VISVESVARAYA TECHNOLOGICAL UNIVERSITY

BELGAUM

PAVEMENT DESIGN

(Subject Code: 18CV825)

LECTURE NOTES

(MODULE-4)

VIII-SEMESTER

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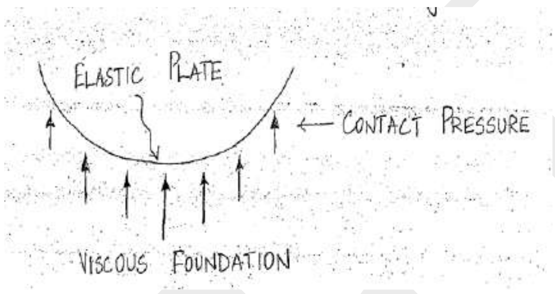
Module - 4
Stresses in Rigid Pavement and Design of Rigid Pavement

Principle

Rigid pavements are pavements with sufficient flexural strength to transmit the wheel load stresses to a wider area below. Compared to flexible pavements, rigid pavements are placed either directly on the prepared subgrade or on a single layer of granular or stabilized material.

In rigid pavement, load is distributed by slab action and the pavement behaves like an elastic plate resting on a viscous medium as shown in the figure below.



They are analyzed by Plate Theory, according to which, the concrete slab acts as a medium thick plate which is plane before loading and to remain plane after loading. The design is based on providing flexural strength in a structural slab to resist destructive action of wheel loads.

Rigid pavement because of its rigidity and high modulus of elasticity tends to distribute the load over a relatively wider area of soil.

Factors affecting the design of Rigid Pavements

The various factors governing the design of rigid pavements are:

1) Wheel load and its repetitions
2) Properties of subgrade
   - Subgrade strength and properties
   - Sub-base provision or omission
3) Properties of concrete
   - Strength
   - Modulus of elasticity
   - Poisson's Ratio
   - Shrinkage properties
   - Fatigue behavior
4) External conditions
   - Temperature changes
   - Friction between slab and subgrade
5) Joints
   - Arrangements of joints
6) Reinforcement
   - Quantity of reinforcement
   - Continuous reinforcement

Wheel Load and its Repetitions

→ Generally, stresses are induced in the slab due to the wheel load. The greater the wheel load, the greater are the stresses. For design purposes in our country, a wheel load of 4100kg is currently adopted.

→ The impact of these stresses, especially at the joints, is a serious concern in the design of concrete slabs. This impact can be minimized if an effective load-transferring device is provided at the joints. For design purposes, the stresses caused by static wheels are increased by 25 percent to account for this impact. If such load transfer devices are not provided, the stresses are increased by 50 percent.

→ The number of repetitions of the wheel load has a significant effect on rigid pavement performance. As the number of repetitions increases, the serviceability index decreases and the concrete suffers fatigue when subjected to repeated load applications.

→ By knowing the present traffic and the expected rate of growth of traffic over the design period, one can assess the load repetitions. IRC suggests a design period of 20 years & 7.5 percent annual rate of growth of traffic.

Properties of Subgrade

1) Subgrade Strength and Properties

→ The strength of the soil subgrade has an important role in the design of a slab in terms of its bearing. The design is simplified, if the soil has uniform bearing power.

→ This bearing power is measured by the plate bearing test using a 75cm dia. circular plate. The property measured is the "modulus of subgrade reaction", K, which is the measure of resistance of the soil to deformation under the pressure caused by the bending slab.

→ Other properties of the subgrade which affect the pavement performance are its drainage characteristics, tendency for volumetric changes with changes in the moisture content & tendency towards frost action.

2) Sub-base Provision or Omission

→ Sub-base is the layer of granular material placed on the subgrade soil and immediately below the concrete pavement. It is also called the base.

→ Sub-base adds to the strength of the sub-grade but it is difficult to evaluate how this strength is provided by the sub-base.

Properties of Concrete

1) Strength of Concrete

→ The design of a concrete pavement slab is dependent upon the strength of the concrete.

→ It is usually measured in terms of crushing strength & rarely fails in compression.

→ Also, the flexural strength of a concrete pavement slab is generally determined by subjecting a beam of concrete (150mm x 150mm x 150mm) to flexure. The span for testing is equal to four times the depth and the load is applied at the third points of the span. The flexural strength is evaluated as modulus of rupture.

→ Indian practice specifies a minimum modulus of rupture of 40 kg/cm^2.

2) Modulus of Elasticity

→ Modulus of Elasticity, E, helps in determining the relative stiffness of the slab. It increases with the strength of concrete.

→ This property is determined by the static method by stress-strain relationship or by dynamic method.

→ This value is about $3x10^5$ kg/cm^3 for concrete having the flexural strength in the range of 38-42 kg/cm^2.

3) Poisson's Ratio

→ Poisson's ratio is determined by static & dynamic method.

→ Its value by static method is around 0.15 and by dynamic method is around 0.24.

4) Shrinkage Properties of Concrete

→ Concrete expands slightly during setting due to hydration of cement and as it dries, it shrinks. Such shrinkage causes some stresses.

→ Changes in moisture content i.e., volumetric changes can also cause shrinkage or expansion.

5) Fatigue Behaviour of Concrete

→ Permanent internal structural damage takes place in concrete as it is subjected to repetitive stresses.

→ As the stress ratio (flexural stress to flexural strength) increases, the concrete is able to resist very few repetitions.

→ When the stress ratio does not exceed 0.55, concrete will withstand unlimited stress repetitions without any reduction in load carrying capacity.

→ For design purposes, the value of stress ratio is taken as 0.50.

External Conditions

1) Temperature Changes

→ Changes in temperature through the slab will cause differential expansion or contraction between the top and bottom of the slab.

→ The slab then tends to warp but is prevented due to slab weight and friction at load transferring devices. Thus, stresses are induced at these cases.

→ The expansion and contraction of the slab due to temperature changes is restrained due to the friction between the subgrade and the slab. This causes stresses in the slab.

2) Friction between Slab and Sub-base

→ The amount of friction between the slab and the sub-base determines the restraint imposed on expansion and contraction due to temperature changes.

→ The spacing of joints is also affected by this friction.

→ Compacted sand and gravel covered with waterproof paper, provides a very smooth surface.

→ Waterbound macadam, soil-gravel mix, rolled lean concrete, etc., give rough surfaces.

Joints

Arrangement of Joints

→ Joints are needed for allowing expansion, contraction and warping of the slab.

→ The width of the CC pavement slab depends on the spacing between the longitudinal joints. The length of the slab depends on the spacing between contraction joints.

→ For a given value of temperature differential, the magnitude of warping stresses in a CC pavement depends on the length and width of the pavement slab.

→ Therefore, in locations where the temperature differential is high, at the design stage it is possible to decrease the warping stress by decreasing the spacing between the contraction joints.
→ The spacing and arrangement of joints govern the stresses induced in the slab.

Reinforcement

→ A slab can be un-reinforced or reinforced. The amount of reinforcement is an important consideration in the design.
→ The function of reinforcement in a CC pavement is not to increase the resisting moment of slab strength.
→ Its role is to control the opening of the cracks. It holds together and ties the two sides of the crack and prevents the crack from widening.
→ It also counteracts the tensile stresses caused by shrinkage and contraction due to changes in temperature and moisture content.
→ Since cracks starting with higher tensile stresses at the top surface are more critical when they tend to open, Indian practice is to place the steel at about 50mm below the surface.

Types of Stresses

Different types of stresses are developed in CC pavements. The major types of stresses in CC pavements consist of: (i) wheel load stresses caused by the heavy wheel loads of vehicles and (ii) warping stresses caused by the temperature differential between the top and bottom of the pavement due to daily variation in temperature.

Other factors which cause additional stresses in CC pavements are (a) due to seasonal variation in temperature resulting in overall expansion and contraction of the slab causing frictional stresses, as the friction force resists the slab movement and (b) due to volumetric changes of the subgrade or supporting layers, which result in non-uniform support or lack of support to the CC pavement slab at certain locations/stretches.

Analysis of Stresses

Westergaard's Analysis

H. M. Westergaard is considered the pioneer in providing the rational approach to the analysis of stresses in rigid pavements. His analysis was based on the following assumptions:

1) The CC pavement slab is homogeneous, isotropic and has uniform elastic properties.
2) The reaction of the subgrade is vertical only and is proportional to the deflection of the slab.
3) The reaction of the subgrade at a point is equal to K x Deflection at that point, where K is the Modulus of Subgrade Reaction. (fig 1)
4) The slab is uniform in thickness.
5) The load in the interior and the corner is circular in shape and the edge loading is semi-circular. (fig 2)

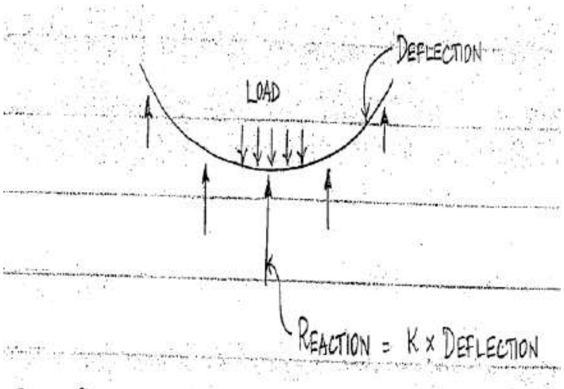


FIG.1: REACTION OF SUBGRADE UNDER CONCRETE SLAB

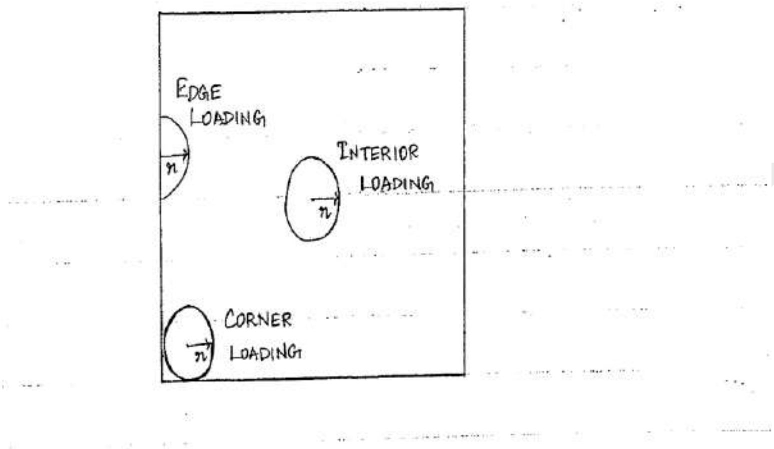


FIG. 2: LOADING POSITIONS FOR RIGID PAVEMENT DESIGN

Important Definitions

1) Modulus of Subgrade Reaction (K)

The modulus of subgrade reaction, K is proportional to the displacement, Δ. The displacement level of Δ is taken as 0.125cm for the determination of K. If 'p' is the pressure sustained in kg/cm^2 by the rigid plate of diameter 75cm at a deflection, Δ = 0.125cm, the modulus of subgrade reaction K is given by:

$$K = \frac{p}{\Delta} = \frac{p}{0.125} \text{kg/cm}^3$$

Thus, Modulus of subgrade reaction, K may be defined as the pressure sustained per unit deformation of subgrade at specified deformation or pressure level, using specified plate size.

2) Radius of Relative Stiffness (*l*)

A certain degree of resistance to slab deflection is offered by the subgrade. The tendency of the slab to deflect is dependent upon its flexural strength, which depends on its thickness and the strength characteristics of the pavement slab. The relative stiffness of the slab with respect to the subgrade support is dependent upon the properties of the slab and the pressure-deformation characteristics of the subgrade material.

Westergaard defined this term as the 'radius of relative stiffness', *l* which is expressed by the equation,

$$l = \left[\frac{Eh^3}{12K\,(1-\mu^2)}\right]^{1/4}$$

where,

l = radius of relative stiffness, cm
h = slab thickness, cm
E = modulus of elasticity of cement concrete kg/cm^2
μ = Poisson's ratio for concrete = 0.15
K = subgrade modulus or modulus of subgrade reaction, kg/cm^3

3) Critical positions of loading

Since the pavement slab has finite length and width, the intensity of maximum stress induced by the application of a given wheel load is dependent on the position of the load on the pavement slab.

Three typical positions, namely the interior, edge and corner were considered by Westergaard with reference to the continuity of the rigid pavement slab. These are termed as 'critical load positions'.

When the load is applied in the interior of the slab surface at any place remote from all the edges, it is called 'interior loading'.

When load is applied on an edge of the slab at any place remote from a corner, it is called 'edge loading'.

When the centre of the load applied is located on the bisector of the corner angle formed by two intersecting edges of the slab and the loaded area is at the corner touching the two corner edges, it is called 'corner loading'.

4) Equivalent radius of resisting section

Considering the case of interior loading, the maximum bending moment occurs at the loaded area and acts radially in all directions. With the load concentrated on a small area of the pavement, to what sectional area of the pavement is effective in resisting the bending moment is the question that arises.

According to Westergaard, the equivalent radius of resisting section is approximated, in terms of radius of load distribution and slab thickness,

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

where, b = equivalent radius of resisting section (cm) when "a" is less than 1.724h
a = radius of wheel load distribution, cm
h = slab thickness, cm
when "a" is greater than 1.724h, the value of b = a

Problems

1) Compute the radius of relative stiffness of 15cm thick C.C. slab using the following data:
Modulus of elasticity of cement concrete = 2.1 x 10^5 kg/cm^2
Poisson's Ratio for concrete = 0.15
Modulus of subgrade reaction, K = (a) 3.0 kg/cm^3, (b) 7.5 kg/cm^3

Solution:

(a) For K = 3.0kg/cm^3

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4} = \left[\frac{2.1 \times 10^5 \times 15^3}{12 \times 3 \times (1-0.15^2)}\right]^{1/4}$$

l = 67cm

(b) For K = 7.5 kg/cm^3

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4} = \left[\frac{2.1 \times 10^5 \times 15^3}{12 \times 7.5 \times (1-0.15^2)}\right]^{1/4}$$

l = 53.3 cm

2) Find the radius of relative stiffness of 20cm thick CC slab. If Modulus of elasticity of cement concrete is 0.21 x 10^6 kg/cm^2, Poisson's ratio 0.15 and Modulus of subgrade reaction (i) 2.5kg/cm^3, (ii) 1 kg/cm^3.

Solution:

(i) For K = 2.5 kg/cm^3

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4} = \left[\frac{0.21 \times 10^6 \times 20^3}{12 \times 2.5 \times (1-0.15^2)}\right]^{1/4}$$

l = 87cm

(ii) For K = 1kg/cm^3

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4} = \left[\frac{0.21 \times 10^6 \times 20^3}{12 \times 1 \times (1-0.15^2)}\right]^{1/4}$$

l = 109.4cm

3) Compute the equivalent radius of resisting section of 20cm thick slab, given that the radius of contact area wheel load is 15cm.

Solution:

Given: h = 20cm

a = 15cm

a/h = 15/20 = 0.75 < 1.724

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

$$= \sqrt{1.6\ x\ 15^2 + 20^2} - 0.675\ x\ 20$$

= 14.07cm

4) Determine the equivalent radius of resisting section of 20cm thick slab given that the ratio of radius of wheel load distributed to the thickness of slab is 0.4.
Solution:
Given: h =20cm

a/h = 0.4

a = 0.4 x h
= 0.4 x 20
= 8cm

As a < 1.724h,

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

$$= \sqrt{1.6\ x\ 8^2 + 20^2} - 0.675\ x\ 20$$

= 8.91cm

Westergaard's Wheel Load Stress Equations

In Westergaard's theoretical analysis of stresses in rigid pavements, the cement concrete slab is assumed to be a homogeneous, thin elastic plate with subgrade reaction being vertical and proportional to the deflection. Westergaard's equations for stresses due to wheel load applied at the three critical locations of interior, edge and corner are given below:

Load stress, S_i due to interior loading,

$$S_i = \frac{0.316P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right]$$

Load stress, S_e due to edge loading,

$$S_e = \frac{0.572P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right]$$

Load stress, S_c due to corner loading,

$$S_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

where,

S_i, S_e, S_c = maximum stress at interior, edge and corner regions of the slab respectively due to applied load P, kg/cm^2

h = slab thickness, cm

P = wheel load, kg

a = radius of wheel load distribution, cm

l = radius of relative stiffness, cm

b = radius of resisting section, cm

If the corner load stress exceeds the flexural strength of the CC slab, a crack is likely to develop across the diagonal on the top surface of the pavement. Maximum stress produced by a wheel load at corner does not exist around the load, but it occurs at some distance 'X' along the diagonal. This distance 'X' from the corner is given by the relation,

$$X = 2.58\sqrt{al}$$

Where, X = Distance from apex of slab corner to section of maximum stress along the corner bisector or diagonal, cm

a = radius of wheel load distribution, cm

l = radius of relative stiffness, cm

Problems

1) Calculate the stresses at interior, edge and corner regions of CC pavement using Westergaard's Analysis. Use the following data:
   Wheel load = 4100kg
   $E_c = 2.1 \times 10^5$ kg/cm^2
   Pavement thickness = 20cm
   Poisson,s ratio of concrete = 0.15
   Modulus of subgrade reaction, K = 2.5 kg/cm^3
   Radius of contact area, a = 8.91cm
   Solution:
   Radius of relative stiffness,

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4} = \left[\frac{2.1 \times 10^5 \times 20^3}{12 \times 2.5 \times (1-0.15^2)}\right]^{1/4} = 87\text{cm}$$

Equivalent radius of resisting section, $b = \sqrt{1.6a^2 + h^2} - 0.675h$

$$= \sqrt{1.6 \times 8.91^2 + 20^2} - 0.675 \times 20$$

$$= 9.46\text{cm}$$

Stress at the interior region,

$$S_i = \frac{0.316P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right]$$

$$S_i = \frac{0.316 \times 4100}{20^2}\left[4\log_{10}\left(\frac{87}{9.46}\right) + 1.069\right]$$

S_i = 15.95 kg/cm^2

Stress at the edge region,

$$S_e = \frac{0.572P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right]$$

$$S_e = \frac{0.572\ x\ 4100}{20^2}\left[4\log_{10}\left(\frac{87}{9.46}\right) + 0.359\right]$$

S_e = 24.70 kg/cm^2

Stress at the corner region,

$$S_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

$$S_c = \frac{3\ x\ 4100}{20^2}\left[1 - \left(\frac{8.91\sqrt{2}}{87}\right)^{0.6}\right]$$

S_c = 21.1 kg/cm^2

2) Using the data given below, calculate the wheel load stresses at (a) interior (b) edge and (c) corner regions of a CC pavement using Westergaard's stress equations. Also determine the probable location where the crack is likely to develop due to corner loading.

Wheel load, P = 5100kg

Ec = 3.0x10^5 kg/cm^2

Pavement thickness, h = 18cm

Poisson's ratio of concrete, μ = 0.15

Modulus of subgrade reaction, K = 6.0 kg/cm^3

Radius of contact area, a = 15cm

Solution:

Radius of relative stiffness,

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4} = \left[\frac{3\ x\ 10^5\ x\ 18^3}{12\ x\ 6\ x\ (1-0.15^2)}\right]^{1/4} = 70.61\text{cm}$$

Equivalent radius of resisting section,

a/h = 15/18 = 0.833 < 1.724

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

$$= \sqrt{1.6\ x\ 15^2 + 18^2} - 0.675\ x\ 18$$

= 14 cm

Stress at the interior region,

$$S_i = \frac{0.316P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right]$$

$$S_i = \frac{0.316 \ x\ 5100}{18^2}\left[4\log_{10}\left(\frac{70.61}{14}\right) + 1.069\right]$$

S_i = 19.3 kg/cm^2

Stress at the edge region,

$$S_e = \frac{0.572P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right]$$

$$S_e = \frac{0.572 \ x\ 5100}{18^2}\left[4\log_{10}\left(\frac{70.61}{14}\right) + 0.359\right]$$

S_e = 28.54 kg/cm^2

Stress at the corner region,

$$S_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

$$S_c = \frac{3\ x\ 5100}{18^2}\left[1 - \left(\frac{15\sqrt{2}}{70.61}\right)^{0.6}\right]$$

S_c = 24.27 kg/cm^2

Location where corner load crack develops:

Distance from the corner of the slab, X = 2.58 $\sqrt{al}$

X = 2.58 x $\sqrt{15 \text{ x } 70.61}$

= 84cm

Modified Westergaard's Equations for Wheel Load Stress

The load stress equations analytically derived by Westergaard were subsequently examined by several other investigators by carrying out experimental studies using suitable instrumentations and they suggested modifications to Westergaard's stress formulas. For example, Westergaard's edge load stress formula was modified by Teller and Sutherland. Similarly, Westergaard's corner load stress formula was modified by Kelley.

Edge load stress equation by Teller and Sutherland

Edge load stress equation was modified by Teller and Sutherland and is given by:

$$S_e = 0.529\,\frac{P}{h^2}\,(1 + 0.54\mu) \times \left[4\log_{10}\left(\frac{l}{b}\right) + \log_{10} b - 0.4048\right]$$

Corner load stress equation by Kelley

Corner load stress equation was modified by Kelley and is given by:

$$S_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right]$$

where,

S_e = load stress at the edge region, kg/cm^2

S_c= load stress at the corner region, kg/cm^2

P = design wheel load, kg

h = thickness of CC pavement slab, cm

μ= Poisson's ratio of the CC slab = 0.15

$a =$ radius of wheel load distribution, cm

l = radius of relative stiffness, cm

b = radius of resisting section, cm

E = modulus of elasticity of the CC, kg/cm^2

K = modulus of subgrade reaction, kg/cm^3

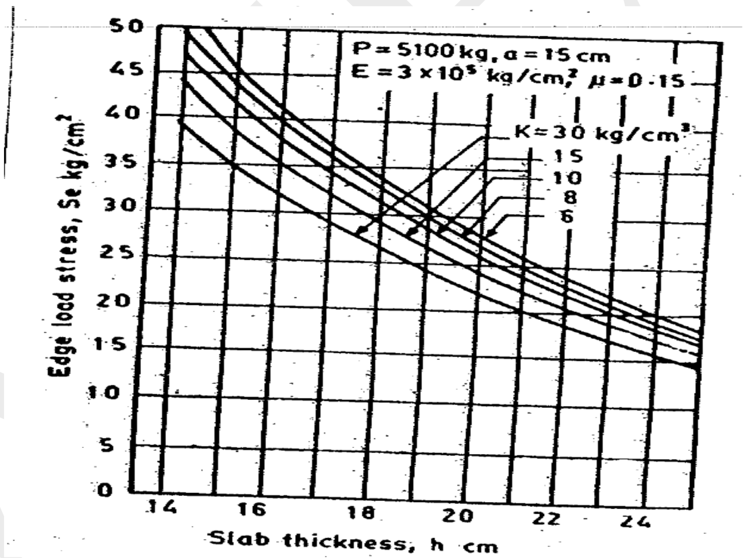


Fig: Edge Load Stress Chart (IRC)

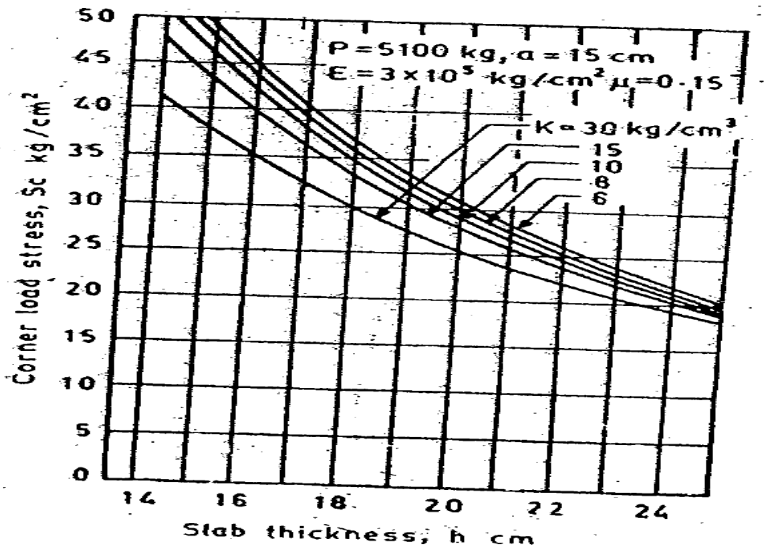


Fig: Corner Load Stress Chart (IRC)

Problems

1) Using the data in the previous problem, determine (a) edge load stresses using modified equation of Teller and Sutherland, (b) corner load stress using modified equation of Kelley.

Solution:

Given: P = 5100kg

h = 18 cm

μ = 0.15

a = 15 cm

l = 70.61cm

b = 14 cm

(a) Edge load stress using equation by Teller and Sutherland,

$S_e = 0.529 \frac{P}{h^2} (1 + 0.54\mu) \times [4 \log_{10} \left(\frac{l}{b}\right) + \log_{10} b - 0.4048]$

$S_e = 0.529 \times \frac{5100}{18^2} (1 + 0.54 \times 0.15) \times [4 \log_{10} \left(\frac{70.61}{14}\right) + \log_{10} 14 - 0.4048]$

$S_e = 31.98$ kg/cm^2

(b) Corner load stress using equation by Kelley,

$S_c = \frac{3P}{h^2} \left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right]$

$S_c = \frac{3 \times 5100}{18^2} \left[1 - \left(\frac{15 \times \sqrt{2}}{70.61}\right)^{1.2}\right]$

$S_c = 36.06$ kg/cm^2

2) A CC pavement of thickness 20cm rests on a WBM base course with modulus of reaction 30 kg/cm^3. Find the load stresses at the edge and corner regions under a wheel load of 5100kg using IRC stress charts. Assume a = 15cm, E = 3 x 10^5 kg/cm^2 and μ = 0.15

Solution:

(a) Using Edge Load Stress Chart, for K = 30 kg/cm^3 & h = 20cm

Edge Load Stress, S_e = 22.0 kg/cm^2

(b) Using Corner Load Stress Chart, for K = 30 kg/cm^3 & h = 20 cm

Corner Load Stress, S_c = 25.5 kg/cm^2

Temperature Stresses

Variation in temperature takes place with change in time, season and weather. When cement concrete pavement slab is exposed to higher temperature, the top surface gets heated up while bottom of the slab has no effect or very little effect. At night, drop in temperature of the top surface is at much faster rate than that at the bottom. This difference in the temperature of top and bottom surface of the slab tends to warp and bend.

Temperature, thus, tends to produce two types of stresses in concrete pavements, namely,

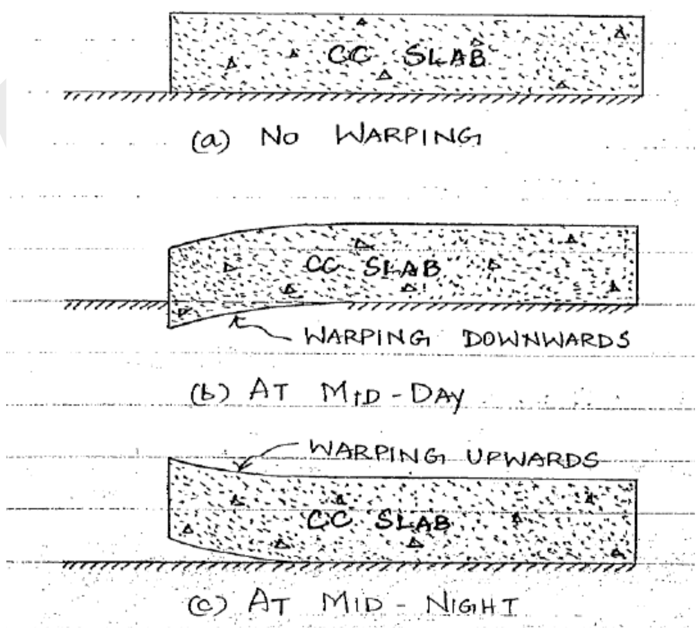
1) Warping Stresses
2) Frictional Stresses

Warping Stresses

Warping Stresses are developed due to daily temperature variations i.e., difference in temperature of top and bottom layers.

→ If top surface is at higher temperature (at mid-day) than that of lower surface, the slab will tend to warp downwards because the top layers expand due to high temperature.

→ At night time, the slab will tend to warp upwards because the top layers contract by larger amount than that of lower layers, which are at lower temperature.



For theoretical computations,

(i) At Interior region,

$$S_{t(i)} = \frac{Eet}{2}\left[\frac{Cx + \mu Cy}{1 - \mu^2}\right]$$

(ii) At Edge region,

$$S_{t(e)} = \frac{Cx\, Eet}{2} \text{ or } \frac{Cy\, Eet}{2} \text{ (whichever is higher)}$$

(iii) At Corner region,

$$S_{t(c)} = \frac{Eet}{3(1-\mu)}\sqrt{\frac{a}{l}}$$

where, $S_{t(i)}$, $S_{t(e)}$, $S_{t(c)}$ = warping stress at interior, edge & corner regions in kg/cm^2

E = modulus of elasticity of concrete, kg/cm^2

e = thermal coefficient of concrete per °C

t = temperature difference between the top and bottom of the slab in °C

L_x and L_y are the length and width of the slab along directions X and Y

C_x = coefficient in direction X, which depends on the ratio, L_x/l

C_y = coefficient in direction Y, which depends on the ratio, L_y/l

μ = Poisson's ratio of CC (which may be taken as 0.15)

The values of the warping stress coefficients C_x and C_y for cement concrete pavement are taken from the chart developed by Bradbury.

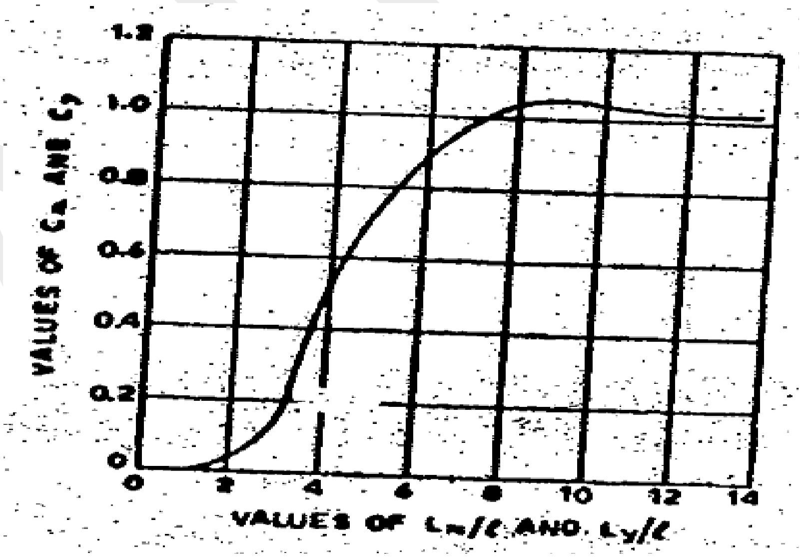


Fig: Bradbury's Warping Stress Coefficient Graph

Chart for Determination of Coefficient C:

L/l or B/l	C	L/l or B/l	C
1	0.000	7	1.030
2	0.040	8	1.077
3	0.175	9	1.080
4	0.440	10	1.075
5	0.720	11	1.050
6	0.920	12	1.000

Fig: Chart for Determination of Coefficient C

Frictional Stresses

Frictional stresses are developed due to seasonal variation in temperature.

→ During summer season, as the temperature of the slab increase, the concrete pavement expands towards the expansion joints. Due to the frictional resistance at the interface of subgrade and the slab, compressive stress is developed at the bottom of the slab as it tends to expand.

→ During winter, the slab contracts causing tensile stresses at the bottom due to the frictional resistance again opposing the movement of the slab.

Stresses developed due to this phenomenon vary with the length of the slab. This is given by,

$$S_f = (W\ L\ f) / (2 \times 10^4)$$

where, S_f = stress developed due to inter-face friction in cement concrete pavement per unit area, kg/cm^2

W = unit weight of concrete, kg/m^3 (2400 kg/m^3)

f = coefficient of friction at the interface (maximum value is about 1.5)

L = spacing between the contraction joint = slab length, m

Combination of Stresses:

The cumulative effect of the different stress gives rise to the following three critical cases:

1. During summer, mid-day: The critical combination at interior and edge regions during mid-day occurs when the slab tends to warp downwards. During this period maximum tensile stress will develop at the bottom fibre due to warping and this is cumulative with the tensile stress due to the loading. However, the frictional stress is compressive during expansion. The load stress at edge region is higher than the interior.

The critical combination of stresses for the edge region is given by,

$S_{critical} = S_e + S_{te} - S_f$

2. During winter, mid-day: During winter, frictional stress developed will be tensile in nature.

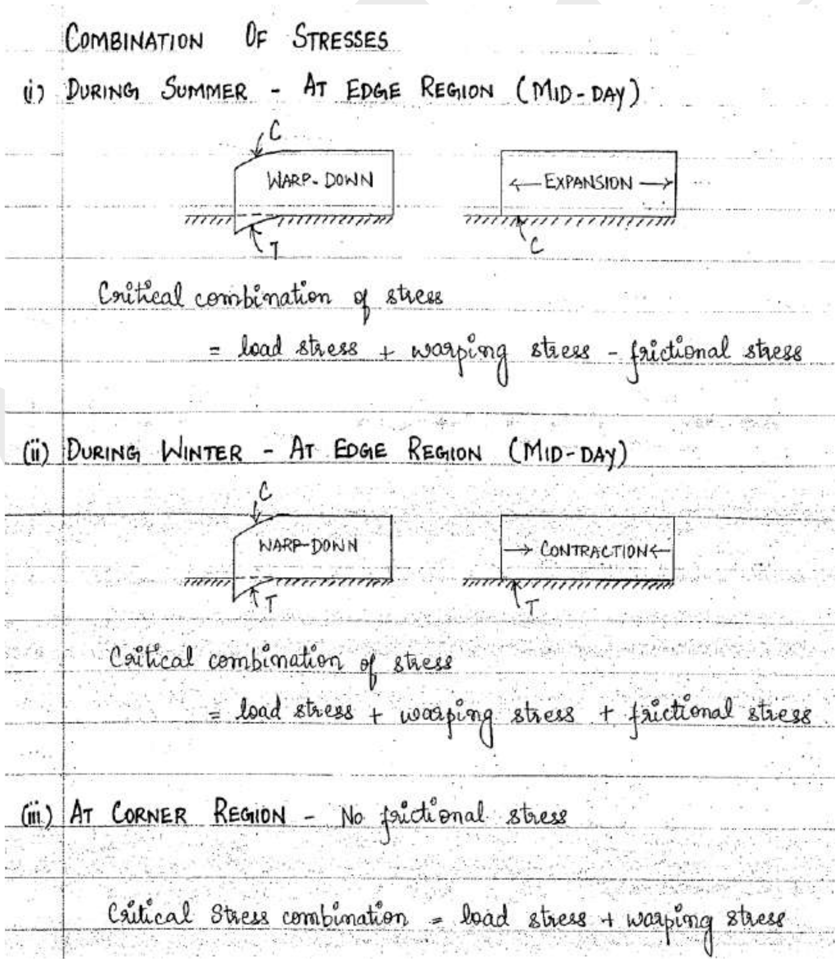
   The critical combination of stresses for the edge region is given by,

   $S_{critical} = S_e + S_{te} + S_f$

3. During summer mid night: During summer mid night, the critical combination of stress occurs at the corner of the slab on the top when the slab tends to warp upwards and is resisted by the self-weight. There is no frictional stress at the corner region.

   The critical stress combination during night at corner region is given by:

   $S_{critical} = S_c + S_{tc}$



Problems

1) Determine the warping stresses at interior, edge and corner regions in a 25cm thick concrete pavement with transverse joints at 11m interval and longitudinal joints at 3.6m intervals. The modulus of subgrade reaction (K) is 6.9 kg/cm^3. Assume temperature differential for day conditions to be 0.6°C per cm slab thickness. Assume radius of loaded area as 15cm for computing warping stress at the corner. Additional data are given below:

   $e = 10 \times 10^{-6}$ /°C

   $E = 3 \times 10^5$ kg/cm^2

   $\mu = 0.15$

   Solution:

   Radius of relative stiffness,

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4} = \left[\frac{3 \times 10^5 \times 25^3}{12 \times 6.9 \times (1-0.15^2)}\right]^{1/4} = 87.24 \text{ cm}$$

   Now, $L_x / l = 1100/87.24 = 12.61$

   $L_y / l = 360/87.24 = 4.13$

   From warping stress co-efficient chart,

   $C_x = 1.02$

   $C_y = 0.60$

   Temperature difference, t = 25 x 0.6 = 15°C

   Warping stress at interior region,

$$S_{t(i)} = \frac{Eet}{2}\left[\frac{Cx+\mu Cy}{1-\mu^2}\right]$$

$$S_{t(i)} = \frac{3 \times 10^5 \times 10 \times 10^{-6} \times 15}{2}\left[\frac{1.02+0.15 \times 0.60}{1-0.15^2}\right]$$

$$S_{t(i)} = 25.55 \text{ kg/cm}^2$$

   Warping stress at edge region, using C_x value,

$$\mathbf{S_{t(e)}} = \frac{Cx\ Eet}{2}$$

$$\mathbf{S_{t(e)}} = \frac{1.02\ x\ \ 3x\ 10^{5} x\ 10x\ 10^{-6} x\ 15}{2}$$

$= 22.95$ kg/cm^2

Warping stress at corner region,

$$S_{t(c)} = \frac{Eet}{3(1-\mu)}\sqrt{\frac{a}{l}}$$

$$S_{t(c)} = \frac{3\ x\ 10^{5}\ x\ 10\ x\ 10^{-6} x\ 15}{3(1-0.15)}\sqrt{\frac{15}{87.24}}$$

$= 7.32$ kg/cm^2

2) A C.C pavement slab of thickness 20cm is constructed over granular sub-base having modulus of reaction 15kg/cm^3. The maximum temperature difference between the top and bottom of the slab during summer day and night is found to be 18°C. The spacing between the transverse contraction joints is 4.5m and that between longitudinal joints is 3.5m. The design wheel load is 5100kg, radius of contact area is 15cm, E of CC is 3x10^5 kg/cm^2, Poisson's ratio is 0.15, and co-efficient of thermal expansion of CC is 10x10^{-6}/°C and co-efficient of friction is 1.5. Using the edge and corner load stress charts given by IRC and the chart for the warping stress co-efficient, find the worst combination of stresses at the edge and corner regions.

Solution:

(i) For Edge region,

(a) Edge Load Stress from chart, for h = 20cm, K = 15 kg/cm^3

$S_e = 24$ kg/cm^2

(b) Warping Stress at edge,

Radius of relative stiffness,

$$l = \left(\frac{Eh^3}{12K\,(1-\mu^2)}\right)^{1/4} = \left(\frac{3 \times 10^5 \times 20^3}{12 \times 15 \times (1-0.15^2)}\right)^{1/4} = 60.77 \text{ cm}$$

Now, $L_x/l = 450/60.77 = 7.4$

$L_y/l = 350/60.77 = 5.76$

From Warping Stress co-efficient chart,

$C_x = 1.02$

$C_y = 0.86$

$$S_{t(e)} = \frac{Cx\ Eet}{2}$$

$$S_{t(e)} = \frac{1.02\ x\ \ 3x\ 10^5 x\ 10x\ 10^{-6} x\ 18}{2}$$

$= 27.54$ kg/cm^2

(c) Frictional stress,

$S_f = W\ L\ f\ /\ 2 \times 10^4$

$= 2400 \times 4.5 \times 1.5\ /\ 2 \times 10^4$

$= 0.81$ kg/cm^2

(d) Combined stress at edge region,

Critical combination of stress Load stress + warping stress – frictional stress

during summer mid-day =

$= 24 + 27.54 – 0.81$

$= 50.73$ kg/cm^2

(ii) For Corner region,

(a) Corner load stress from chart, for h = 20cm, K = 15 kg/cm^3

Sc = 28 kg/cm^2

(b) Warping stress at corner,

$$S_{t(c)} = \frac{Eet}{3(1-\mu)}\sqrt{\frac{a}{l}}$$

$$S_{t(c)} = \frac{3\ x\ 10^5\ x\ 10\ x\ 10^{-6} x\ 18}{3(1-0.15)}\sqrt{\frac{15}{60.77}}$$

$= 10.52$ kg/cm^2

(c) Frictional Stress is zero at corner region

(d) Combined stress at the corner region,

Critical combination of stress = Load stress + Warping stress

in summer mid-night

= 28 + 10.52

= 38.52 kg/cm^2

Reinforcement in Slab

The function of reinforcement in a concrete pavement is not to increase the resisting moment of slab strength. Its role is to control the opening of the cracks. It holds together and ties the two sides of the crack and prevents the crack from widening. It also counteracts the tensile stresses caused by shrinkage and contraction due to changes in temperature and moisture content.

Reinforcement Design

The design of steel reinforcement in concrete pavement can only be done on an empirical basis. Some guidelines for such a design can be taken from the following considerations:

1. The maximum tension in the steel across the crack equals the force required to overcome friction between the pavement and the foundation, from the crack to the nearest joint or free edge.
2. The force is greatest when the crack occurs at the middle of the slab. Thus the greater the joint spacing, the greater is the reinforcement required.
3. In order to obtain the greatest advantage from a given weight of reinforcement, a major part of this weight should be in the longitudinal direction which has the greatest dimension in a slab. The longitudinal bars should thus be heavier and more closely spaced than the cross bars.

The following formula is popularly used to design the longitudinal and transverse steel:

$$A = L\,f\,W/2S$$

where, A = area of steel in cm^2 required per m width or length of slab

L = distance in m between transverse joints or free longitudinal joints

W = weight of slab in kg/m^2

f = coefficient of friction

S = allowable working stress in steel in kg/cm^2

Design of Dowel bars

Dowel bars of expansion joints are mild steel round bars of short length. Half length of this bar is bonded in one cement concrete slab and another half is embedded in the adjacent slab. Dowel bars are kept free for the movement during expansion and contraction of the slab. Dowel bars will maintain the slab edges at the same level, and the load transferred from one slab to another.

If dowel bars are not provided in between two slabs at the transverse joint, the slab loaded with wheel load will deflect more, say d_1. When two slabs are connected with dowel bars, the wheel load acting on one slab will be transferred to another slab and hence the effect of deflection will be less as the load will be taken by both the slabs. In this case, deflection d_2 will be less than d_1.

In the design of dowel bars, the load transferred is worked out by considering the capacity of the dowel bar system. The capacity depends on pavement thickness, subgrade modulus, relative stiffness, spacing and diameter of dowel bars.

IRC recommends that dowel bar system can be designed based on Bradbury's analysis for load transfer capacity of a single dowel bar in shear, bending and bearing in concrete.

The formulae to calculate the load transfer capacity of a single dowel bar (P) in kg is given as below:

For shear in the bar, $P_s = 0.785\, d^2 F_s \longrightarrow (1)$

For bending in the bar, $P_f = \dfrac{2d^3 F_f}{(L_d + 8.8\delta)} \longrightarrow (2)$

For bearing in concrete, $P_b = \dfrac{F_b L_d^2 d}{12.5(L_d + 1.5\delta)} \longrightarrow (3)$

where, P = load transfer capacity of a single dowel bar in shear s, bending f and bearing b(kg)

d = diameter of dowel bar (cm)

L_d = length of embedment of dowel bar (cm)

δ = joint width (cm)

F_s = permissible shear stress in dowel bar (kg/cm^2)

F_f = permissible bending stress in dowel bar (kg/cm^2)

F_b = permissible bearing stress on concrete (kg/cm^2)

Design Procedure

Step 1 – Find the length of the dowel bar embedded in slab L_d by equating equation 2 & 3

$$\frac{2d^3F_f}{(L_d + 8.8\delta)} = \frac{F_bL_d^2d}{12.5(L_d + 1.5\delta)}$$

$$L_d = 5d\sqrt{\frac{F_f\,(L_d + 1.5\delta)}{F_b\,(L_d + 8.8\delta)}}$$

Step 2 – Find the load transfer capacities P_s, P_f and P_b of single dowel bar with the Ld.

Step 3 – Assume load capacity of dowel bar is 40 percent wheel load, find the load capacity factor f as

$$\max\left\{\frac{0.4P}{P_s}, \frac{0.4P}{P_f}, \frac{0.4P}{P_b}\right\}$$

Step 4 – Spacing of dowel bars

- Effective distance upto which effective load transfer takes place is given by 1.8 *l*, *where l* is the radius of relative stiffness.
- Assume a linear variation of capacity factor of 1.0 under load to 0 at a distance of 1.8 *l*.
- Assume dowel spacing and find the capacity factor of the above spacing.
- Actual capacity factor should be greater than the required capacity factor.
- If not, do one more iteration with new spacing.

The length of dowel bars is generally 0.5m, the diameter being 25mm. The spacing is of the order of 200mm for 15cm thick slab and 300mm for 20cm thick slab.

Problems

1) Design size and spacing of dowel bars at an expansion joint of concrete pavement of thickness 25cm. Given the radius of relative stiffness of 80cm, design wheel load 5000kg, load capacity of the dowel system is 40 percent of design wheel load, joint width is 2.0cm and the permissible stress in shear, bending and bearing stress in dowel bars are 1000, 1400 and 100 kg/cm^2 respectively.

Solution:

Given: P = 5000 kg, l = 80cm, h = 25cm, δ = 2cm, F_s = 1000 kg/cm^2, F_f = 1400 kg/cm^2, F_b = 100 kg/cm^2, assume d = 2.5cm

Step 1 – Length of the dowel bar L_d

$$L_d = 5d \sqrt{\frac{F_f(L_d + 1.5\delta)}{F_b(L_d + 8.8\delta)}}$$

$$L_d = 5 \text{ x } 2.5 \sqrt{\frac{1400(L_d + 1.5x\ 2)}{100(L_d + 8.8x2)}}$$

$$L_d = 12.5 \sqrt{\frac{14(L_d + 3)}{(L_d + 17.6)}}$$

Solve for Ld by trial and error:

Put L_d = 45.00 => L_d = 40.95

Put L_d = 45.95 => L_d = 41.04

Put L_d = 45.50 => L_d = 41.00

Minimum length of the dowel bar is L_d + δ = 40.5 + 2.0 = 42.5cm. So, provide 45cm long and 2.5cm ø.

Therefore, L_d = 45-2 = 43cm

Step 2 – Find the total load transfer capacity of single dowel bar

$$P_s = 0.785\, d^2 F_s \quad = 0.785 \text{ x } 2.5^2 \text{ x } 1000 = 4906.25\text{kg}$$

$$P_f = \frac{2d^3F_f}{(L_d + 8.8\delta)} = \frac{2\ x\ 2.5^3 x1400}{43 + 8.8\ x\ 2} = 722\text{kg}$$

$$P_b = \frac{F_b L_d^2 d}{12.5(L_d + 1.5\delta)} = \frac{100x43^2 x\ 2.5}{12.5\ (43 + 1.5x2)} = 804\text{kg}$$

Step 3 – Therefore, the required load transfer capacity

$$\max \left\{ \frac{0.4P}{P_s},\ \frac{0.4P}{P_f},\ \frac{0.4P}{P_b} \right\}$$

$$\max\left\{\frac{0.4x5000}{4906.25}, \frac{0.4x5000}{722}, \frac{0.4x5000}{804}\right\}$$

max {0.41, 2.77, 2.487} = 2.77

Step 4 – Find the required spacing:

Effective distance of load transfer = 1.8 l = 1.8 x 80 = 144cm.

Assuming 35cm spacing,

Actual capacity is

$$1 + \frac{144-35}{144} + \frac{144-70}{144} + \frac{144-105}{144} + \frac{144-140}{144}$$

= 2.57 < 2.77 (the required capacity)

Therefore, assume 30cm spacing and now the actual capacity is

$$1 + \frac{144-30}{144} + \frac{144-60}{144} + \frac{144-90}{144} + \frac{144-120}{144}$$

= 2.92 > 2.77 (the required capacity)

Therefore provide 2.5cm ø mild steel dowel bars of length 45cm @ 30cm center to center.

2) Design size and spacing of dowel bars at an expansion joint of concrete pavement of thickness 20 cm. Given the radius of relative stiffness of 90cm, design wheel load 4000kg, load capacity of the dowel system is 40 percent of design wheel load, joint width is 3.0cm and the permissible stress in shear, bending and bearing stress in dowel bars are 1000, 1500 and 100 kg/cm^2 respectively.

Solution:

Given: P = 4000 kg, l = 90cm, h = 20cm, δ = 3cm, F_s = 1000 kg/cm^2, F_f = 1500 kg/cm^2, F_b = 100 kg/cm^2, assume d = 2.5cm diameter

Step 1 – Length of the dowel bar L_d

$$L_d = 5d \sqrt{\frac{F_f(L_d + 1.5\delta)}{F_b(L_d + 8.8\delta)}}$$

$$L_d = 5 \text{ x } 2.5 \sqrt{\frac{1500(L_d + 1.5x\ 3)}{100(L_d + 8.8x3)}}$$

$$L_d = 12.5 \sqrt{\frac{15(L_d + 4.5)}{(L_d + 26.4)}}$$

Solving for L_d by trial and error, it is = 39.5cm.

Minimum length of the dowel bar is $L_d + \delta$ = 39.5 + 3.0 = 42.5cm. So, provide 45cm long and 2.5cm ø.

Therefore, L_d = 45-3 = 42cm

Step 2 – Find the total load transfer capacity of single dowel bar

$$P_s = 0.785\, d^2 F_s \quad = 0.785 \times 2.5^2 \times 1000 = 4906.25\text{kg}$$

$$P_f = \frac{2d^3 F_f}{(L_d + 8.8\delta)} \quad = \frac{2 \times 2.5^3 \times 1500}{42 + 8.8 \times 3} = 685.307\text{kg}$$

$$P_b = \frac{F_b L_d^2 d}{12.5(L_d + 1.5\delta)} \quad = \frac{100 x 42^2 x\ 2.5}{12.5\ (42 + 1.5x3)} = 758.71\text{kg}$$

Step 3 – Therefore, the required load transfer capacity

$$\max \left\{ \frac{0.4P}{P_s}, \frac{0.4P}{P_f}, \frac{0.4P}{P_b} \right\}$$

$$\max \left\{ \frac{0.4x4000}{4906.25}, \frac{0.4x4000}{685.307}, \frac{0.4x4000}{758.71} \right\}$$

max {0.326, 2.335, 2.10} = 2.335

Step 4 – Find the required spacing:

Effective distance of load transfer = 1.8 *l* = 1.8 x 90 = 162cm.

Assuming 35cm spacing,

Actual capacity is

$$1 + \frac{162-35}{162} + \frac{162-70}{162} + \frac{162-105}{162} + \frac{162-140}{162}$$

= 2.83

Assuming 40cm spacing, capacity is

$$1 + \frac{162-40}{162} + \frac{162-80}{162} + \frac{162-120}{162} + \frac{162-160}{162}$$

$= 2.52$

So, consider 2.52 > 2.335 as it is greater and more near to other value.

Therefore provide 2.5cm ø mild steel dowel bars of length 45cm @ 40cm center to center.

Design of Tie bars

Tie bars are used across the longitudinal joints of CC pavements. Tie bars ensure the adjacent slabs to remain firmly together. These bars are not designed to act as load transfer devices and hence tie bars are designed to withstand tensile stresses. Maximum tensile force in tie bars will be equal to the force required to overcome frictional force between the bottom of the adjoining pavement slabs and the soil subgrade.

Step 1 – Diameter and spacing:

The diameter and the spacing is first found out by equating the total sub-grade friction to the total tensile stress for a unit length (one meter). Hence the area of steel per one meter in cm^2 is given by:

$$A_s \times S_s = b \times h \times W \times f$$

$$A_s = \frac{b \times h \times W \times f}{100\, S_s}$$

where, b = width of the pavement panel (m)

h = depth of the pavement (cm)

f = coefficient of friction between pavement and subgrade, usually taken as 1.5

W = unit weight of the concrete (2400 kg/cm^2)

S_s = allowable working tensile stress in steel, (assume 1750 kg/cm^2)

ø = 0.8 to 1.5 cm dia bars for the design

Step 2 – Length of the tie bar:

Length of the tie bar is twice the length needed to develop bond stress equal to the working tensile stress and is given by:

$$L_t = \frac{d\, S_s}{2\, S_b}$$

where, L = length of tie bar (cm)

d = diameter of the bar

S_s = allowable tensile stress, kg/cm^2

S_b = allowable bond stress and can be assumed for plain and deformed bars respectively as 17.5 and 24.6 kg/cm^2

Problems

1) A cement concrete pavement of thickness 18cm, has two lanes of 7.2m with a joint. Design the tie bars.

Solution:

Given: h = 18cm, b = 7.2/2 = 3.6m, S_s = 1750 kg/cm^2, W = 2400 kg/cm^2, f = 1.5, S_b = 24.6 kg/cm^2

Step 1 – Diameter and spacing

$$A_s = \frac{b \times h \times W \times f}{100\, S_s}$$

$$A_s = \frac{3.6 \times 18 \times 2400 \times 1.5}{100 \times 1750} = 1.33\ \text{cm}^2/\text{m}$$

Assume ø = 1cm, => A = 0.785cm^2.

Therefore, spacing is $\frac{100 \times 0.785}{1.33}$ = 59cm, say 55cm

Step 2 – Length of the bar

Get L_t from

$$L_t = \frac{d\, S_s}{2\, S_b} = \frac{1 \times 1750}{2 \times 24.6} = 36\text{cm}$$

Use 1cm ø tie bars of length of 36cm @ 55cm c/c.

2) Design the length and spacing of tie bars given that the pavement thickness is 20cm and width of the road is 7m with one longitudinal joint. The unit weight of concrete is 2400 kg/m^3, the coefficient of friction is 1.5, allowable working tensile stress in steel is 1750 kg/cm^2 and bond stress of deformed bars is 24.6 kg/cm^2.

Solution:

Given: h = 20cm, b = 7/2 = 3.5m, S_s = 1750 kg/cm^2, W = 2400 kg/cm^2, f = 1.5, S_b = 24.6 kg/cm^2

Step 1 – Diameter and spacing

$$A_s = \frac{b \times h \times W \times f}{100\ S_s}$$

$$A_s = \frac{3.5 \times 20 \times 2400 \times 1.5}{100 \times 1750} = 1.44\ \text{cm}^2/\text{m}$$

Assume ø = 1cm, => A = 0.785cm^2.

Therefore, spacing is $\frac{100 \times 0.785}{1.44}$ = 54.51cm, say 55cm

Step 2 – Length of the bar

Get L_t from

$$L_t = \frac{d\ S_s}{2\ S_b} = \frac{1 \times 1750}{2 \times 24.6} = 36\text{cm}$$

Use 1cm ø tie bars of length of 36cm @ 55cm c/c.

Question Bank

1. As per IRC 58-2002, explain the procedure of design of rigid pavements.
2. Explain how warping stresses are formed in cc pavements. Describe the Bradbury's equations to calculate warping stresses at critical locations.
3. Explain the analysis of stresses by using Westergaard's equation.
4. Describe design of dowel bars.
5. Explain design of tie bars