$$V_e = V_u + 1.6(T_u/b) \quad \text{Eq (12)}$$

where V_e = equivalent shear,

V_u = actual shear,

T_u = actual torsional moment,

b = breadth of beam.

- (b) The equivalent nominal shear stress τ_{ve} is determined from:

$$\tau_{ve} = (V_e / bd) \quad \text{Eq (13)}$$

However, τ_{ve} shall not exceed $\tau_{c,max}$ given in Table 20 of IS 456

- (c) Minimum shear reinforcement is to be provided as per cl. 26.5.1.6 of IS 456, if the equivalent nominal shear stress τ_{ve} obtained from Eq.13 does not exceed τ_c given in Table 19 of IS 456.
- (d) If τ_{ve} exceeds τ_c given in Table 19, both longitudinal and transverse reinforcement shall be provided in accordance with Cl 41.4.

3. Reinforcement in Members subjected to Torsion

- (a) Reinforcement for torsion shall consist of longitudinal and transverse reinforcement as mentioned
- (b) The longitudinal flexural tension reinforcement shall be determined to resist an equivalent bending moment M_{e1} as given below:

$$M_{e1} = M_u + M_t$$

where M_u = bending moment at the cross-section, and

$$M_t = (T_u/1.7) \{1 + (D/b)\} \dots\dots\dots (14)$$

where T_u = torsional moment,

D = overall depth of the beam, and

b = breadth of the beam.

(c) The longitudinal flexural compression reinforcement shall be provided if the numerical value of M_t as defined above in Eq.14 exceeds the numerical value of M_u . Such compression reinforcement should be able to resist an equivalent bending moment M_{e2} as given below:

$$M_{e2} = M_t - M_u \dots\dots\dots (15)$$

The M_{e2} will be considered as acting in the opposite sense to the moment M_u .

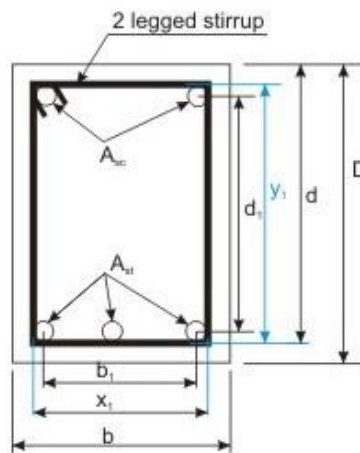


Fig 3.7 Stirrups in beams

(d) The transverse reinforcement consisting of two legged closed loops (Fig.3.7) enclosing the corner longitudinal bars shall be provided having an area of cross-section A_{sv} given below:

$$A_{sv} = \frac{T_u s_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y)} \quad (16)$$

However, the total transverse reinforcement shall not be less than the following:

$$A_{sv} \geq (\tau_{ve} - \tau_c) b s_v / (0.87 f_y)$$

where T_u = torsional moment,

V_u = shear force,

s_v = spacing of the stirrup reinforcement,

b_1 = centre to centre distance between corner bars in the direction of the width,

d_1 = centre to centre distance between corner bars

b = breadth of the member,

f_y = characteristic strength of the stirrup reinforcement,

τ_{ve} = equivalent shear stress as specified in Eqs.12 and 13, and

τ_c = shear strength of concrete as per Table 19 of IS 456

4. Requirements of Reinforcement

Beams subjected to bending moment, shear and torsional moment should satisfy the following requirements:

(a) *Tension reinforcement (cl. 26.5.1.1 of IS 456)*

The minimum area of tension reinforcement should be governed by

$$A_s / (bd) = 0.85 / f_y \quad (17)$$

where A_s = minimum area of tension reinforcement,

b = breadth of rectangular beam or breadth of web of T-beam,

d = effective depth of beam,

f_y = characteristic strength of reinforcement in N/mm^2 .

The maximum area of tension reinforcement shall not exceed $0.04 bD$, where D is the overall depth of the beam.

(b) *Compression reinforcement (cl. 26.5.1.2 of IS 456)*

The maximum area of compression reinforcement shall not exceed $0.04 bD$. They shall be enclosed by stirrups for effective lateral restraint.

(c) Side face reinforcement (cls. 26.5.1.3 and 26.5.1.7b)

Beams exceeding the depth of 750 mm and subjected to bending moment and shear shall have side face reinforcement. However, if the beams are having torsional moment also, the side face reinforcement shall be provided for the overall depth exceeding 450 mm. The total area of side face reinforcement shall be at least 0.1 per cent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness, whichever is less

(d) Transverse reinforcement (cl. 26.5.1.4 of IS 456)

The transverse reinforcement shall be placed around the outer-most tension and compression bars. They should pass around longitudinal bars located close to the outer face of the flange in T- and I-beams.

(e) Maximum spacing of shear reinforcement (cl. 26.5.1.5 of IS 456)

The centre to centre spacing of shear reinforcement shall not be more than $0.75 d$ for vertical stirrups and d for inclined stirrups at 45° , but not exceeding 300 mm, where d is the effective depth of the section.

(f) Minimum shear reinforcement (cl. 26.5.1.6 of IS 456)

(g) Distribution of torsion reinforcement (cl. 26.5.1.7 of IS 456)

The transverse reinforcement shall consist of rectangular close stirrups placed perpendicular to the axis of the member. The spacing of stirrups shall not be more than the least of x_1 , $(x_1 + y_1)/4$ and 300 mm, where x_1 and y_1 are the short and long dimensions of the stirrups (Fig.6.16.2).

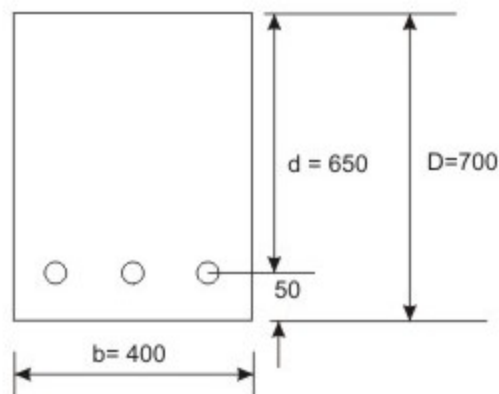
Longitudinal reinforcements should be placed as close as possible to the corners of the cross-section.

(h) Reinforcement in flanges of T- and L-beams (cl. 26.5.1.8 of IS 456)

For flanges in tension, a part of the main tensile reinforcement shall be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is smaller. For effective flange width greater than one-tenth of the span, nominal longitudinal reinforcement shall be provided to the outer portion of the flange.

Problem 1

Determine the reinforcement required of a ring beam (Fig.6.16.3) of $b = 400$ mm, $d = 650$ mm, $D = 700$ mm and subjected to factored $M_u = 200$ kNm, factored $T_u = 50$ kNm and factored $V_u = 100$ kN. Use M 20 and Fe 415 for the design.



Solution:

The solution of the problem is illustrated in seven steps below.

Step 1: Check for the depth of the beam

From Eq.6.22, we have the equivalent shear

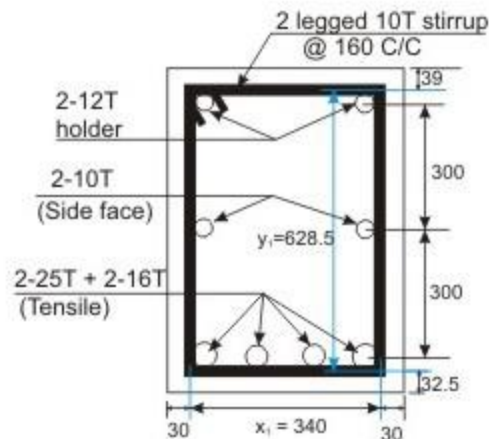
$$V_e = V_u + 1.6(T_u/b) = 100 + 1.6(50/0.4) = 300 \text{ kN}$$

$$\text{the equivalent shear stress } \tau_{ve} = V_e/bd = 300/(0.4)(0.65) = 1.154 \text{ N/mm}^2$$

(Table 20 of IS 456), $\tau_{cmax} = 2.8 \text{ N/mm}^2$. Hence, the section does not need any revision.

Step 2: Check if shear reinforcement shall be required.

Assuming percentage of tensile steel as 0.5, Table 6.1 of Lesson 13 (Table 19 of IS 456) gives $\tau_c = 0.48 \text{ N/mm}^2 < \tau_{ve} < \tau_{cmax}$. So, both longitudinal and transverse reinforcement shall be required.

Step 3: Longitudinal tension reinforcement

$$M_{e1} = M_u + M_t = M_u + (T_u/1.7) \{1 + (D/b)\}$$

$$= 200 + (50/1.7) \{1 + (700/400)\} = 200 + 80.88 = 280.88 \text{ kNm}$$

$$M_{e1}/bd_2 = (280.88) (106) / (400) (650) (650) = 1.66 \text{ N/mm}^2$$

From Table 2 of SP-16, corresponding to $M_u/bd_2 = 1.66 \text{ N/mm}^2$, we have by linear interpolation $p_t = 0.5156$. So,

$$A_{st} = 0.5156(400) (650)/100 = 1340.56 \text{ mm}^2.$$

Provide 2-25T and 2-16T = 981 + 402 = 1383 mm². This gives percentage of tensile reinforcement = 0.532, for which τ_c from Table 6.1 of Lesson 13 is 0.488 N/mm².

From Eq.6.29, minimum percentage of tension reinforcement = $(0.85/f_y)(100) = 0.205$ and, the maximum percentage of tension reinforcement is 4.0. So, 2-25T and 2-16T bars satisfy the requirements (Fig. 6.16.4).

Step 4: Longitudinal compression reinforcement

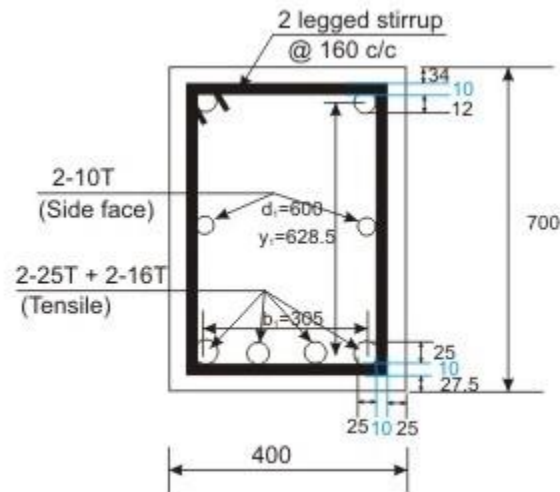
Here, in this problem, the numerical value of M_t (= 80.88 kNm) is less than that of M_u (200 kNm). So, longitudinal compression reinforcement shall not be required.

Step 5: Longitudinal side face reinforcement

Side face reinforcement shall be provided as the depth of the beam exceeds 450 mm. Providing 2-10 mm diameter bars (area = 157 mm²) at the mid-depth of the beam and one on

each face (Fig.6.16.4), the total area required as per sec.6.16.8c, $0.1(400) (300)/100 = 120 \text{ mm}^2 < 157 \text{ mm}^2$. Hence o.k.

Step 6: Transverse reinforcement



Providing two-legged, 10 mm diameter stirrups (area = 157 mm^2), we have

$$d_1 = 700 - 50 - 50 = 600 \text{ mm}$$

$$b_1 = 400 - 2(25 + 10 + 12.5) = 305 \text{ mm}$$

$$0.87 f_y A_{sv}/s_v = (T_u/b_1 d_1) + (V_u/2.5 d_1)$$

Using the numerical values of T_u , b_1 , d_1 and V_u , we have

$$0.87 f_y A_{sv}/s_v = 339.89 \text{ N/mm}$$

we have,

$$0.87 f_y A_{sv}/s_v \geq (1.154 - 0.48) 400 \geq 269.6 \text{ N/mm}$$

we get for 2 legged 10 mm stirrups ($A_{sv} = 157 \text{ mm}^2$),

$$s_v = 0.87(415) (157)/339.89 = 166.77 \text{ mm}$$

Step 7: Check for s_v

Figure shows the two-legged 10 mm diameter stirrups for which $x_1 = 340$ mm and $y_1 = 628.5$ mm. The maximum spacing s_v should be the least of x_1 , $(x_1 + y_1)/4$ and 300 mm.

Here, $x_1 = 340$ mm, $(x_1 + y_1)/4 = 242.12$ mm. So, provide 2 legged 10 mm T stirrups @ 160 mc/c

Important Question

1. A rectangular RC beam of size 250mm * 600 mm of effective simply supported span of 7 m has a support service load of 26.25 kN/m excluding self-weight. The effective cover = 50 mm. Design the beam for flexure and shear. Check the beam depth for control of deflection using empirical method. Design the stress value for different strain in steel is given below

Strain	Stress (N/mm ²)
0.00276	351.8
0.00380	360.9

2. A reinforced concrete beam is to be designed over an effective span of 5m to support a service load of 8 kN/m. Adopt M20 grade concrete and Fe 415 steel. Design a beam to satisfy the collapse and serviceability limit states.
3. A hall of 16 m * 6m supported by beams spaced 4 m C/C thickness of slab is 120 mm UDL 4 kN/m. Design a T beam using M20 Concrete and Fe 415 steel for flexure and shear. Take bearing as 500 mm. Also show check for deflection and bond
4. A doubly reinforced concrete beam 250 mm wide 500mm deep is required to support 40 kN/m including self-weight with effective span 5m. Effective cover of 50 mm using M20 concrete and Fe 415 steel find steel for flexure and shear