VISVESVARAYA TECHNOLOGICAL UNIVERSITY

BELGAUM

DESIGN OF RC STRUCTURAL ELEMENTS

(Subject Code: 21CV53)

LECTURE NOTES

(MODULE-4)

V-SEMESTER

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Module – 4
LIMIT STATE DESIGN OF SLABS

4.1Introduction

A slab is a flat two-dimensional planar structural element having thickness small compared to its other two dimensions. It provides a working flat surface or a covering shelter in buildings. It primarily transfers the load by bending in one or two directions. Reinforced concrete slabs are used in floors, roofs and walls of buildings and as the decks of bridges. The floor system of a structure can take many forms such as in situ solid slab, ribbed slab or pre-cast units. Slabs maybe supported on monolithic concrete beam, steel beams, walls or directly over the columns. Concrete slab behaves primarily as flexural members and the design is similar to that of beams.

4.2 CLASSIFICATION OF SLABS

Slabs are classified based on many aspects

1) **Based of shape:** Square, rectangular, circular and polygonal in shape.

2) **Based on type of support:** Slab supported on walls, Slab supported on beams, Slab supported on columns (Flat slabs).

3) **Based on support or boundary condition:** Simply supported, Cantilever slab, Overhanging slab, Fixed or Continues slab.

4) **Based on use:** Roof slab, Floor slab, Foundation slab, Water tank slab.

5) **Basis of cross section or sectional configuration:** Ribbed slab /Grid slab, Solid slab, Filler slab, Folded plate

6) **Basis of spanning directions:**

One-way slab – Spanning in one direction

Two-way slab _ Spanning in two directions

In general, rectangular one way and two-way slabs are very common and are discussed in detail.

4.2 One-way and Two-way Slabs

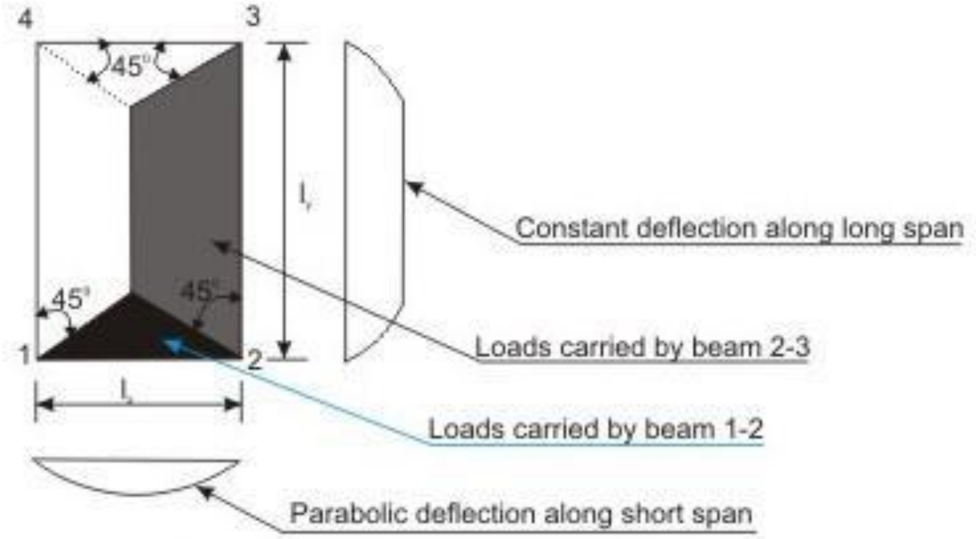

Fig 4.1 a One-way slab ($l_y/l_x > 2$) Gtg

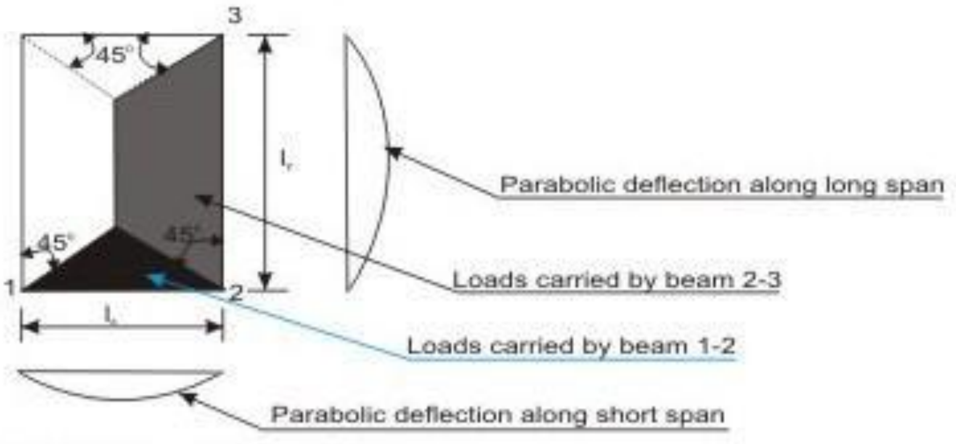

Fig. 8.18.4(b): Two-way slab ($l_y/l_x <= 2$)

Fig. 8.18.4: Sharing of loads

Figures 4.1a and b explain the share of loads on beams supporting solid slabs along four edges when vertical loads are uniformly distributed. It is evident from the figures that the share of loads on beams in two perpendicular directions depends upon the aspect ratio l_y / l_x of the slab, l_x being the shorter span. For large values of l_y, the triangular area is much less than the trapezoidal area (Fig.4.1a). Hence, the share of loads on beams along shorter span will gradually reduce with increasing ratio of l_y / l_x. In such cases, it may be said that the loads are primarily taken by beams along longer span. The deflection profiles of the slab along both directions are also shown in the figure. The deflection profile is found to be constant along the longer span except near the edges for the slab panel of Fig.4.1a. These slabs are designated as one-way slabs as they span in one direction (shorter one) only for a large part of the slab when $l_y / l_x > 2$. When a slab is supported only on two parallel apposite edges, it spans only in the direction perpendicular to two supporting edges. Such a slab is also called one-way slab

On the other hand, for square slabs of *ly /lx* = 1 and rectangular slabs of *ly /lx* up to 2, the deflection profiles in the two directions are parabolic (Fig.4.1b). Thus, they are spanning in two directions and these slabs with l_y /l_x up to 2 are designated as two-way slabs, when supported on all edges.

It would be noted that an entirely one-way slab would need lack of support on short edges. Also, even for $l_y /l_x < 2$, absence of supports in two parallel edges will render the slab one-way. In Fig. 4.1b, the separating line at 45 degree is tentative serving purpose of design. Actually, this angle is a function of l_y /l_x.

4.3 Design Shear Strength of Concrete in Slabs

Experimental tests confirmed that the shear strength of solid slabs up to a depth of 300 mm is comparatively more than those of depth greater than 300 mm. Accordingly, cl.40.2.1.1 of IS 456 stipulates the values of a factor *k* to be multiplied with τ_c given in Table 19 of IS 456 for different overall depths of slab. Table 4.1 presents the values of *k* as a ready reference below:

Table 4.1 Values of the multiplying factor *k*

Overall depth of slab (mm)	300 or more	275	250	225	200	175	150 or less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

Thin slabs, therefore, have more shear strength than that of thicker slabs. It is the normal practice to choose the depth of the slabs so that the concrete can resist the shear without any stirrups for slab subjected to uniformly distributed loads. However, for deck slabs, culverts, bridges and fly over, shear reinforcement should be provided as the loads are heavily concentrated in those slabs. Though, the selection of depth should be made for normal floor and roof slabs to avoid stirrups, it is essential that the depth is checked for the shear for these slabs taking due consideration of enhanced shear strength as discussed above depending on the overall depth of the slabs.

4.4 ONE WAY CONTINUOUS SLAB

The slabs spanning in one direction and continuous over supports are called one-way continuous slabs. These are idealized as continuous beam of unit width. For slabs of uniform section which support substantially UDL over three or more spans which do not differ by more than 15% of the longest, the B.M and S.F are obtained using the coefficients available in Table 12 and Table 13 of IS 456-2000. For moments at supports where two unequal spans meet or in case where the slabs are not equally loaded, the average of the two values for the negative moments at supports may be taken. Alternatively, the moments may be obtained by moment distribution or any other methods.

For slabs built into a masonry wall developing only partial restraint, the negative moment at the face of the support should be taken as *Wl*/24, where *W* is the total design loads on unit width and *l* is the effective span. The shear coefficients, given in Table 13 of IS 456, in such a situation, may be increased by 0.05 at the end support as per cl.22.5.2 of IS 456.

Table 12 Bending Moment Coefficients
(*Clause* 22.5.1)

Type of Load	Span Moments		Support Moments	
	Near Middle of End Span	At Middle of Interior Span	At Support Next to the End Support	At Other Interior Supports
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$

NOTE — For obtaining the bending moment, the coefficient shall be multiplied by the total design load and effective span.

Table 13 Shear for Coefficients
(*Clauses* 22.5.1 *and* 22.5.2)

Type of Load	At End Support	At Support Next to the End Support		At All Other Interior Supports
		Outer Side	Inner Side	
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	0.4	0.6	0.55	0.5
Imposed load (not fixed)	0.45	0.6	0.6	0.6

NOTE — For obtaining the shear force, the coefficient shall be multiplied by the total design load.

4.4.5 Design Considerations

The primary design considerations of both one and two-way slabs are strength and deflection. The depth of the slab and areas of steel reinforcement are to be determined from these two aspects. The detailed procedure of design of one-way slab is taken up in the next section. However, the following aspects are to be decided first.

(a) Effective span (cl.22.2 of IS 456)

The effective span of a slab depends on the boundary condition. Table 4.2 gives the guidelines stipulated in cl.22.2 of IS 456 to determine the effective span of a slab.

Table 4.2 Effective span of slab (cl.22.2 of IS 456)

Sl.No.	Support condition	Effective span
1	Simply supported not built integrally with its supports	Lesser of (i) clear span + effective depth of slab, and (ii) centre to centre of supports
2	Continuous when the width of the support is < 1/12th of clear span	Lesser of (i) clear span + effective depth of slab, and (ii) centre to centre of supports.
3	Continuous when the width of the support is > lesser of 1/12th of clear span or 600 mm (i) for end span with one end fixed and the other end continuous or for intermediate spans, (ii) for end span with one end free and the other end continuous, (iii) spans with roller or rocker bearings.	(i) Clear span between the supports (ii) Lesser of (a) clear span + half the effective depth of slab, and (b) clear span + half the width of the discontinuous support (iii) The distance between the centres of bearings
4	Cantilever slab at the end of a continuous slab	Length up to the centre of support
5	Cantilever span	Length up to the face of the support + half the effective depth
6	Frames	Centre to centre distance

(b) Effective span to effective depth ratio (cls.23.2.1a-e of IS 456)

The deflection of the slab can be kept under control if the ratios of effective span to effective depth of one-way slabs are taken up from the provisions in cl.23.2.1a-e of IS 456. These stipulations are for the beams and are also applicable for one-way slabs as they are designed considering them as beam of unit width.

(c) Nominal cover (cl.26.4 of IS 456)

The nominal cover to be provided depends upon durability and fire resistance requirements. Table 16 and 16A of IS 456 provide the respective values. Appropriate value of the nominal cover is to be provided from these tables for the particular requirement of the structure.

(d) Minimum reinforcement (cl.26.5.2.1 of IS 456)

Both for one and two-way slabs, the amount of minimum reinforcement in either direction shall not be less than 0.15 and 0.12 per cents of the total cross-sectional area for mild steel (Fe 250) and high strength deformed bars (Fe 415 and Fe 500)/welded wire fabric, respectively.

(e) Maximum diameter of reinforcing bars (cl.26.5.2.2)

The maximum diameter of reinforcing bars of one and two-way slabs shall not exceed one-eighth of the total depth of the slab.

(f) Maximum distance between bars (cl.26.3.3 of IS 456)

The maximum horizontal distance between parallel main reinforcing bars shall be the lesser of (i) three times the effective depth, or (ii) 300 mm. However, the same for secondary/distribution bars for temperature, shrinkage etc. shall be the lesser of (i) five times the effective depth, or (ii) 450 mm.

Depth of slab:
The depth of slab depends on bending moment and deflection criterion. the trail depth can be obtained using:

- Effective depth d= Span /((l/d) Basic x modification factor)
- For obtaining modification factor, the percentage of steel for slab can be assumed from 0.2 to 0.5%
- The effective depth d of two-way slabs can also be assumed using cl.24.1, IS 456 provided short span is _ 3.5m and loading class is < 3.5KN/m^2

Type of support	Fe-250	Fe-415
Simply supported	l/35	l/28
continuous	l/40	l/32

OR

The following thumb rules can be used

• One-way slab d=(l/22) to (l/28).

• Two way simply supported slab d=(l/20) to (l/30)

• Two-way restrained slab d=(l/30) to (l/32)

4.6 Design of One-way Slabs

The procedure of the design of one-way slab is the same as that of beams. However, the amounts of reinforcing bars are for one metre width of the slab as to be determined from either the governing design moments (positive or negative) or from the requirement of minimum reinforcement. The different steps of the design are explained below.

Step 1: Selection of preliminary depth of slab

The depth of the slab shall be assumed from the span to effective depth ratios

Step 2: Design loads, bending moments and shear forces

The total factored (design) loads are to be determined adding the estimated dead load of the slab, load of the floor finish, given or assumed live loads etc. after multiplying each of them with the respective partial safety factors. Thereafter, the design positive and negative bending moments and shear forces are to be determined using the respective coefficients given in Tables 12 and 13 of IS 456

Step 3: Determination/checking of the effective and total depths of slabs

The effective depth of the slab shall be determined employing Eq.in Annexure G section c of IS 456.

The total depth of the slab shall then be determined adding appropriate nominal cover (Table 16 and 16A of cl.26.4 of IS 456) and half of the diameter of the larger bar if the bars are of different sizes. Normally, the computed depth of the slab comes out to be much less than the assumed depth in Step 1. However, final selection of the depth shall be done after checking the depth for shear force.

Step 4: Determination of areas of steel

Area of steel reinforcement along the direction of one-way slab should be determined employing Eq.1.2 of sec.3.5.5 of Lesson 5 and given below as a ready reference.

$$M_u = 0.87\, f_y\, A_{st}\, d\, \{1 - (A_{st})(f_y)/(f_{ck})(bd)\} \qquad \ldots.\ (1.23)$$

The above equation is applicable as the slab in most of the cases is under-reinforced due to the selection of depth larger than the computed value in Step 3. The area of steel so determined should be checked whether it is at least the minimum area of steel as mentioned in cl.26.5.2.1 of IS 456.

The amount of steel reinforcement along the large span shall be the minimum amount of steel as per cl.26.5.2.1 of IS 456.

Step 5: Check for shear in 1000mm strip

The design shear strength of concrete is found by considering the percentage of steel. The value of τ_c shall be modified after knowing the multiplying factor *k* from cl.40.2.1.1 of IS 456.

Step 6: Selection of diameters and spacings of reinforcing bars (cls.26.5.2.2 and 26.3.3 of IS 456)

The diameter and spacing of bars are to be determined as per cls.26.5.2.2 and 26.3.3 of IS 456.

4.7 Detailing of Reinforcement

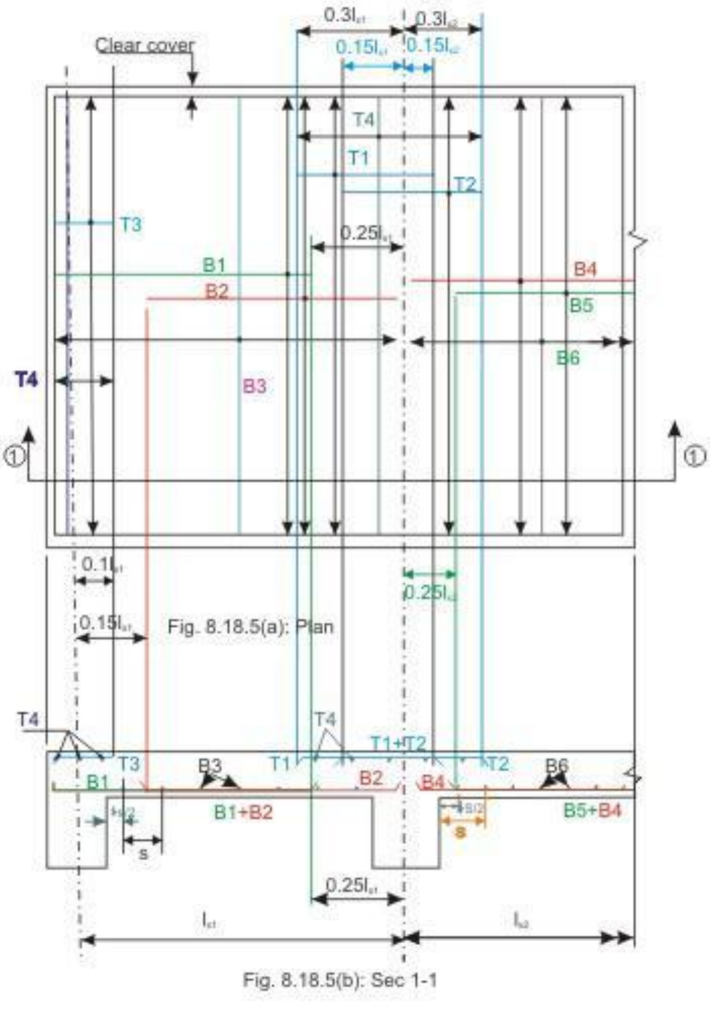

Fig 4.2 Reinforcement of one-way slab

Figures 4.2 a and b present the plan and section 1-1 of one-way continuous slab showing the different reinforcing bars in the discontinuous and continuous ends (DEP and CEP, respectively) of end panel and continuous end of adjacent panel (CAP). The end panel has three bottom bars B1, B2 and B3 and four top bars T1, T2, T3 and T4. Only three bottom bars B4, B5 and B6 are shown in the adjacent panel. Table 8.3 presents these bars mentioning the respective zone of their placement (DEP/CEP/CAP), direction of the bars (along x or y) and the resisting moment for which they shall be designed or if to be provided on the basis of minimum reinforcement. These bars are explained below for the three types of ends of the two panels.

Table 4.3 Steel bars of one-way slab (Figs.4..5a and b)

Sl.No.	Bars	Panel	Along	Resisting moment
1 2	B1, B2 B3	DEP DEP	x y	+ 0.5 M_x for each, Minimum steel
3 4	B4, B5 B6	CAP CAP	x y	+ 0.5 M_x for each, Minimum steel
5 6	T1, T2 T3	CEP DEP	x x	- 0.5 M_x for each, + 0.5 M_x
7	T4	DEP	y	Minimum steel

Notes: (i) DEP = Discontinuous End Panel

(ii) CEP = Continuous End Panel

(iii) CAP = Continuous Adjacent Panel

Discontinuous End Panel (DEP)

- Bottom steel bars B1 and B2 are alternately placed such that B1 bars are curtailed at a distance of 0.25 lx_1 from the adjacent support and B2 bars are started from a distance of 0.15lx_1 from the end support. Thus, both B1 and B2 bars are present in the middle zone covering 0.6lx_1, each of which is designed to resist positive moment 0.5M_x. These bars are along the direction of x and are present from one end to the other end of l_y.
- Bottom steel bars B3 are along the direction of y and cover the entire span lx_1 having the minimum area of steel. The first bar shall be placed at a distance

not exceeding $s/2$ from the left discontinuous support, where s is the spacing of these bars in y direction.

- Top bars T3 are along the direction of x for resisting the negative moment which is numerically equal to fifty per cent of positive M_x. These bars are continuous up to a distance of $0.1lx_1$ from the centre of support at the discontinuous end.
- Top bars T4 are along the direction of y and provided up to a distance of $0.1lx_1$ from the centre of support at discontinuous end. These are to satisfy the requirement of minimum steel.

Continuous End Panel (CEP)

- Top bars T1 and T2 are along the direction of x and cover the entire l_y. They are designed for the maximum negative moment M_x and each has a capacity of $-0.5M_x$. Top bars T1 are continued up to a distance of $0.3lx_1$, while T2 bars are only up to a distance of $0.15lx_1$.
- Top bars T4 are along y and provided up to a distance of $0.3lx_1$ from the support. They are on the basis of minimum steel requirement.

Continuous Adjacent Panel (CAP)

- Bottom bars B4 and B5 are similar to B1 and B2 bars of (i) above.
- Bottom bars B6 are similar to B3 bars of (i) above.

Detailing is an art and hence structural requirement can be satisfied by more than one mode of detailing each valid and acceptable.

4.8 Importance of bond, anchorage length

<u>4.8.1 Introduction</u>

The bond between steel and concrete is very important and essential so that they can act together without any slip in a loaded structure. With the perfect bond between them, the plane section of a beam remains plane even after bending. The length of a member required to develop the full bond is called the anchorage length. The bond is measured by bond stress. The local bond stress varies along a member with the variation of bending moment. The average value throughout its anchorage length is designated as the average bond stress Thus, a tensile member has to be anchored properly by providing additional length on either side of the point of maximum tension, which is known as ‘Development length in tension’. Similarly, for compression members also, we have ‘Development length L_d in compression’.

It is worth mentioning that the deformed bars are known to be superior to the smooth mild steel bars due to the presence of ribs. In such a case, it is needed to check for the sufficient development length Ld only rather than checking both for the local bond stress and development length as required for the smooth mild steel bars. Accordingly, IS 456, cl. 26.2 stipulates the requirements of proper anchorage of reinforcement in terms of development length Ld only employing design bond stress τ_{bd}.

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4.8.2Design Bond Stress τ_{bd}

(a) Definition

The design bond stress τbd is defined as the shear force per unit nominal surface area of reinforcing bar. The stress is acting on the interface between bars and surrounding concrete and along the direction parallel to the bars.

This concept of design bond stress finally results in additional length of a bar of specified diameter to be provided beyond a given critical section. Though, the overall bond failure may be avoided by this provision of additional development length Ld, slippage of a bar may not always result in overall failure of a beam. It is, thus, desirable to provide end anchorages also to maintain the integrity of the structure and thereby, to enable it carrying the loads. Clause 26.2 of IS 456 stipulates, “The calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or by a combination thereof.”

(b) Design bond stress – values

Local bond stress varies along the length of the reinforcement while the average bond stress gives the average value throughout its development length. This average bond stress is still used in the working stress method and IS 456 has mentioned about it in cl. B-2.1.2. However, in the limit state method of design, the average bond stress has been designated as design bond stress τ_{bd} and the values are given in cl. 26.2.1.1. The same is given below as a ready reference.

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design Bond Stress τ_{bd} in N/mm^2	1.2	1.4	1.5	1.7	1.9

(c)Development Length

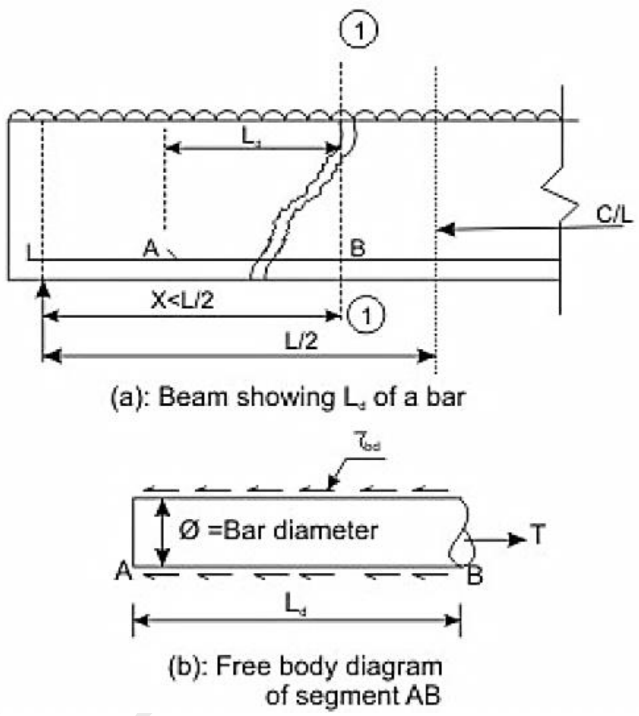


Fig4.3

(a) A single bar

Figure 4.3 (a) shows a simply supported beam subjected to uniformly distributed load. Because of the maximum moment, the Ast required is the maximum at x = L/2. For any section 1-1 at a distance x < L/2, some of the

tensile bars can be curtailed. Let us then assume that section 1-1 is the theoretical cut-off point of one bar. However, it is necessary to extend the bar for a length Ld as explained earlier. Let us derive the expression to determine Ld of this bar.

Figure 4.3(b) shows the free body diagram of the segment AB of the bar. At B, the tensile force *T* trying to pull out the bar is of the value T = ($\pi\ \varphi^2\ \sigma s$ /4), where φ is the nominal diameter of the bar and σ_s is the tensile stress in bar at the section considered at design loads. It is necessary to have the resistance force to be developed by τbd for the length Ld to overcome the tensile force. The resistance force = π φ (L_d) (τ_{bd}). Equating the two, we get

$$\pi \varphi (L_d) (\tau_{bd}) = (\pi \varphi^2 \sigma_s /4)$$

From the above Equation, thus gives

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

The above equation is given in cl. 26.2.1 of IS 456 to determine the development length of bars. The example taken above considers round bar in tension. Similarly, other sections of the bar should have the required L_d as determined for such sections. For bars in compression, the development length is reduced by 25 per cent as the design bond stress in compression τbd is 25 per cent more than that in tension. Following the same logic, the development length of deformed bars is reduced by 60 per cent of that needed for the plain round bars.

(b) Bars bundled in contact

The respective development lengths of each of the bars for two, three or four bars in contact are determined following the same principle. However, cl. 26.2.1.2 of IS 456 stipulates a simpler approach to determine the development length directly under such cases and the same is given below:

"The development length of each bar of bundled bars shall be that for the individual bar, increased by 10 per cent for two bars in contact, 20 per cent for three bars in contact and 33 per cent for four bars in contact."

However, while using bundled bars the provision of cl. 26.1.1 of IS 456 must be satisfied. According to this clause:

- In addition to single bar, bars may be arranged in pairs in contact or in groups of three or four bars bundled in contact.
- Bundled bars shall be enclosed within stirrups or ties to ensure the bars remaining together.
- Bars larger than 32 mm diameter shall not be bundled, except in columns.

Curtailment of bundled bars should be done by terminating at different points spaced apart by not less than 40 times the bar diameter except for bundles stopping at support (cl. 26.2.3.5 of IS 456).

(c) Checking of Development Lengths of Bars in Tension

The following are the stipulation of cl. 26.2.3.3 of IS 456.

(i) At least one-third of the positive moment reinforcement in simple members and one-fourth of the positive moment reinforcement in continuous members shall be extended along the same face of the member into the support, to a length equal to Ld/3.

(ii) Such reinforcements of (i) above shall also be anchored to develop its design stress in tension at the face of the support, when such member is part of the primary lateral load resisting system.

(iii) The diameter of the positive moment reinforcement shall be limited to a diameter such that the L_d computed for σs = f_d does not exceed the following:

$$(L_d)_{\text{when } \sigma_s = f_d} \leq \frac{M_1}{V} + L_o$$

where M_1 = moment of resistance of the section assuming all reinforcement at the section to be stressed to fd,

$f_d = 0.87\ f_y$,

V = shear force at the section due to design loads,

L_o = sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support. At a point of inflection, Lo is limited to the effective depth of the member or 12φ, whichever is greater, and

φ = diameter of bar.

It has been further stipulated that M_1/V in the above expression may be increased by 30 per cent when the ends of the reinforcement are confined by a compressive reaction.

(d) Anchoring Reinforcing Bars

Bars may be anchored in combination of providing development length to maintain the integrity of the structure. Such anchoring is discussed below under three sub-sections for bars in tension, compression and shear respectively, as stipulated in cl. 26.2.2 of IS 456.

(a) Bars in tension (cl. 26.2.2.1 of IS 456)

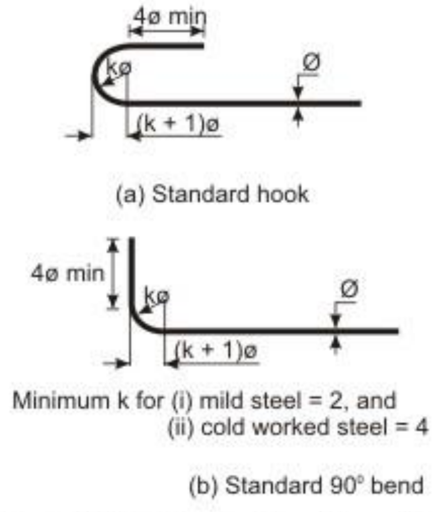


Fig 4.4

The salient points are:

- Deformed bars may not need end anchorages if the development length requirement is satisfied.
- Hooks should normally be provided for plain bars in tension.
- Standard hooks and bends should be as per IS 2502 or as given in Table 67 of SP-16, which are shown in Figs.4.4 a and b.
- The anchorage value of standard bend shall be considered as 4 times the diameter of the bar for each 45^{o} bend subject to a maximum value of 16 times the diameter of the bar.
- The anchorage value of standard U-type hook shall be 16 times the diameter of the bar.

(b) Bars in compression (cl. 26.2.2.2 of IS 456)

Here, the salient points are:

- The anchorage length of straight compression bars shall be equal to its development length as mentioned in sec. 6.15.3.
- The development length shall include the projected length of hooks, bends and straight lengths beyond bends, if provided

(c) Bars in shear (cl. 26.2.2.4 of IS 456)

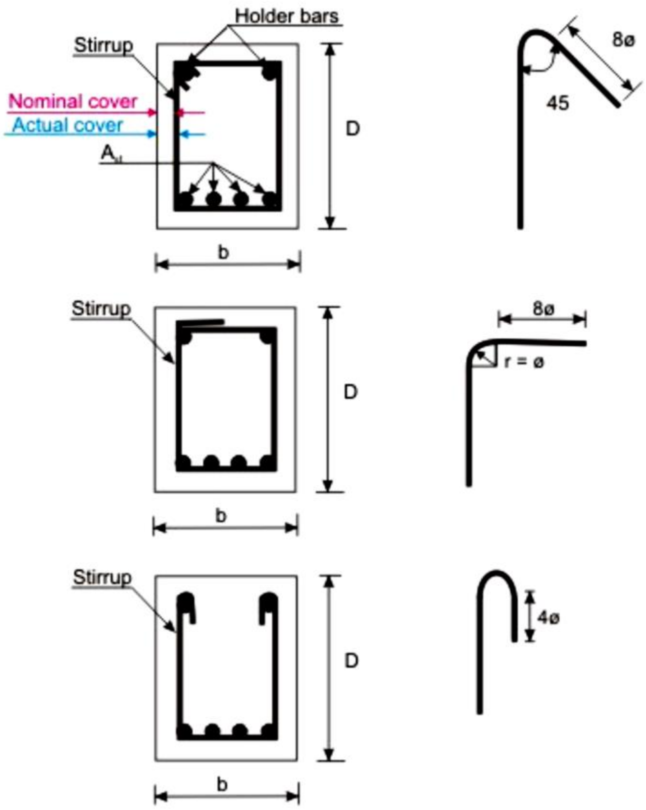


Fig 4.5 Anchorage of Stirrups

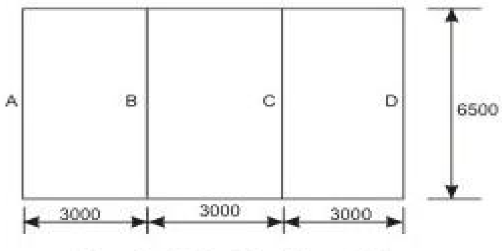
The salient points are:

- Inclined bars in tension zone will have the development length equal to that of bars in tension and this length shall be measured from the end of sloping or inclined portion of the bar.
- Inclined bars in compression zone will have the development length equal to that of bars in tension and this length shall be measured from the mid-depth of the beam.
- For stirrups, transverse ties and other secondary reinforcement, complete development length and anchorage are considered to be satisfied if prepared

Numerical Problems

(a) Problem 4.1

Design the one-way continuous slab of Fig.4.6 subjected to uniformly distributed imposed loads of 5 kN/m^2 using M 20 and Fe 415. The load of floor finish is 1 kN/m^2. The span dimensions shown in the figure is effective spans. The width of beams at the support = 300 mm.



Solution:

Step 1: Selection of preliminary depth of slab

The basic value of span to effective depth ratio for the slab having simple support at the end and continuous at the intermediate is (20+26)/2 = 23 (cl.23.2.1 of IS 456).

Modification factor with assumed p = 0.5 and f_s = 240 N/mm^2 is obtained as 1.18 from Fig.4 of IS 456.

Therefore, the minimum effective depth = 3000/23(1.18) = 110.54 mm. Let us take the effective depth d = 115 mm and with 25 mm cover, the total depth D = 140 mm.

Step 2: Design loads, bending moment and shear force

Dead loads of slab of 1 m width = 0.14(25) = 3.5 kN/m

Dead load of floor finish =1.0 kN/m

Factored dead load = 1.5(4.5) = 6.75 kN/m

Factored live load = 1.5(5.0) = 7.50 kN/m

Total factored load = 14.25 kN/m

Maximum moments and shear are determined from the coefficients given in Tables 12 and 13 of IS 456.

Maximum positive moment = 14.25(3)(3)/12 = 10.6875 kNm/m

Maximum negative moment = 14.25(3)(3)/10 = 12.825 kNm/m

Maximum shear V_u = 14.25(3) (0.4) = 17.1 kN

Step 3: Determination of effective and total depths of slab

From Eq.3.25 of sec. 3.5.6 of Lesson 5, we have

$M_{u,lim} = R_{,lim}\, bd^2$ where $R_{,lim}$ is 2.76 N/mm^2 .

So, $d = \{12.825(10^6)/ (2.76) (1000)\}^{0.5}$ = 68.17 mm

Since, the computed depth is much less than that determined in Step 1, let us keep D = 140 mm and d = 115 mm

Step 4: Depth of slab for shear force

Table 19 of IS 456 gives τ_c = 0.28 N/mm^2 for the lowest percentage of steel in the slab.

Further for the total depth of 140 mm, let us use the coefficient k of cl. 40.2.1.1 of IS 456 as 1.3 to get $\tau_c = k\, \tau_c$ = 1.3(0.28) = 0.364 N/mm^2.

Table 20 of IS 456 gives τ c max= 2.8 N/mm^2. For this problem τ_v= V /bd = (17.1x1000)/(1000x115) = 0.148 N/mm2. Since, $\tau_v < \tau_c < \tau_{c\,max}$, the effective depth d = 115 mm is acceptable.

Step 5: Determination of areas of steel

we have,

$M_u = 0.87\, f_y\, A_{st}\, d\, \{1 - (A_{st})(f_y)/(f_{ck})(bd)\}$

(i) For the maximum negative bending moment

12825000 = 0.87(415) (A_{st}) (115) {1 – (A_{st}) (415)/ (1000) (115) (20)}

or A_{st}^2 - 5542.16 Ast + 1711871.646 = 0

Solving the quadratic equation, we have the negative A_{st} = 328.34 mm^2

(ii) For the maximum positive bending moment

10687500 = 0.87(415) A_{st} (115) {1 – (A_{st}) (415)/ (1000) (115) (20)}

or A_{st}^2 - 5542.16 Ast + 1426559.705 = 0

Solving the quadratic equation, we have the positive A_{st} = 270.615 mm^2

<u>Alternative approach: Use of Table 2 of SP-16</u>

(i) For negative bending

moment M_u/bd^2 = 0.9697

Table 2 of SP-16 gives: p_s = 0.2859 (by linear interpolation). So, the area of negative steel = 0.2859(1000) (115)/100 = 328.785 mm^2.

(ii) For positive bending

moment M_u/bd^2 = 0.8081

Table 2 of SP-16 gives: p_s = 0.23543 (by linear interpolation). So, the area of positive steel = 0.23543(1000) (115)/100 = 270.7445 mm^2.

These areas of steel are comparable with those obtained by direct computation using Eq.3.23.

Distribution steel bars along longer span l_y

Distribution steel area = Minimum steel area = 0.12(1000) (140)/100 = 168 mm^2. Since, both positive and negative areas of steel are higher than the minimum area, we provide:

(a) For negative steel: 10 mm diameter bars @ 230 mm c/c for which A_{st} = 341 mm^2 giving p_s = 0.2965.

(b) For positive steel: 8 mm diameter bars @ 180 mm c/c for which A_{st} = 279 mm^2 giving p_s = 0.2426

(c) For distribution steel: Provide 8 mm diameter bars @ 250 mm c/c for which A_{st} (minimum) = 201 mm^2.

Step 6: Selection of diameter and spacing of reinforcing bars

The diameter and spacing already selected in step 5 for main and distribution bars are checked below:

For main bars (cl. 26.3.3.b.1 of IS 456), the maximum spacing is the lesser of 3*d* and 300 mm i.e., 300 mm. For distribution bars (cl. 26.3.3.b.2 of IS 456), the maximum spacing is the lesser of 5*d* or 450 mm i.e., 450 mm. Provided spacings, therefore, satisfy the requirements.

Maximum diameter of the bars (cl. 26.5.2.2 of IS 456) shall not exceed 140/8 = 17 mm is also satisfied with the bar diameters selected here.

Figure 4.7 presents the detailing of the reinforcement bars. The abbreviation B1 to B3 and T1 to T4 are the bottom and top bars, respectively which are shown in Fig.4.5 for a typical one-way slab.

The above design and detailing assume absence of support along short edges. When supports along short edges exist and there is eventual clamping top reinforcement would be necessary at shorter supports also.

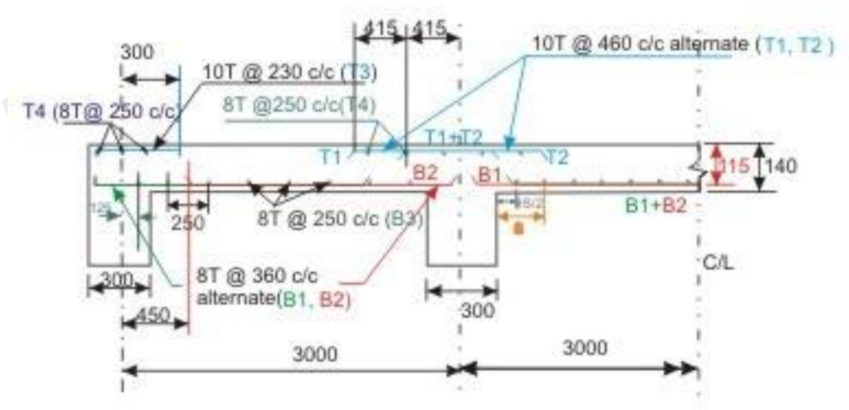


Fig 4.7

4.9 Two-way Slabs

Two-way slabs subjected mostly to uniformly distributed loads resist them primarily by bending about both the axis. However, as in the one-way slab, the depth of the two-way slabs should also be checked for the shear stresses to avoid any reinforcement for shear. Moreover, these slabs should have sufficient depth for the control deflection. Thus, strength and deflection are the requirements of design of two-way slabs.

4.8 Design Shear Strength of Concrete

Design shear strength of concrete in two-way slabs is to be determined incorporating the multiplying factor k as in case of one-way slab.

4.9.1 Computation of shear force

Shear forces are computed following the procedure stated below with reference to Fig.4.8.

The two-way slab of Fig. 4.8 is divided into two trapezoidal and two triangular zones by drawing lines from each corner at an angle of 45o. The loads of triangular segment A will be transferred to beam 1-2 and the same of trapezoidal segment B will be beam 2-3. The shear forces per unit width of the strips aa and bb are highest at the ends of strips. Moreover, the length of half the strip bb is equal to the length of the strip aa. Thus, the shear forces in both strips are equal and we can write,

$$V_u = W\,(l_x/2)$$

where W = intensity of the uniformly distributed loads. The nominal shear stress acting on the slab is then determined from

$$\tau_v = Vu / bd$$

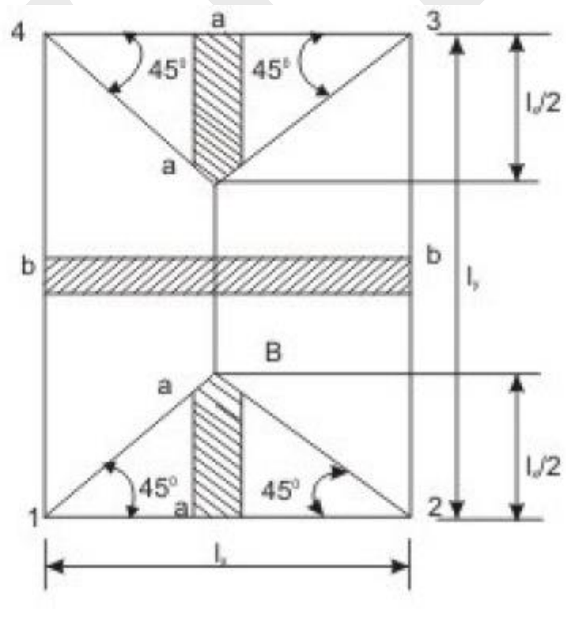


Fig 4.8

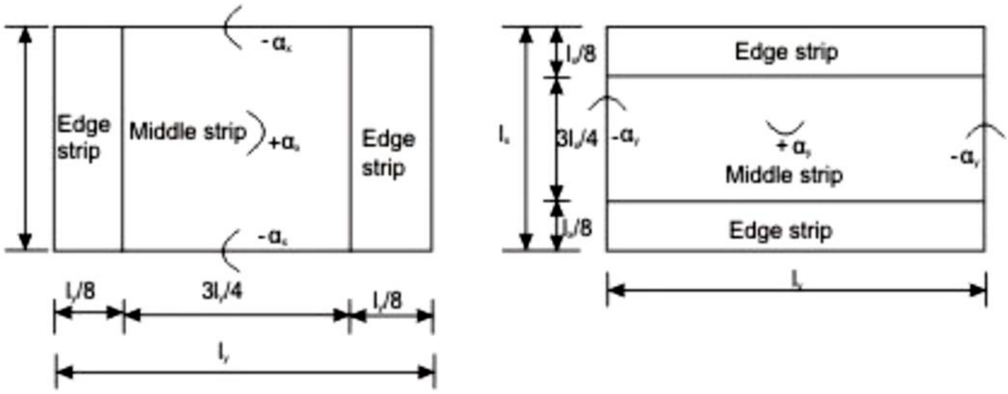
4.9.2 Computation of bending moments

Pigeoud's or Wester-guard's theories are the suggested elastic methods and Johansen's yield line theory is the most commonly used in the limit state of collapse method and suggested by IS 456 in the note of cl. 24.4. Alternatively, Annex D of IS 456 can be employed to determine the bending moments in the two directions for two types of slabs: (i) restrained slabs, and (ii) simply supported slabs. The two methods are explained below:

(i) Restrained slabs

Restrained slabs are those whose corners are prevented from lifting due to effects of torsional moments. These torsional moments, however, are not computed as the amounts of reinforcement are determined from the computed areas of steel due to positive bending moments depending upon the intensity of torsional moments of different corners. Thus, it is essential to determine the positive and negative bending moments in the two directions of restrained slabs depending on the various types of panels and the aspect ratio l_y/lx.

Restrained slabs are considered as divided into two types of strips in each direction: (i) one middle strip of width equal to three-quarters of the respective length of span in either directions, and (ii) two edge strips, each of width equal to one-eighth of the respective length of span in either directions. Figures 4.9a and b present the two types of strips for spans l_x and l_y separately.



a) for span lx b) for span lx

Fig 4.9

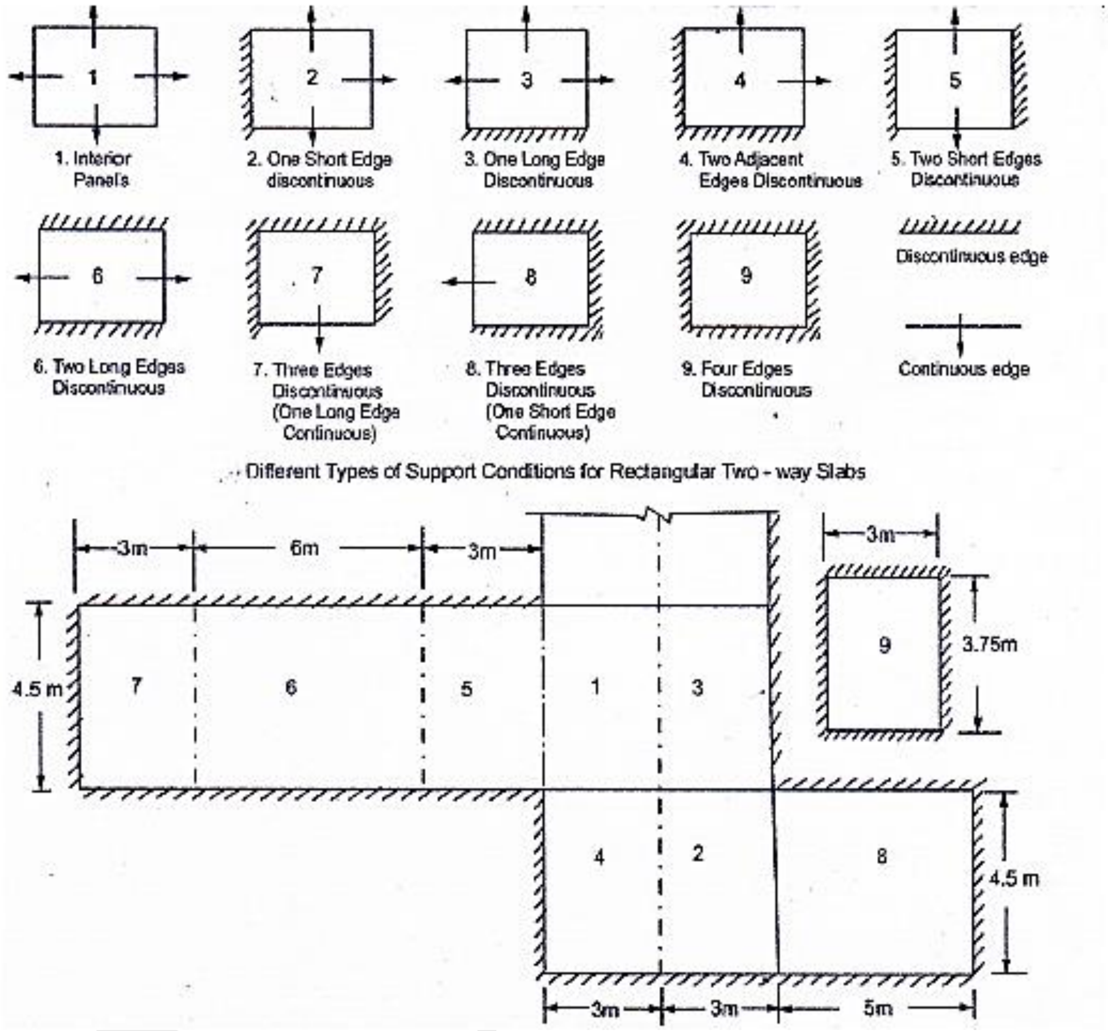


Fig 4.9 a Different Boundary conditions of Two-way Restrained slabs

The maximum positive and negative moments per unit width in a slab are determined from

$$M_{x} = \alpha_{x} w l_x^2$$

$$M_{y} = \alpha_{y} w l_y^2$$

where α_x and α_y are coefficients given in Table 26 of IS 456, Annex D, cl. D-1.1. Total design load per unit area is w and lengths of shorter and longer spans are represented by l_x and l_y, respectively. The values of α_x and α_y, given in Table 26 of IS 456, are for nine types of panels having eight aspect ratios of l_y/l_x from one to two at an interval of 0.1. The above maximum bending moments are applicable only to the middle strips and no redistribution shall be made.

Tension reinforcing bars for the positive and negative maximum moments are to be provided in the respective middle strips in each direction. Figure 4.9 shows the positive and negative coefficients α_x and α_y.

The edge strips will have reinforcing bars parallel to that edge following the minimum amount as stipulated in IS 456.

The detailing of all the reinforcing bars for the respective moments and for the minimum amounts as well as torsional requirements are discussed in separate section.

(ii) Simply supported slabs

The maximum moments per unit width of simply supported slabs, not having adequate provision to resist torsion at corners and to prevent the corners from lifting, are determined from equation used for restrained slab, where α_x and α_y are the respective coefficients of moments as given in Table 27 of IS 456, cl. D-2.

4.10 Design Considerations

The design considerations mentioned for one-way slabs are applicable for the two-way slabs also. However, the effective span to effective depth ratio is different from those of one-way slabs. Accordingly, this item for the two-way slabs is explained below.

Effective span to effective depth ratio (cl. 24.1 of IS 456)

The following are the relevant provisions given in Notes 1 and 2 of cl.24.1.

- The shorter of the two spans should be used to determine the span to effective depth ratio.
- For spans up to 3.5 m and with mild steel reinforcement, the span to overall depth ratios satisfying the limits of vertical deflection for loads up to 3 kN/m^2 are as follows:

Simply supported slabs	35
Continuous slabs	40

The same ratios should be multiplied by 0.8 when high strength deformed bars (Fe 415) are used in the slabs.

Design of Two-way Slabs

The procedure of the design of two-way slabs will have all the six steps mentioned for the design of one-way slabs except that the bending moments and shear forces are determined by different methods for the two types of slab.

While the bending moments and shear forces are computed from the coefficients given in Tables 12 and 13 (cl. 22.5) of IS 456 for the one-way slabs, the same are obtained from

Tables 26 or 27 for the bending moment in the two types of two-way slabs and the shear forces are computed for the two-way slabs.

Further, the restrained two-way slabs need adequate torsional reinforcing bars at the corners to prevent them from lifting. There are three types of corners having three different requirements. Accordingly, the determination of torsional reinforcement is discussed in Step 7, as all the other six steps are common for the one and two-way slabs.

Step 7: Determination of torsional reinforcement

Three types of corners, C1, C2 and C3, shown in Fig.4.10, have three different requirements of torsion steel as mentioned below.

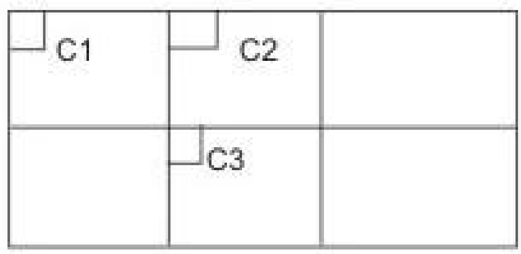


Fig 4.10 Three types of corners

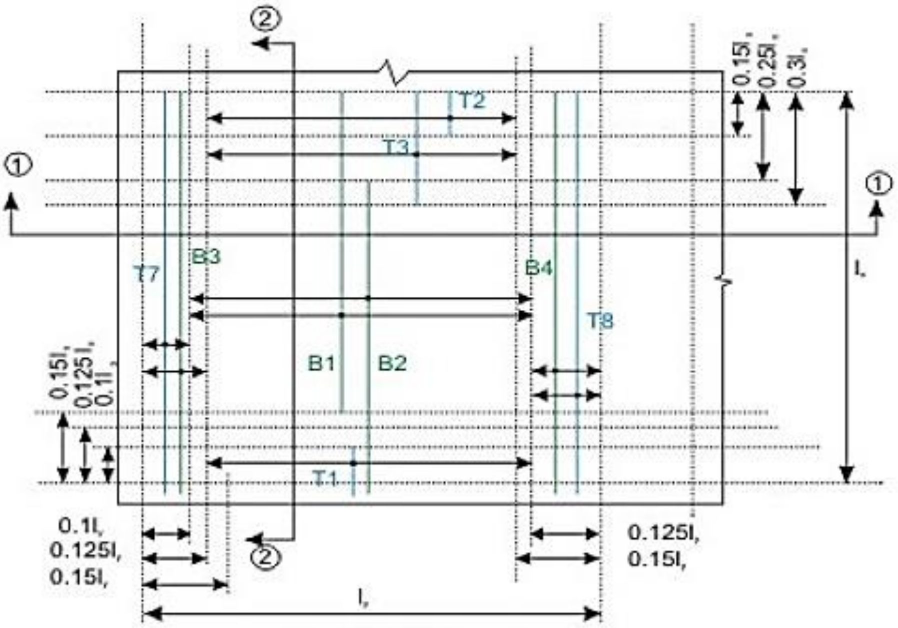
i. At corner C1 where the slab is discontinuous on both sides, the torsion reinforcement shall consist of top and bottom bars each with layers of bar placed parallel to the sides of the slab and extending a minimum distance of one-fifth of the shorter span from the edges. The amount of reinforcement in each of the four layers shall be 75 per cent of the area required for the maximum mid-span moment in the slab. This provision is given in cl. D-1.8 of IS 456.

ii. At corner C2 contained by edges over one of which is continuous, the torsional reinforcement shall be half of the amount of (a) above. This provision is given in cl. D-1.9 of IS 456.

iii. At corner C3 contained by edges over both of which the slab is continuous, torsional reinforcing bars need not be provided, as stipulated in cl. D-1.10 of IS 456.

4.10.1 Detailing of Reinforcement

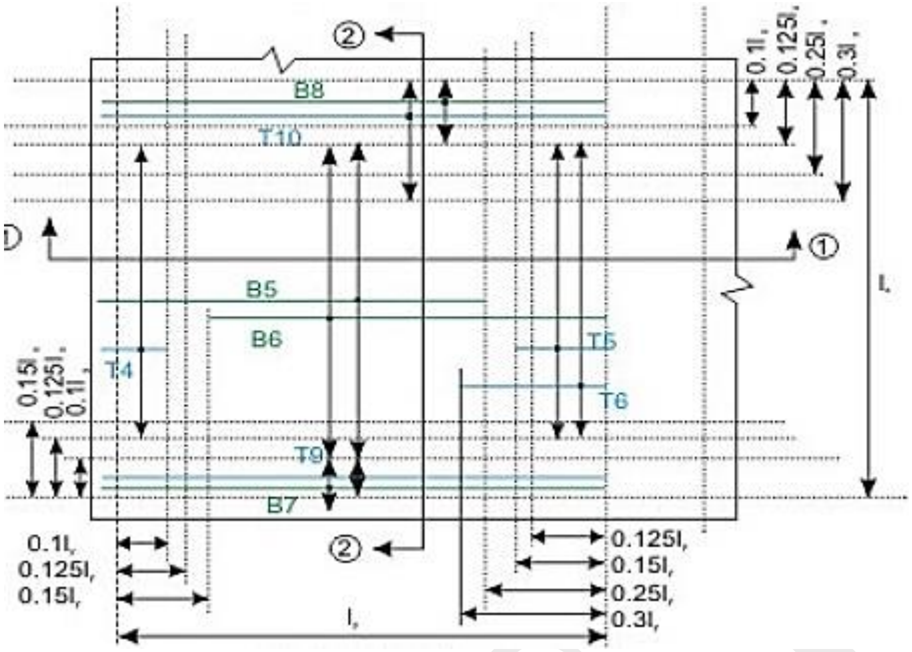
Determining the areas steel for corners of restrained slab depending on the type of corner. In the following, the detailing of reinforcing bars for (i) restrained slabs and (ii) simply



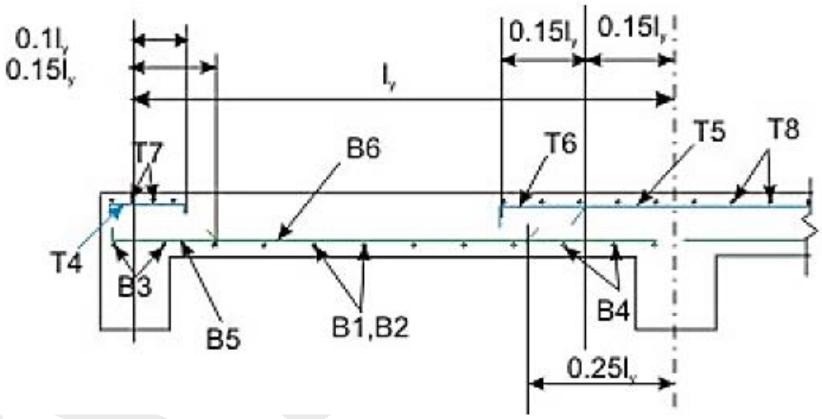
supported slabs are discussed separately for the bars either for the maximum positive or negative bending moments or to satisfy the requirement of minimum amount of steel.

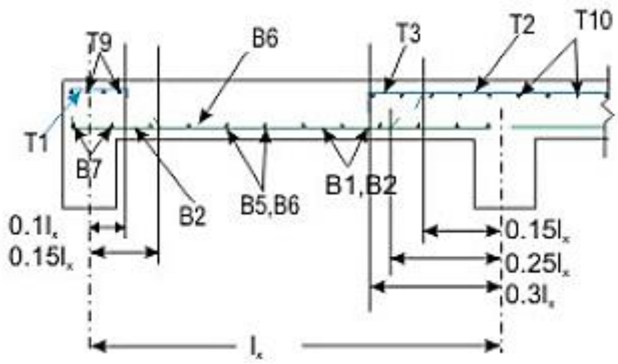
(i) Restrained slabs

The maximum positive and negative moments per unit width of the slab are applicable only to the respective middle strips. There shall be no redistribution of these moments. The reinforcing bars so calculated from the maximum moments are to be placed satisfying the following stipulations of IS 456.



4.11 Reinforcement for two-way slab except Torsional reinforcement





Reinforcement for two-way slab

ii. Bottom tension reinforcement bars of mid-span in the middle strip shall extent in the lower part of the slab to within 0.25*l* of a continuous edge, or 0.15*l* of a discontinuous edge (cl. D-1.4 of IS 456). Bars marked as B1, B2, B5 and B6 in Figs.4.11 a and b are these bars.

iii. Top tension reinforcement bars over the continuous edges of middle strip shall extend in the upper part of the slab for a distance of 0.15*l* from the support, and at least fifty per cent of these bars shall extend a distance of 0.3*l* (cl. D-1.5 of IS 456). Bars marked as T2, T3, T5 and T6 in Figs4.11 a and b are these bars.

iv. To resist the negative moment at a discontinuous edge depending on the degree of fixity at the edge of the slab, top tension reinforcement bars equal to fifty per cent of that provided at mid-span shall extend 0.1*l* into the span (cl. D-1.6 of IS 456). Bars marked as T1 and T4 in Figs.4.11 a and b are these bars.

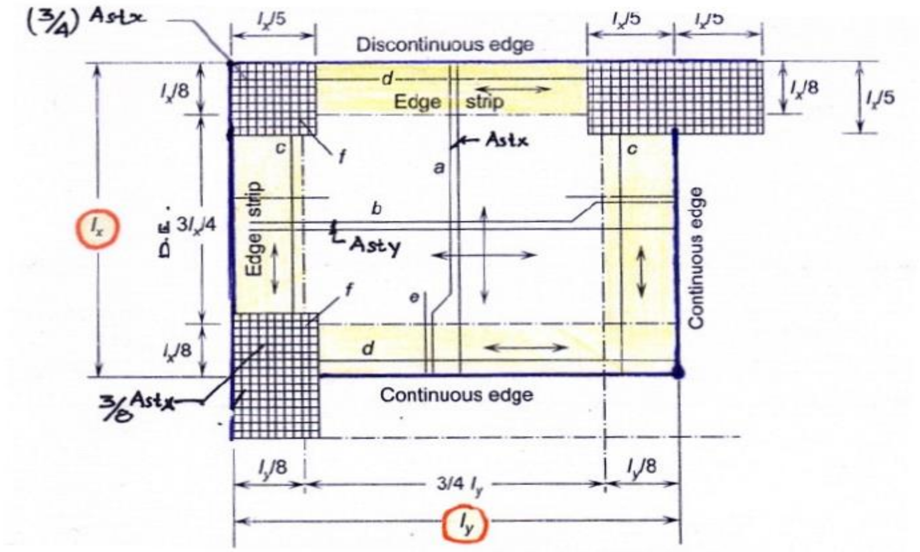
v. The edge strip of each panel shall have reinforcing bars parallel to that edge satisfying the requirement of minimum amount as specified in cl. 26.5.2.1 of IS 456 and the requirements for torsion, explained in cls. D-1.7 to D-1.10 of IS 456). The bottom and top bars of the edge strips are explained below.

vi. Bottom bars B3 and B4 (Fig.4.11 a) are parallel to the edge along lx for the edge strip for span ly, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).

vii. Bottom bars B7 and B8 (Fig.4.11 b) are parallel to the edge along ly for the edge strip for span lx, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).

viii. Top bars T7 and T8 (Fig.4.11 a) are parallel to the edge along lx for the edge strip for span ly, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).

ix. Top bars T9 and T10 (Fig.4.11 b) are parallel to the edge along ly for the edge strip for span lx, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).



The above explanation reveals that there are eighteen bars altogether comprising eight bottom bars (B1 to B8) and ten top bars (T1 to T10). Tables 4.3 and 4.4 present them separately for the bottom and top bars, respectively, mentioning the respective zone of their placement (MS/LDES/ACES/BDES to designate Middle Strip/Left Discontinuous Edge Strip/Adjacent Continuous Edge Strip/Bottom Discontinuous Edge Strip), direction of the bars (along x or y), the resisting moment for which they shall be determined or if to be provided on the basis of minimum reinforcement clause number of IS 456 and Fig. 4.11No. For easy understanding, plan views in (a) and (b) of Fig.4.11. show all the bars separately along x and y directions, respectively. Two sections (1-1 and 2-2), however, present the bars shown in the two plans. Torsional reinforcements are not included in Tables 4.3 and 4.4 and Figs.4.11 a and b.

Table 4.3 Details of eight bottom bars

Sl. No.	Bars	Into	Along	Resisting Moment	Cl.No. of IS 456	Fig.No.
1	B1, B2	MS	x	Max. + M_x	D-1.3,1.4	8.19.5a, c, d
2	B3	LDES	x	Min. Steel	D-1.7	8.19.5a, c
3	B4	ACES	x	Min. Steel	D-1.7	8.19.5a, c
4	B5, B6	MS	y	Max. + M_y	D-1.3,1.4	8.19.5b, c, d
5	B7	BDES	y	Min. Steel	D-1.7	8.19.5b, d
6	B8	ACES	y	Min. Steel	D-1.7	8.19.5b, d

Notes: (i) MS = Middle Strip

- LDES = Left Discontinuous Edge Strip

- ACES = Adjacent Continuous Edge Strip
- BDES = Bottom Discontinuous Edge Strip

Table 4.4 Details of eight top bars

Sl.No.	Bars	Into	Along	Resisting Moment	Cl.No. of IS 456	Fig.No.
1	T1	BDES	x	+ 0.5 M_x	D-1.6	8.19.5a, d
2	T2, T3	ACES	x	- 0.5 M_x for each	D-1.5	8.19.5a, d
3	T4	LDES	y	+ 0.5 M_y	D-1.6	8.19.5b, c
4	T5, T6	ACES	y	-0.5 M_y for each	D-1.5	8.19.5b, c
5	T7	LDES	x	Min. Steel	D-1.7	8.19.5a, c
6	T8	ACES	x	Min. Steel	D-1.7	8.19.5a, c
7	T9	LDES	y	Min. Steel	D-1.7	8.19.5b, d
8	T10	ACES	y	Min. Steel	D-1.7	8.19.5b, d

Notes: (i) MS = Middle Strip

(ii)LDES = Left Discontinuous Edge Strip

ACES = Adjacent Continuous Edge Strip

BDES = Bottom Discontinuous Edge Strip

(ii) Simply supported slabs

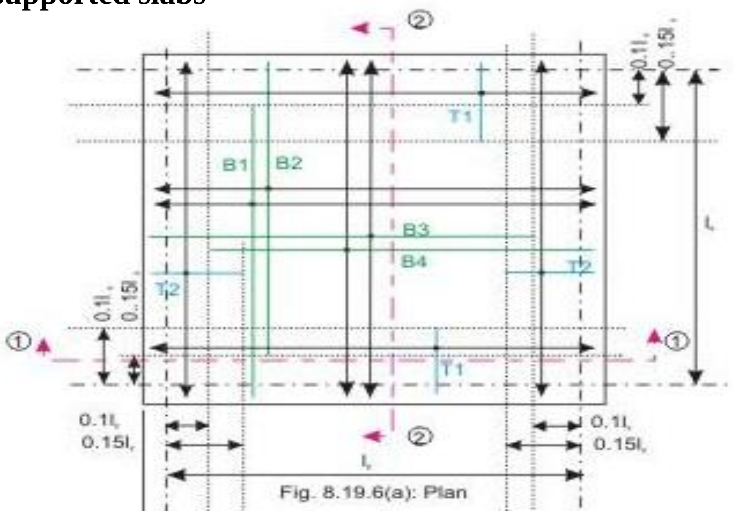


Fig 4.12

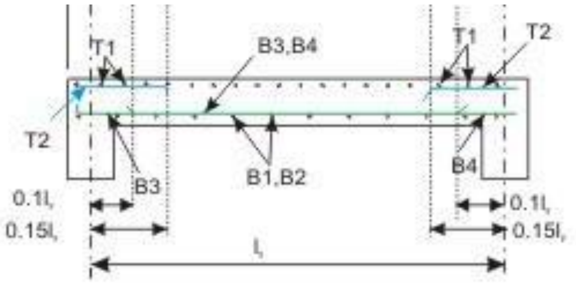


Fig. 8.19.6(b): Section 1-1

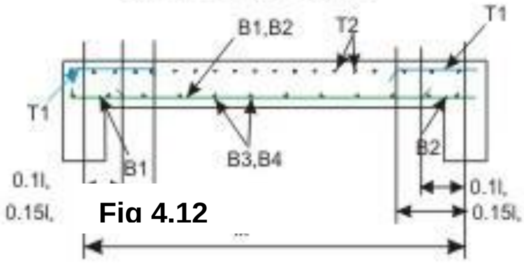


Fig 4.12

Fig. 8.19.6(c): Section 2-2

Figures 4.12 a, b and c present the detailing of reinforcing bars of simply supported slabs not having adequate provision to resist torsion at corners and to prevent corners from lifting. Clause D-2.1 stipulates that fifty per cent of the tension reinforcement provided at mid-span should extend to the supports. The remaining fifty per cent should extend to within 0.1lx or 0.1ly of the support, as appropriate.

Numerical Problems

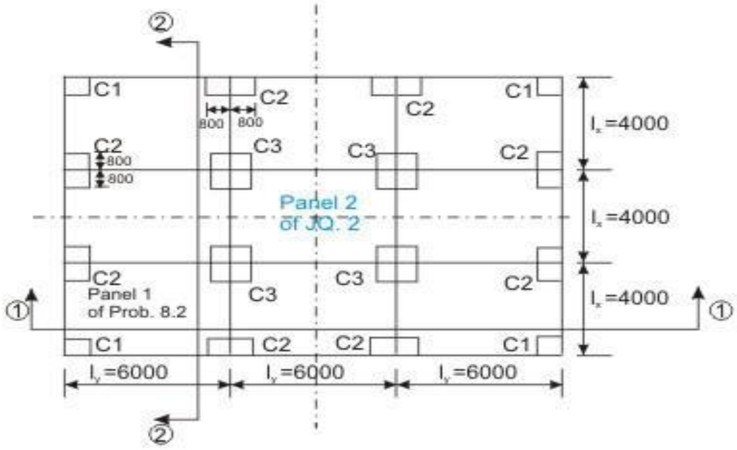


Fig. 8.19.7: Problem 8.2 (panel 1) and TQ 2 (panel 2)

Fig 4.16

Design the slab panel 1 of Fig.4.16 subjected to factored live load of 8 kN/m^2 in addition to its dead load using M 20 and Fe 415. The load of floor finish is 1 kN/m^2. The spans shown in figure are effective spans. The corners of the slab are prevented from lifting.

Solution

Step 1: Selection of preliminary depth of slab

The span to depth ratio with Fe 415 is taken from cl. 24.1, Note 2 of IS 456 as 0.8 (35 + 40) / 2 = 30. This gives the minimum effective depth d = 4000/30 = 133.33 mm, say 135 mm. The total depth D is thus 160 mm.

Step 2: Design loads, bending moments and shear forces

Dead load of slab (1 width) = 0.16(25) = 4.0 kN/m

Dead load of floor finish (given) = 1.0 kN/m^2

Factored dead load = 1.5(5) = 7.5 kN/m^2

Factored live load (given) = 8.0 kN/m^2

Total factored load = 15.5 kN/m^2

The coefficients of bending moments and the bending moments M_x and M_y per unit width (positive and negative) are determined as per cl. D-1.1 and Table 26 of IS 456 for the case 4, "Two adjacent edges discontinuous" and presented in Table 8.6. The l_y / l_x for this problem is 6/4 = 1.5.

Table 8.6 Maximum bending moments of Problem 8.2

For	Short span		Long span	
	α_x	M_x (kNm/m)	α_y	M_y (kNm/m)
Negative moment at continuous edge	0.075	18.6	0.047	11.66
Positive moment at mid-span	0.056	13.89	0.035	8.68

Maximum shear force in either direction is determined from Eq.8.1 (Fig.8.19.1) as

Vu = w(lx/2) = 15.5 (4/2) = 31 kN/m

Step 3: Determination/checking of the effective depth and total depth of slab

Using the higher value of the maximum bending moments in x and y directions from Table 8.6, we get from Eq.3.25 of Lesson 5 (sec. 3.5.5):

$M_{u,lim} = R_{,lim}\, bd^2$

or d = [(18.6) (10^6) {2.76(10^3)}] $^{1/2}$ = 82.09 mm,

where 2.76 N/mm^2 is the value of $R_{,lim}$ taken from Table 3.3 of Lesson 5 (sec. 3.5.5). Since, this effective depth is less than 135 mm assumed in

Step 1, we retain d = 135 mm and D = 160 mm.

Step 4: Depth of slab for shear force

Table 19 of IS 456 gives the value of τ_c = 0.28 N/mm^2 when the lowest percentage of steel is provided in the slab. However, this value needs to be modified by multiplying with k of cl. 40.2.1.1 of IS 456. The value of k for the total depth of slab as 160 mm is 1.28. So, the value of τ_c is 1.28(0.28) = 0.3584 N/mm^2.

Table 20 of IS 456 gives $\tau_{c\ max}$ = 2.8 N/mm^2. The computed shear stress

$\tau_v = V_u/bd$ = 31/135 = 0.229 N/mm^2.

Since, $\tau_v < \tau_c < \tau_{c\ max}$, the effective depth of the slab as 135 mm and the total depth as 160 mm are safe.

Step 5: Determination of areas of steel

The respective areas of steel in middle and edge strips are to be determined employing Eq.3.23 of Step 5 of sec. 4.6 of Lesson 18. However, in Problem 8.1 of Lesson 18, it has been shown that the areas of steel computed from Eq.3.23 and those obtained from the tables of SP-16 are in good agreement. Accordingly, the areas of steel for this problem are computed from the respective Tables 40 and 41 of SP-16 and presented in Table 8.7. Table 40 of SP-16 is for the effective depth of 150 mm, while Table 41 of SP-16 is for the effective depth of 175 mm. The following results are, therefore, interpolated values obtained from the two tables of SP-16.

Table 8.7 Reinforcing bars of Problem 8.2

Particulars	Short span l_x			Long span l_y		
	Table No.	M_x (kNm/m)	Dia. & spacing	Table No.	M_y (kNm/m)	Dia. & spacing
Top steel for negative moment	40,41	18.68 > 18.6	10 mm @ 200 mm c/c	40,41	12.314 > 11.66	8 mm @ 200 mm c/c
Bottom steel for positive moment	40,41	14.388 > 13.89	8 mm @ 170 mm c/c	40,41	9.20 > 8.68	8 mm @ 250 mm c/c

The minimum steel is determined from the stipulation of cl. 26.5.2.1 of IS 456 and is

A_s = (0.12/100) (1000) (160) = 192 mm^2

and 8 mm bars @ 250 mm c/c (= 201 mm^2) is acceptable. It is worth mentioning that the areas of steel as shown in Table 8.7 are more than the minimum amount of steel.

Step 6: Selection of diameters and spacings of reinforcing bars

The advantages of using the tables of SP-16 are that the obtained values satisfy the requirements of diameters of bars and spacings. However, they are checked as ready reference here. Needless to mention that this step may be omitted in such a situation.

Maximum diameter allowed, as given in cl. 26.5.2.2 of IS 456, is 160/8 = 20 mm, which is more that the diameters used here.

The maximum spacing of main bars, as given in cl. 26.3.3(1) of IS 456, is the lesser of 3(135) and 300 mm. This is also satisfied for all the bars.

The maximum spacing of minimum steel (distribution bars) is the lesser of 5(135) and 450 mm. This is also satisfied.

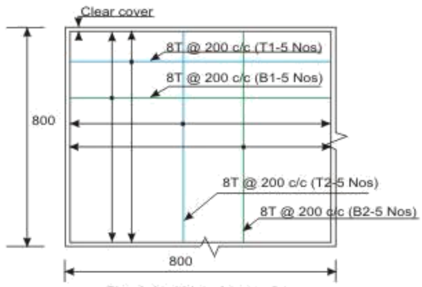


Fig. 8.19.10(a): Corner C1

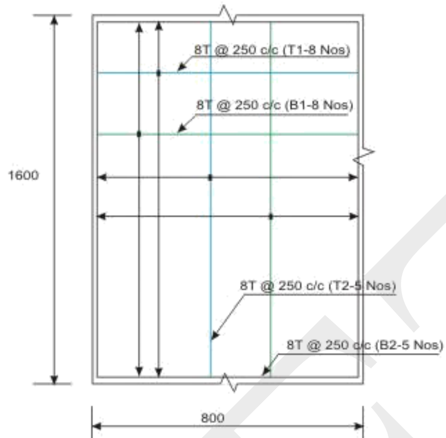


Fig. 8.19.10(b): Corners C2

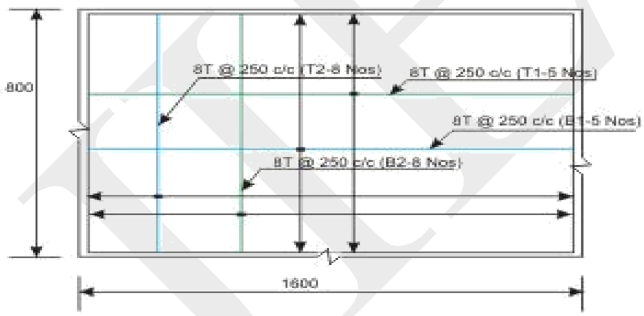


Fig. 8.19.10(c): Corners C2

Fig. 8.19.10: Torsion reinforcement bars of Problem 8.2

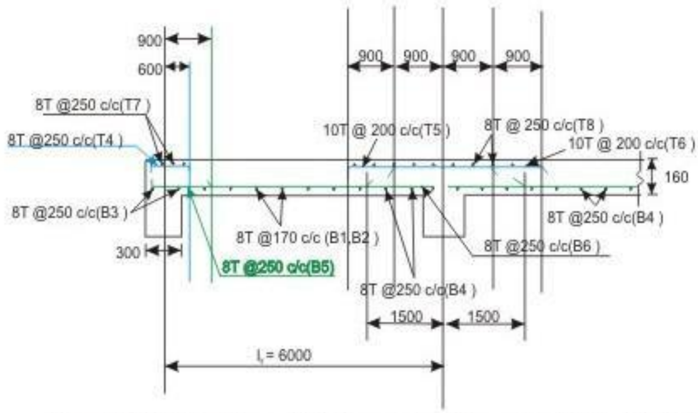


Fig. 8.19.8: Problem 8.2, Sec 1-1 of Panel 1 of Fig. 8.19.7

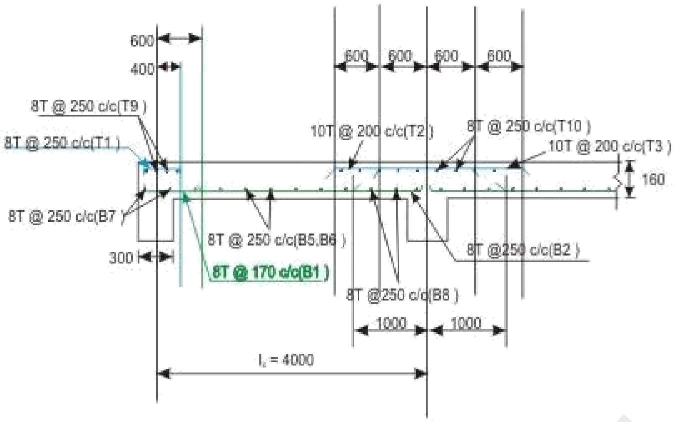


Fig. 8.19.9: Problem 8.2, Sec 2-2 of Panel 1 of Fig. 8.19.7

Step 7: Determination of torsional reinforcement

Torsional reinforcing bars are determined for the three different types of corners as explained in sec. 8.19.6 (Fig.8.19.4). The length of torsional strip is 4000/5 = 800 mm and the bars are to be provided in four layers. Each layer will have 0.75 times the steel used for the maximum positive moment. The C1 type of corners will have the full amount of torsional steel while C2 type of corners will have half of the amount provided in C1 type. The C3 type of corners do not need any torsional steel. The results are presented in Table 8.8 and Figs.8.19.10 a, b and c.

Table 8.8 Torsional reinforcement bars of Problem 8.2

Type	Dimensions along		Bar diameter & spacing	No. of bars along		Cl. no. of IS 456
	x (mm)	*y* (mm)		*x*	*y*	
C1	800	800	8 mm @ 200 mm c/c	5	5	D-1.8
C2	800	1600	8 mm @ 250 mm c/c	5	8	D-1.9
C2	1600	800	8 mm @ 250 mm c/c	8	5	D-1.9

STAIRCASES

Staircase is an important component of a building providing access to different floors and roof of the building. It consists of a flight of steps (stairs) and one or more intermediate landing slabs between the floor levels. Different types of staircases can be made by arranging stairs and landing slabs. Staircase, thus, is a structure enclosing a stair. The design of the main components of a staircase-stair, landing slabs and supporting beams or wall – are already covered in earlier lessons. The design of staircase, therefore, is the application of the designs of the different elements of the staircase.

4.11 Types of Staircases

Figures 9.20.1a to e present some of the common types of staircases based on geometrical configurations:

- Single flight staircase (Fig. 14.20a)
- Two flight staircases (Fig. 14.20b)
- Open-well staircase (Fig. 14.20c)
- Spiral staircase (Fig. 14.20d)
- Helicoidal staircase (Fig. 14.20e)

Architectural considerations involving aesthetics, structural feasibility and functional requirements are the major aspects to select a particular type of the staircase. Other influencing parameters of the selection are lighting, ventilation, comfort, accessibility, space etc.

Fig 4.20

Fig 4.20

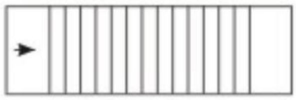


Fig. 9.20.1(a): Single flight staircase

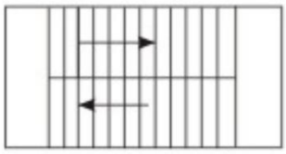


Fig. 9.20.1(b): Two flight staircase

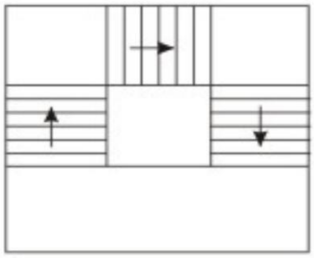


Fig. 9.20.1 (c): Open-well staircase

Fig. 9.20.1: Types of staircases

Fig 4.20

Fig 4.20

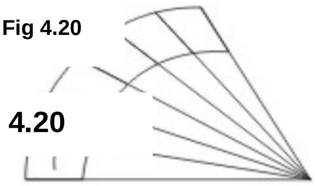


Fig. 9.20.1(e): Helicoidal staircase

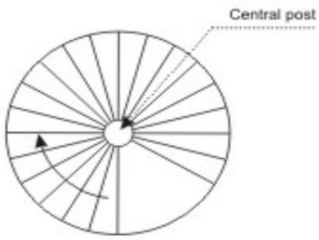


Fig. 9.20.1(d): Spiral staircase

Fig. 9.20.1: Types of staircases

4.12 A Typical Flight

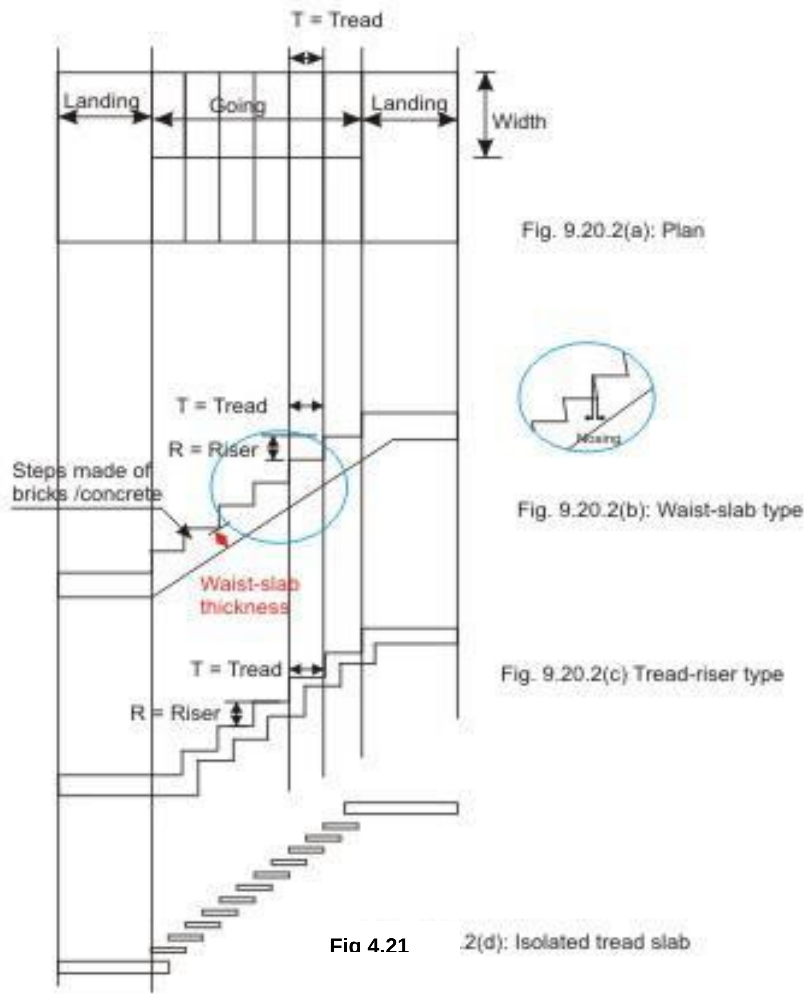


Fig 4.21 : A typical flight

Figures 14.21a to d present plans and sections of a typical flight of different possibilities. The different terminologies used in the staircase are given below:

- Tread: The horizontal top portion of a step where foot rests (Fig.14.21b) is known as tread. The dimension ranges from 270 mm for residential buildings and factories to 300 mm for public buildings where large number of persons use the staircase.
- Nosing: In some cases, the tread is projected outward to increase the space. This projection is designated as nosing (Fig.14.21b).
- Riser: The vertical distance between two successive steps is termed as riser (Fig.14.21b). The dimension of the riser ranges from 150 mm for public buildings to 190 mm for residential buildings and factories.

- Waist: The thickness of the waist-slab on which steps are made is known as waist (Fig.14.21b). The depth (thickness) of the waist is the minimum thickness perpendicular to the soffit of the staircase (cl. 33.3 of IS 456). The steps of the staircase resting on waist-slab can be made of bricks or concrete.
- Going: Going is the horizontal projection between the first and the last riser of an inclined flight (Fig.14.21a).
    - The flight shown in Fig.14.21a has two landings and one going. Figures 9.2b to d present the three ways of arranging the flight as mentioned below:

- waist-slab type (Fig.14.21b),
- tread-riser type (Fig.14.21c), or free-standing staircase, and
- isolated tread type (Fig.14.21d).

4.13 General Guidelines

The following are some of the general guidelines to be considered while planning a staircase:

ii The respective dimensions of tread and riser for all the parallel steps should be the same in consecutive floor of a building.

iii The minimum vertical headroom above any step should be 2 m.

iv Generally, the number of risers in a flight should be restricted to twelve.

v The minimum width of stair (Fig.14.2a) should be 850 mm, though it is desirable to have the width between 1.1 to 1.6 m. In public building, cinema halls etc., large widths of the stair should be provided.

4.14 Structural Systems

Different structural systems are possible for the staircase, shown in Fig. 9.20.3a, depending on the spanning direction. The slab component of the stair spans either in the direction of going i.e., longitudinally or in the direction of the steps, i.e., transversely. The systems are discussed below:

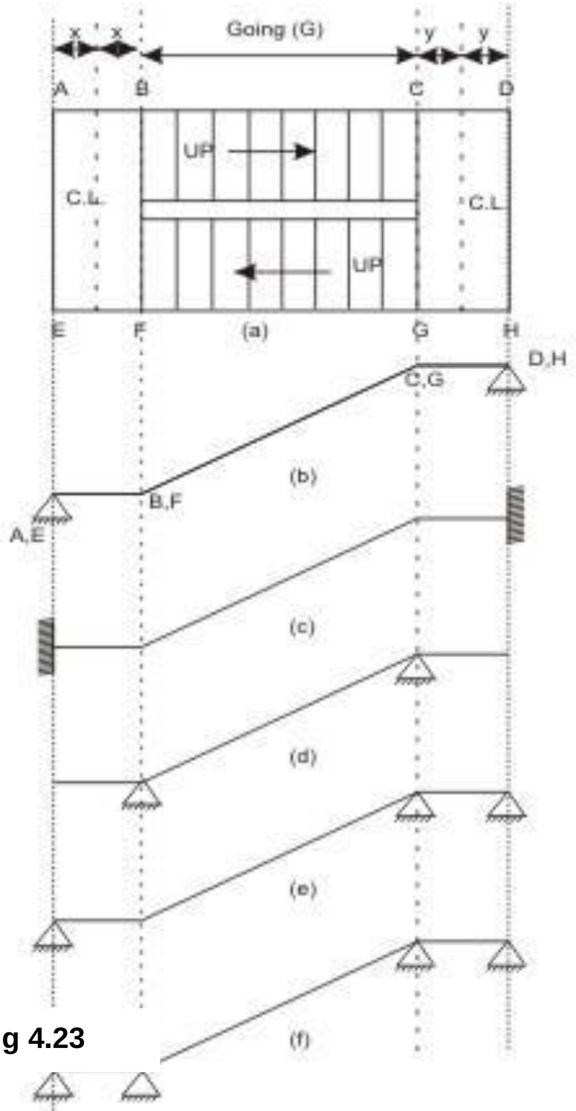


Fig 4.23

Fig. 9.20.3: Longitudinally spanning staircases

(A) Stair slab spanning longitudinally

Here, one or more supports are provided parallel to the riser for the slab bending longitudinally. Figures 14.23b to f show different support arrangements of a two-flight stair of Fig.14.23a:

- Supported on edges AE and DH (Fig.14.23b)
- Clamped along edges AE and DH (Fig.14.23c)
- Supported on edges BF and CG (Fig.14.23d)
- Supported on edges AE, CG (or BF) and DH (Fig.14.23e)
- Supported on edges AE, BF, CG and DH (Fig.14.23f)

Cantilevered landing and intermediate supports (Figs.14.23d, e and f) are helpful to induce negative moments near the supports which reduce the positive moment and thereby the depth of slab becomes economic.

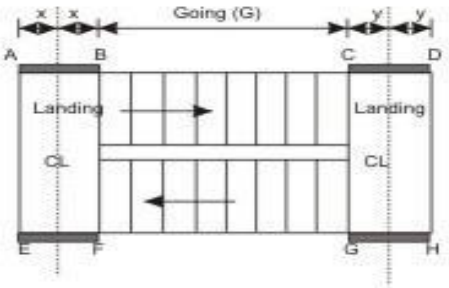


Fig. 9.20.4(a): Beams at two ends of landings

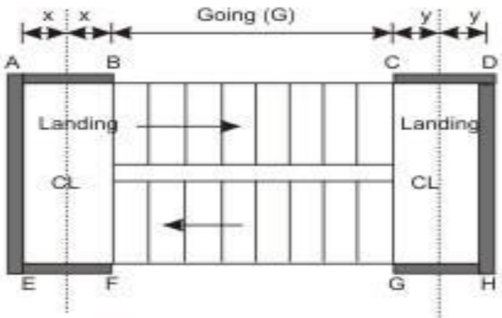


Fig. 9.20.4(b): Beams at three ends of landings

Fig. 9.20.4: Staircases (spanning longitudinally) and landings (spanning transversely)

Fig 4.24

In the case of two flight stair, sometimes the flight is supported between the landings which span transversely (Figs.14.24a and b). It is worth mentioning that some of the above-mentioned structural systems are statically determinate while others are statically indeterminate where deformation conditions have to take into account for the analysis.

Longitudinal spanning of stair slab is also possible with other configurations including single flight, open-well helicoidal and free-standing staircases.

(B) Stair slab spanning transversely

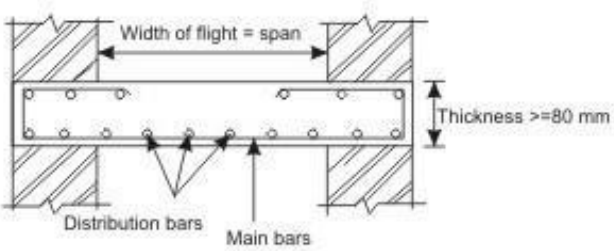


Fig 4.25 **(a)**: Slabs supported between two stringer beams or walls

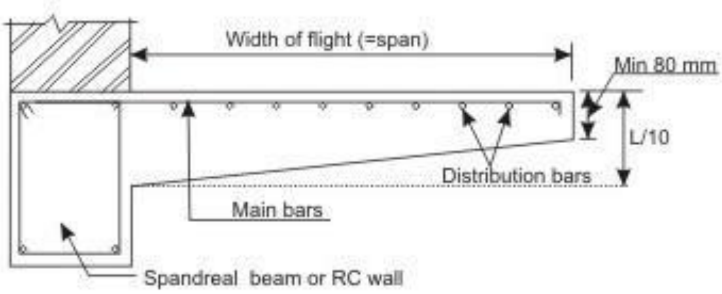


Fig 4.25 **(b)**: Cantilever slab from a spandreal beam or wall

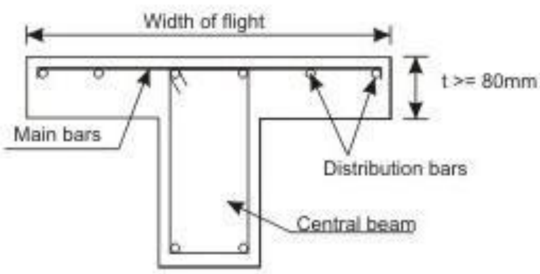


Fig 4.25 **(c)**: Doubly cantilever slab from a central beam

Fig 4.25 Transversely spanning staircases

Here, either the waist slabs or the slab components of isolated tread- slab and trade-riser units are supported on their sides or are cantilevers along the width direction from a central beam. The slabs thus bend in a transverse vertical plane. The following are the different arrangements:

- Slab supported between two stringer beams or walls (Fig.14.25a)
- Cantilever slabs from a spandreal beam or wall (Fig.14.25b)
- Doubly cantilever slabs from a central beam (Fig.14.25c)

4.15 Effective Span of Stairs

The stipulations of clause 33 of IS 456 are given below as a ready reference regarding the determination of effective span of stair. Three different cases are given to determine the effective span of stairs without stringer beams.

(ii) The horizontal centre-to-centre distance of beams should be considered as the effective span when the slab is supported at top and bottom risers by beams spanning parallel with the risers.

(iii)The horizontal distance equal to the going of the stairs plus at each end either half the width of the landing or one meter, whichever is smaller when the stair slab is spanning on to the edge of a landing slab which spans parallel with the risers. See Table 9.1 for the effective span for this type of staircases shown in Fig.14.23a.

Table 9.1 Effective span of stairs shown in Fig.14.23a

Sl. No.	x	y	Effective span in metres
1	< 1 m	< 1 m	$G + x + y$
2	< 1 m	≥ 1 m	$G + x + 1$
3	≥ 1 m	< 1 m	$G + y + 1$
4	≥ 1 m	≥ 1 m	$G + 1 + 1$

Note: G = Going, as shown in Fig. 14.23a

4.15 Distribution of Loadings on Stairs

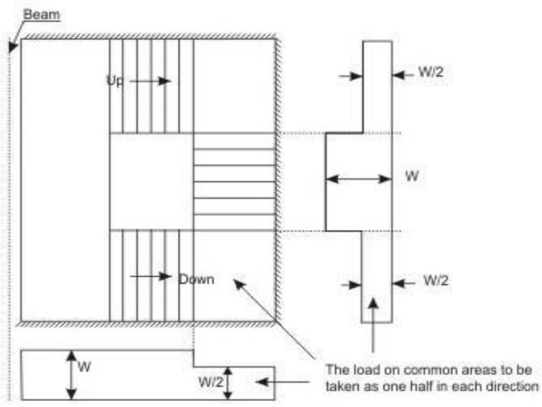


Fig. 9.20.6: Loadings on open-well staircases

Fig 4.26

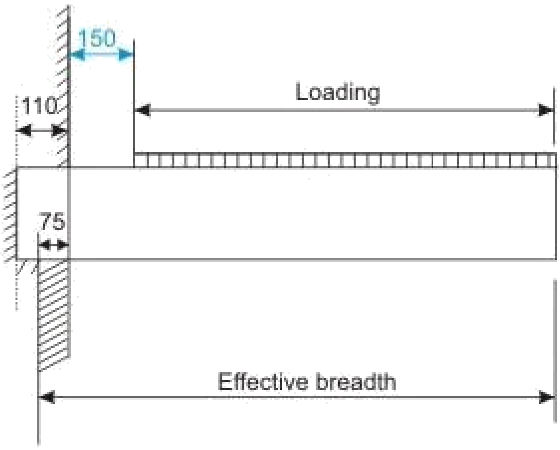


Fig. 9.20.7: Loading on staircases built into walls

Fig 4.27

Figure 4.26 shows one open-well stair where spans partly cross at right angle. The load in such stairs on areas common to any two such spans should be taken as fifty per cent in each direction as shown in Fig.4.27. Moreover, one 150 mm strip may be deducted from the loaded area and the effective breadth of the section is increased by 75 mm for the design where flights or landings are embedded into walls for a length of at least 110 mm and are designed to span in the direction of the flight (Fig.4.27).

4.16 Structural Analysis

Most of the structural systems of stair spanning longitudinally or transversely are standard problems of structural analysis, either statically determinate or indeterminate. Accordingly, they can be analysed by methods of analysis suitable for a particular system. However, the rigorous analysis is difficult and involved for a trade-riser type or free-standing staircase where the slab is repeatedly folded. This type of staircase has drawn special attraction due to its aesthetic appeal and, therefore, simplified analysis for this type of staircase spanning longitudinally is explained below. It is worth mentioning that certain idealizations are made in the actual structures for the applicability of the simplified analysis. The designs based on the simplified analysis have been found to satisfy the practical needs.

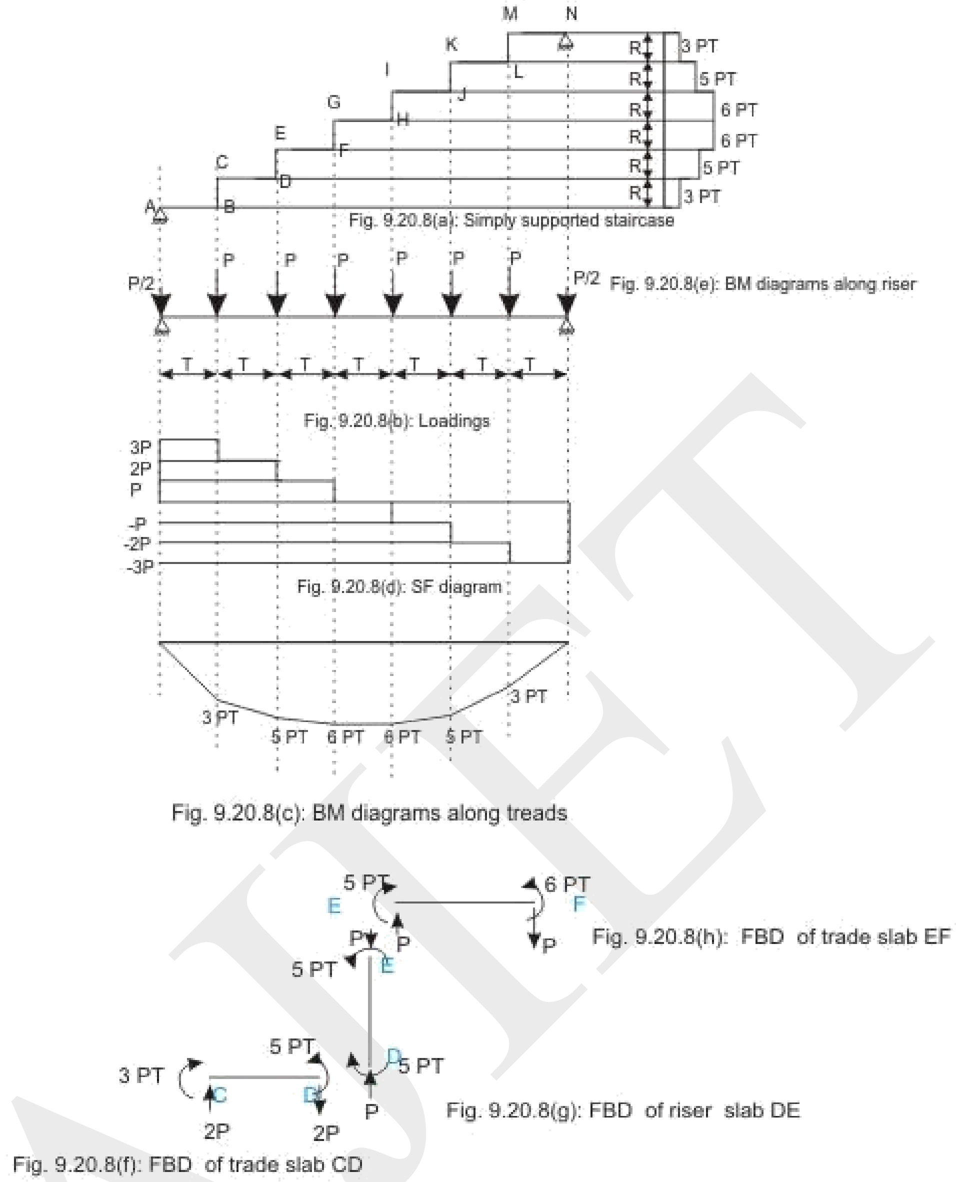


Fig. 9.20.8: Structural analysis of simply supported trade-riser staircase

Figure 14.28a shows the simply supported trade-riser staircase. The uniformly distributed loads are assumed to act at the riser levels (Fig.14.28b). The bending moment and shear force diagrams along the treads and the bending moment diagram along the risers are shown in Figs.14.28c, d and e, respectively. The free body diagrams of CD, DE and EF are shown in Figs.14.28f, g and h, respectively. It is seen that the trade slabs are subjected to varying bending moments and constant shear force (Fig.14.28f). On the other hand the riser slabs are subjected to a constant bending moment and axial force (either compressive or tensile). The assumption is that the riser and trade slabs are rigidly connected. It has been observed that both trade and riser slabs may be designed for bending moment alone as the shear stresses in trade slabs and axial forces in riser slabs are comparatively low. The slab thickness of the trade and risers should be kept the same and equal to span/25 for simply supported and span/30 for continuous stairs.

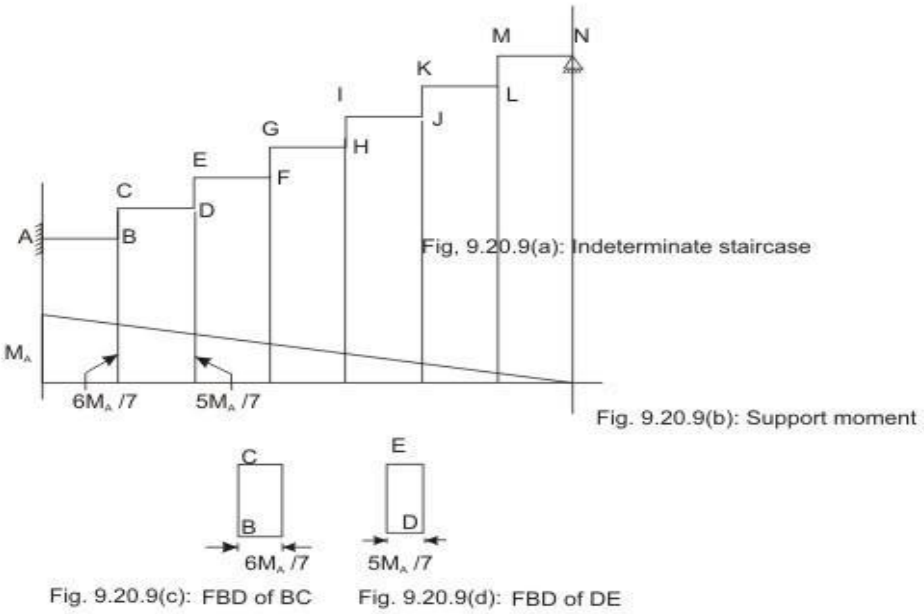


Fig. 9.20.9: Structural analysis of an indeterminate trade-riser staircase

Fig 4.29

Figure 14.29a shows an indeterminate trade-riser staircase. Here, the analysis can be done by adding the effect of the support moment MA (Fig.14.29b) with the results of earlier simply supported case. However, the value of MA can be determined using the moment-area method. The free body diagrams of two vertical risers BC and DE are show in Figs.14.29c and d, respectively.

Illustrative Examples

Two typical examples of waist-slab and trade-riser types spanning longitudinally are taken up here to illustrate the design.

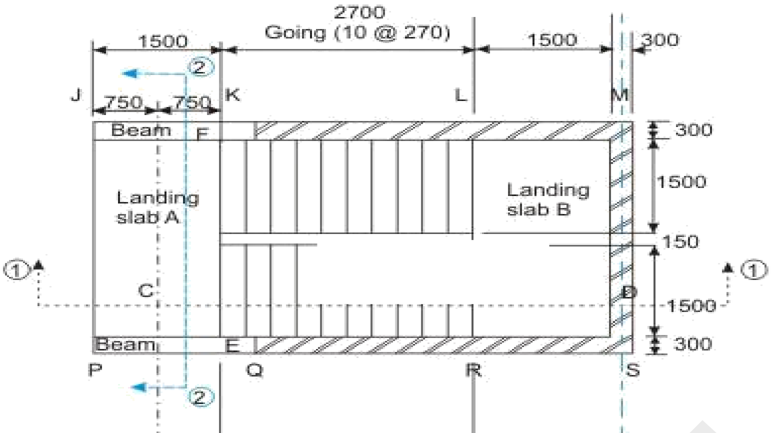


Fig. 9.20.10: Example 9.1

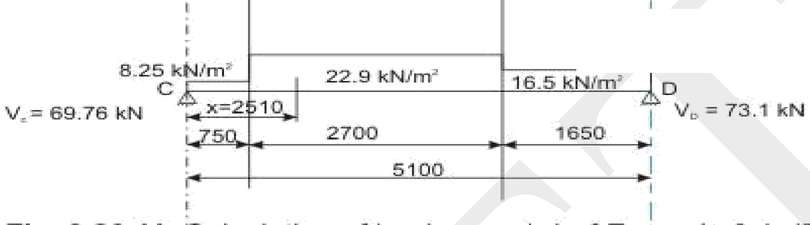


Fig. 9.20.11: Calculation of loads, sec 1-1 of Example 9.1, (Fig. 9.20.10)

Example .1:

Design the waist-slab type of the staircase of Fig.9.20.10. Landing slab A is supported on beams along JK and PQ, while the waist-slab and landing slab B are spanning longitudinally as shown in Fig.9.20.10. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Use riser R = 160 mm, trade T = 270 mm, concrete grade = M 20 and steel grade = Fe 415.

Solution:

With R = 160 mm and T = 270 mm, the inclined length of each step = $\{(160)^2 + (270)^2\}^{½}$ = 313.85 mm.

(iv) Design of going and landing slab B

Step 1: Effective span and depth of slab

The effective span (cls. 33.1b and c) = 750 + 2700 + 1500 + 150 = 5100

(ii) The depth of waist slab = 5100/20 = 255 mm. Let us assume total depth of 250 mm and effective depth = 250 – 20 – 6 = 224 mm (assuming cover = 20 mm and diameter of main reinforcing bar = 12 mm). The depth of landing slab is assumed as 200 mm and effective depth = 200 – 20 – 6 = 174 mm.

Step 2: Calculation of loads

(iii)Loads on going (on projected plan area)

Self-weight of waist-slab = 25(0.25) (313.85)/270 = 7.265 kN/m^2

Self-weight of steps = 25(0.5) (0.16) = 2.0 kN/m^2

Finishes (given) = 1.0 kN/m^2

Live loads (given) = 5.0 kN/m^2

Total = 15.265 kN/m^2

Total factored loads = 1.5(15.265) = 22.9 kN/m^2

Loads on landing slab A (50% of estimated loads)

Self-weight of landing slab = 25(0.2) = 5 kN/m^2

Finishes (given) = 1 kN/m^2

Live loads (given) = 5 kN/m^2

Total = 11 kN/m^2

Factored loads on landing slab A = 0.5(1.5) (11) = 8.25 kN/m^2

Factored loads on landing slab B = (1.5) (11) = 16.5 kN/m^2 the loads are drawn in Fig.9.20.11.

Step 3: Bending moment and shear force

Total loads for 1.5 m width of flight = 1.5{8.25(0.75) + 22.9(2.7) + 16.5(1.65)} = 142.86 kN

V_C = 1.5{8.25(0.75) (5.1 – 0.375) + 22.9(2.7) (5.1 – 0.75 – 1.35)

+ 16.5(1.65) (1.65) (0.5)}/5.1 = 69.76 kN

V_D = 142.86 – 69.76 = 73.1 kN

The distance x from the left where shear force is zero is obtained from:

(b) = {69.76 – 1.5(8.25) (0.75) + 1.5(22.9) (0.75)}/ (1.5) (22.9) = 2.51 m

The maximum bending moment at x = 2.51 m is

= 69.76(2.51) – (1.5) (8.25) (0.75) (2.51 – 0.375)

- (1.5) (22.9) (2.51 – 0.75) (2.51 – 0.75) (0.5) = 102.08 kNm.

For the landing slab B, the bending moment at a distance of 1.65 m from D

= 73.1(1.65) – 1.5(16.5) (1.65) (1.65) (0.5) = 86.92 kNm

Step 4: Checking of depth of slab

From the maximum moment, we get d = {102080/2(2.76)}^{½} = 135.98

d < 224 mm for waist-slab and < 174 mm for landing slabs. Hence, both the depths of 250 mm and 200 mm for waist-slab and landing slab are more than adequate for bending.

For the waist-slab, τ_v = 73100/1500(224) = 0.217 N/mm^2. For the waist-slab of depth 250 mm, k = 1.1 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456

τ_c = 1.1(0.28) = 0.308 N/mm^2. Table 20 of IS 456, $\tau_{c\ max}$ = 2.8 N/mm^2.

Since $\tau_v < \tau_c < \tau_{c\ max}$, the depth of waist-slab as 250 mm is safe for shear.

For the landing slab, τ_v = 73100/1500(174) = 0.28 N/mm^2. For the landing slab of depth 200 mm, k = 1.2 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456, τ_c = 1.2(0.28) = 0.336 N/mm^2 and from Table 20 of IS 456, $\tau_{c\ max}$ = 2.8 N/mm^2. Here also $\tau_v < \tau_c < \tau_{c\ max}$, so the depth of landing slab as 200 mm is safe for shear.

Step 5: Determination of areas of steel reinforcement

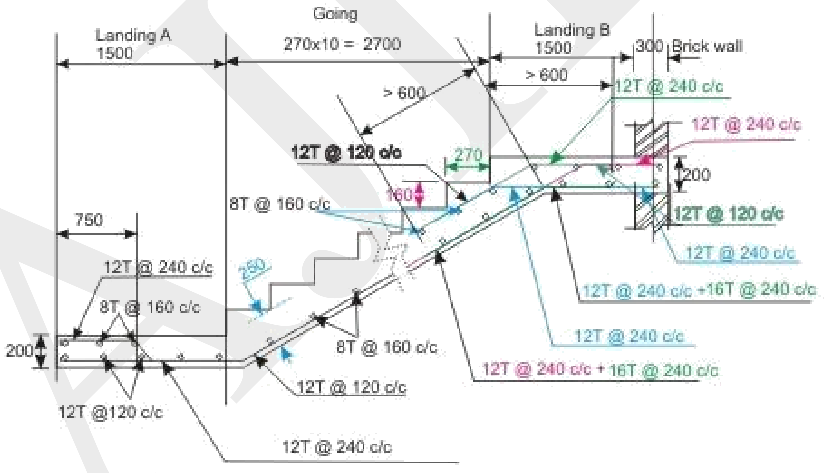


Fig. 9.20.12: Reinforcing bars of Example 9.1, sec 1-1 of Fig. 9.20.10

1. Waist-slab: M_u/bd^2 = 102080/ (1.5)224(224) = 1.356 N/mm^2. Table 2 of SP-16 gives p = 0.411.

The area of steel = 0.411(1000) (224)/ (100) = 920.64 mm^2. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m).

8. Landing slab B: M_u/bd^2 at a distance of 1.65 m from V_D (Fig. 9.20.11) = 86920/ (1.5) (174) (174) = 1.91 N/mm^2. Table 2 of SP-16 gives: p = 0.606. The area of steel

= 0.606(1000) (174)/100 = 1054 mm^2/m. Provide 16 mm diameter @ 240 mm c/c and 12 mm dia. @ 240 mm c/c (1309 mm^2) at the bottom of landing slab B of which 16 mm bars will be terminated at a distance of 500 mm from the end and will continue up to a distance of 1000 mm at the bottom of waist slab

Distribution steel: The same distribution steel is provided for both the slabs as calculated for the waist-slab. The amount is = 0.12(250) (1000)/100 = 300 mm^2/m. Provide 8 mm diameter @ 160 mm c/c (= 314 mm^2/m).

Step 6: Checking of development length and diameter of main bars

Development length of 12 mm diameter bars = 47(12) = 564 mm, say 600 mm and the same of 16 mm dia. Bars = 47(16) = 752 mm, say 800 mm.

(i) For waist-slab

M_1 for 12 mm diameter @ 120 mm c/c (= 942 mm^2) = 942(102.08)/920.64

(b) 104.44 kNm. With V (shear force) = 73.1 kN, the diameter of main bars ≤ {1.3(104440)/73.1}/47 ≤ 39.5 mm. Hence, 12 mm diameter is o.k.

(ii) For landing-slab B

M_1 for 16 mm diameter @ 120 mm c/c (= 1675 mm^2) = 1675(102.08)/1650.88 = 103.57 kNm. With V (shear force) = 73.1 kN, the diameter of main bars ≤ {1.3(103570)/73.1}/47 = 39.18 mm. Hence, 16 mm diameter is o.k.

(B) Design of landing slab A

Step 1: Effective span and depth of slab

The effective span is lesser of (i) (1500 + 1500 + 150 + 174), and (ii) (1500

20. 1500 + 150 + 300) = 3324 mm. The depth of landing slab = 3324/20 = 166 mm, < 200 mm already assumed. So, the depth is 200 mm.

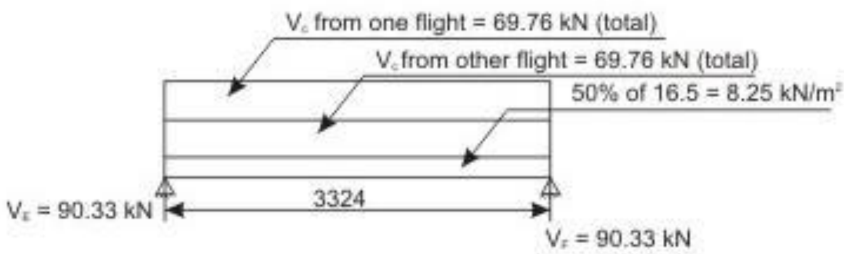


Fig. 9.20.13: Calculation of loads, sec 2-2 of Example 9.1, (Fig. 9.20.10)

Step 2: Calculation of loads (Fig.9.20.13)

The following are the loads:

Factored load on landing slab A (see Step 2 of A @ 50%) = 8.25 kN/m^2

Factored reaction V_C (see Step 3 of A) = 69.76 kN as the total load of one flight

Factored reaction V_C from the other flight = 69.76 kN

Thus, the total load on landing slab A

= (8.25) (1.5) (3.324) + 69.76 + 69.76 = 180.65 kN

Due to symmetry of loadings, VE = VF = 90.33 kN. The bending moment is maximum at the centre line of EF.

Step 3: Bending moment and shear force (width = 1500 mm)

Maximum bending moment = (180.65) (3.324)/8 = 75.06 kNm

Maximum shear force = 0.5(180.65) = 90.33 kN

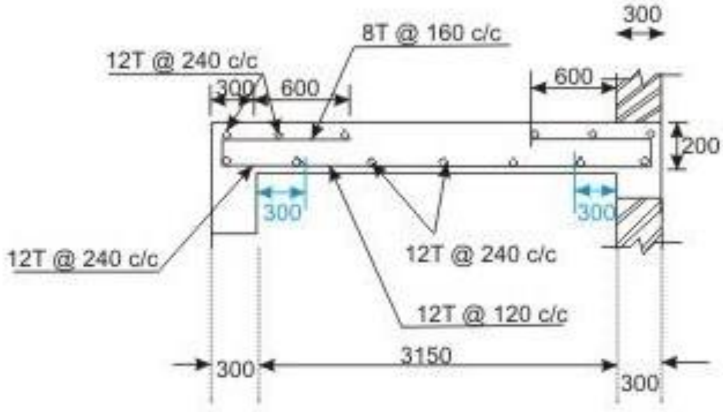
Step 4: Checking of depth of slab

In Step 3 of A, it has been observed that 135.98 mm is the required depth for bending moment = 102.08 kNm. So, the depth of 200 mm is safe for this bending moment of 75.06 kNm. However, a check is needed for shear force.

d_v = 90330/1500(174) = 0.347 N/mm^2 > 0.336 N/mm^2

The above value of τ c = 0.336 N/mm2 for landing slab of depth 200 mm has been obtained in Step 4 of A. However, here τ_c is for the minimum tensile steel in the slab. The checking of depth for shear shall be done after determining the area of tensile steel as the value of τ v is marginally higher.

Step 5: Determination of areas of steel reinforcement



For Mu/bd^2 = 75060/ (1.5) (174) (174) = 1.65 N/mm^2, Table 2 of SP-16 gives p = 0.512.

The area of steel = (0.512) (1000) (174)/100 = 890.88 mm2/m. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m). With this area of steel p = 942(100)/1000(174) = 0.541.

Distribution steel = The same as in Step 5 of A i.e., 8 mm diameter @ 160 mm c/c.

Step 6: Checking of depth for shear

Table 19 and cl. 40.2.1.1 gives: τ c = (1.2) (0.493) = 0.5916 N/mm2. τ v = 0.347 N/mm2 (see Step 3 of B) is now less than τ c (= 0.5916 N/mm2). Since, τ v

τ c < τ c max, the depth of 200 mm is safe for shear. The reinforcing bars are shown in Fig. 9.20.14.

4.18.6 Bearing Stresses at Bends (cl. 26.2.2.5 of IS 456)

The bearing stress inside a bend is to be calculated from the expression:

Bearing stress = Fbt/ rφ

where Fbt = tensile force due to design loads in a bar or group of bars,

r = internal radius of the bend, and

φ = size of the bar or bar of equivalent area in bundled bars. The calculated bearing stress of Eq.6.20 shall not exceed the following:

Calculated bearing stress >/ 1.5 fck/1 + 2 φ /a where fck = characteristic cube strength of concrete

a = center to center distance between bars or groups of bars perpendicular to the plane of the bend. For bars adjacent to the face of the member, a shall be taken as cover plus size of the bar φ.

4.18.9 Change in Direction of Reinforcement (cl. 26.2.2.6 of IS 456)

In some situations, the change in direction of tension or compression reinforcement induces a resultant force. This force may have a tendency to split the concrete and, therefore, should be taken up by additional links or stirrups. Normally, this aspect is taken care while detailing of bars is carried out.

4.18.10 Reinforcement Splicing (cl. 26.2.5 of IS 456)

Reinforcement is needed to be joined to make it longer by overlapping sufficient length or by welding to develop its full design bond stress. They should be away from the sections of maximum stress and be staggered. IS 456 (cl. 26.2.5) recommends that splices in flexural members should not be at sections where the bending moment is more than 50 per cent of the moment of resistance and not more than half the bars shall be spliced at a section.

(a) Lap Splices (cl. 26.2.5.1 of IS 456) The following are the salient points:

- They should be used for bar diameters up to 36 mm.
- They should be considered as staggered if the centre-to-centre distance of the splices is at least 1.3 times the lap length calculated as mentioned below.
- The lap length including anchorage value of hooks for bars in flexural tension shall be Ld or 30φ, whichever is greater. The same for direct tension shall be 2Ld or 30φ, whichever is greater.
- The lap length in compression shall be equal to Ld in compression but not less than 24φ.
- The lap length shall be calculated on the basis of diameter of the smaller bar when bars of two different diameters are to be spliced.
- Lap splices of bundled bars shall be made by splicing one bar at a time and all such individual splices within a bundle shall be staggered.

(b) Strength of Welds (cl. 26.2.5.2 of IS 456)

The strength of welded splices and mechanical connections shall be taken as 100 per cent of the design strength of joined bars for compression splices.

For tension splices, such strength of welded bars shall be taken as 80 per cent of the design strength of welded bars. However, it can go even up to 100 per cent if welding is strictly supervised and if at any cross-section of the member not more than 20 per cent of the tensile reinforcement is welded. For mechanical connection of tension splice, 100 per cent of design strength of mechanical connection shall be taken.

IMPORTANT QUESTIONS

1. Explain the structural action of one way and two-way slabs with the help of sketches

2. Design a RC slab for a hall measuring 6.5 m x 5 m. The slab is cast monolithically over the beam with comers held down. The bearing is 250 mm. The slab carries superimposed load 3 kN/rn^2. Use M20 and Fe 415 steel. Sketch the details of steel.

3. Design RC panel discontinuous and restrained all-round, has an effective span 3.5m x 5.0m Live load is 2 kN/m^2 and floor finish is 0.6 kN/m^2. Use M20 grade Fe-415 grade steel. All corners are held down

4. The main stair of an office building has to be located in a stair case room measuring 2.5m x 5.6m. The vertical distance between the floors is 3.75m. Live load on stairs id 5 kN/m^2. Design the flight slab using M20 and Fe 415 if flight slab and landing slab span in the same direction.

5. Explain Importance of Bond, Anchorage length

6. What is development length? Obtain the expression for development length in tension?